Introduction

Classification groups P.D.E. with similar properties together.

Example: inviscid fluid flow past a wing cross-section:

Subsonic region $M < 1$: smooth solutions, unlimited region of dependence, iterative solution. These features are indicative of an elliptic PDE.

Supersonic region $M > 1$: singularities, characteristic lines, spatial marching solution. Typical for a hyperbolic PDE.

One set of PDEs that has a unambiguous classification are 2D second order quasilinear equations:

$$au_{xx} + 2bu_{xy} + cu_{yy} + d = 0$$

where $a = a(x, y, u, u_x, u_y)$, $b = b(x, y, u, u_x, u_y)$, $c = c(x, y, u, u_x, u_y)$, and $d = d(x, y, u, u_x, u_y)$.

The classification for these equations is:

- $b^2 - ac > 0$: hyperbolic
- $b^2 - ac = 0$: parabolic
- $b^2 - ac < 0$: elliptic