2D Coordinate Changes

More powerful simplifications are possible in 2D.

In the initial independent coordinates x, y:

$$au_{xx} + 2bu_{xy} + cu_{yy} + d = 0$$

In the new independent coordinates ξ, η

$$a'u_{\xi\xi} + 2b'u_{\xi\eta} + c'u_{\eta\eta} + d' = 0$$

The new coefficients may be found by writing out the transformation formulae from the introduction for the two-dimensional case, and are:

$$\begin{aligned}
a' &= a\left(\xi_x\right)^2 + 2b\left(\xi_x\right)\left(\xi_y\right) + c\left(\xi_y\right)^2 \\
b' &= a\left(\xi_x\right)\left(\eta_x\right) + b\left(\xi_x\right)\left(\eta_y\right) + b\left(\xi_y\right)\left(\eta_x\right) + c\left(\xi_y\right)\left(\eta_y\right) \\
c' &= a\left(\eta_x\right)^2 + 2b\left(\eta_y\right)\left(\eta_y\right) + c\left(\eta_y\right)^2 \\
d' &= d + \left(a\xi_{xx} + 2b\xi_{xy} + c\xi_{yy}\right)u_\xi + \left(a\eta_{xx} + 2b\eta_{xy} + c\eta_{yy}\right)u_\eta
\end{aligned}$$

The trick now is to demand that a', b' and c' are as simple as possible and from that compute what the required ξ and η are.