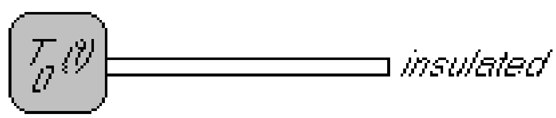
Introduction

1 §1 Examples

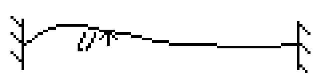
Partial differential equations:

- Standard examples:
  - Steady heat conduction in a plate:

  - Unsteady heat conduction in a bar:



  - Vibrations of a string:

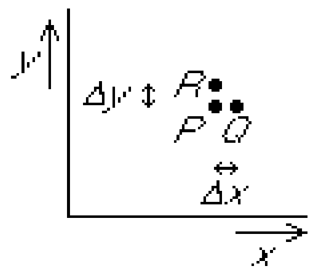


- Fluid mechanics;
- Heat transfer;
- Solid mechanics;
- Dynamics;
- Electro-magnetodynamics;
- Geometry;
- Optics;
- ...

2 §2 Notations

- Ordinary differential equations: one independent variable
- Partial differential equations: more independent variables
- Partial derivative:

$$u_{xP} \equiv \left(\frac{\partial u}{\partial x}\right)_P = \lim_{\Delta x \to 0} \frac{u_Q - u_P}{\Delta x} \qquad u_{yP} \equiv \left(\frac{\partial u}{\partial y}\right)_P = \lim_{\Delta y \to 0} \frac{u_R - u_P}{\Delta y}$$

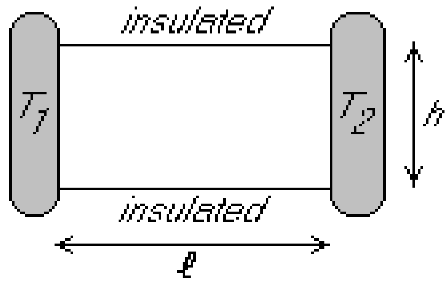


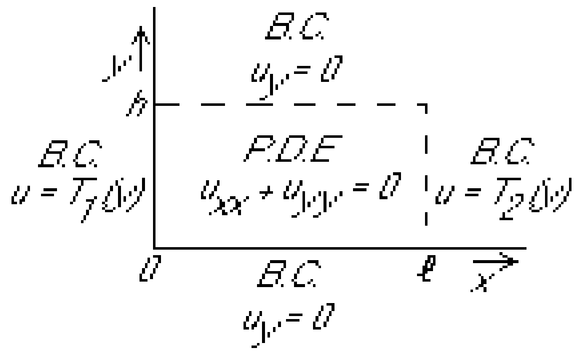
- Order: order of the highest derivative
- Degree: highest degree of the dependent variable
- Linear: first degree
- Domain Ω: the spatial region, i.e.
  - Plate (rectangle in the x, y-plane
  - Bar (line segment $0 < x < \ell$)
  - String (line segment $0 < x < \ell$)
- Boundary $\delta\Omega$: the edges of the domain, i.e.
  - Perimeter of the plate
  - Ends of the bar
  - End points of the string

3 §3 Standard Examples

You must know by heart:

- *The Laplace equation.* Steady heat conduction in a plate:

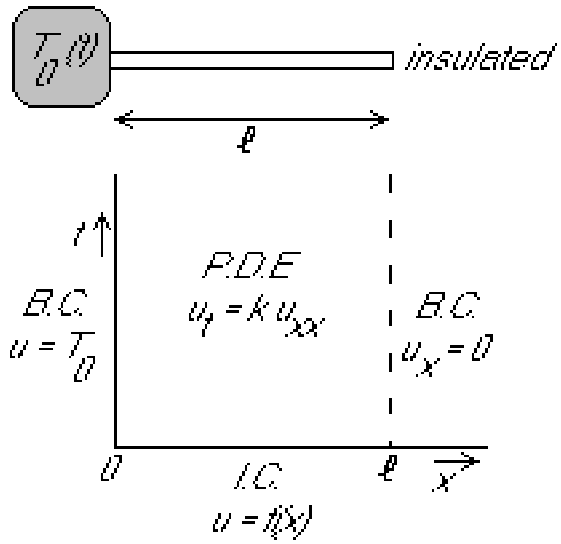




Also describes ideal flows, unidirectional flows, membranes, electro and magnetostatics, complex functions, ...
In any number of dimensions: $\nabla^2 u = 0$.
Properties:

- Smooth solutions.
- Boundary-value problems.
- Maximum property.
- Unlimited region of influence.
- A simple example of an *elliptic* equation.

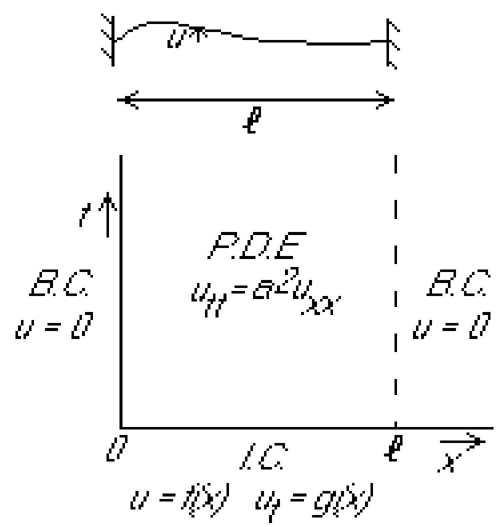
- *The heat equation.* Unsteady heat conduction in a bar:



Also describes unsteady unidirectional flow, ...
In any number of dimensions $u_t = k\nabla^2 u$.

  - Smooth solutions.
  - Initial/boundary value problems.
  - Maximum property.
  - Unlimited region of influence in space.
  - A simple example of a *parabolic* equation.

- Vibrations of a string: the wave equation:



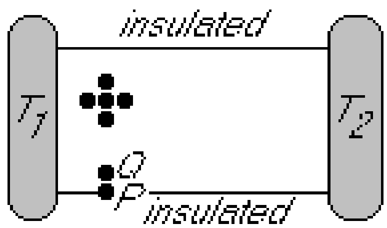
Also describes acoustics in a pipe, steady supersonic flow, water waves, optics, ...
In any number of dimensions $u_{tt} = a^2\nabla^2 u$.

- Propagating waves.
- Propagates singularities.
- Initial/boundary value problems.
- Energy conservation.
- Finite propagation speed.
- A simple example of a *hyperbolic* equation.

4 §4 Boundary Conditions

Boundary condition types:

- Dirichlet: u is given on the boundary



- Neumann: $\partial u/\partial n$ is given on the boundary

$$\frac{\partial u}{\partial n} = \vec{n} \cdot \nabla u$$

- Mixed: a combination of u and $\partial u/\partial n$ is given on the boundary
- ...

5 §5 Properly Posedness

A P.D.E. problem is properly posed if

- a solution exists;
- it is unique
- small changes in the conditions produce correspondingly small changes in the solution.