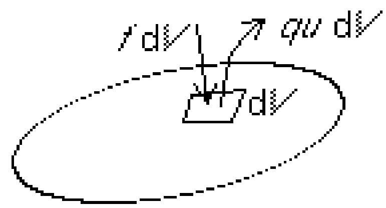
2.21 (b)

1 2.21 (b), §1 Asked

Classify:

$$u_t - \nabla \cdot (p\nabla u) + qu = f$$



2 2.21 (b), §2 Solution

$$u_t - \nabla \cdot (p\nabla u) + qu = f$$

Written out:

$$\nabla = \hat\imath \frac{\partial}{\partial x} + \hat\jmath \frac{\partial}{\partial y} + \hat k \frac{\partial}{\partial z}$$

$$\nabla u \equiv \text{grad } u = \hat\imath u_x + \hat\jmath u_y + \hat k u_z$$

$$\nabla \cdot \vec v \equiv \text{div } \vec v = v_{1x} + v_{2y} + v_{3z}$$

$$u_t - (pu_x)_x - (pu_y)_y - (pu_z)_z + qu = f$$

Highest derivatives of u:

$$-pu_{xx} - pu_{yy} - pu_{zz} + \ldots = f$$

Coefficient matrix A

$$A = \begin{pmatrix} -p & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues are $\lambda_1 = \lambda_2 = \lambda_3 = -p$ and $\lambda_4 = 0$.