Introduction

Wave equation:

$$u_{tt} = a^2 u_{xx}$$

Characteristics:

$$x + at = \xi \qquad x - at = \eta$$

General solution:

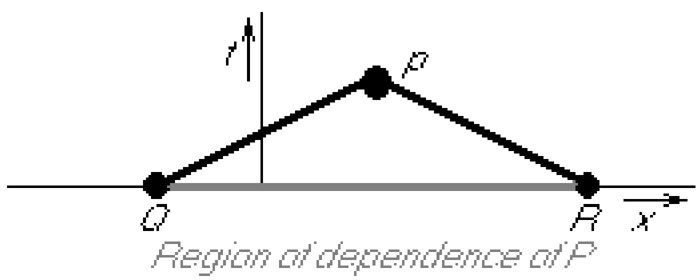
$$u(x,t) = f_1(x - at) + f_2(x + at)$$

Here $f_1(x - at)$ is a function that moves to the right with speed a; a 'right-going wave'. And $f_2(x - at)$ is a function that moves to the left with speed a; a 'left-going wave'.

D'Alembert solution:

Assume no boundaries, $(-\infty < x < \infty)$. Then we can solve for f_1 and f_2 in terms of the given initial string displacement $f(x) = u(x,0)$ and initial velocity $g(x) = u_t(x,0)$ to give:

$$u(x,t) = \frac{f(x-at) + f(x+at)}{2} + \frac{1}{2a} \int_{x-at}^{x+at} g(\xi)\,\mathrm{d}\xi$$



$$u_P = \frac{u_Q + u_R}{2} + \frac{1}{2a} \int_Q^R u_t\,\mathrm{d}\xi$$

This is derived in example 4.10 in the book.

If x is restricted by finite boundaries, we must somehow extend the problem to doubly infinite x without boundaries. But our solution without boundaries should still satisfy the boundary conditions we are given. That is often possible by clever use of symmetry.