Analysis II Spring 2010
Homework Problems

Do not print out this page. Keep checking for changes. Complete assignment will normally be available the day after the last lecture whose material is included in the assignment (Monday, normally).

1 01/15 F

Use vector analysis wherever possible.

1. 1st Ed: p13, q31a-f,h-j, 2nd Ed: p17, q31a-i. *if they can be vectors, count them as such.*
2. 1st Ed: p13, q32, 2nd Ed: p17, q32. Do it both graphically and analytically.
3. 1st Ed: p14, q48, 2nd Ed: p14, q46. Use vector calculus only, no trig.
4. 1st Ed: p32, q66, 2nd Ed: p38, q66.
5. 1st Ed: p32, q69, 2nd Ed: p40, q69a.
6. 1st Ed: p32, q82, 2nd Ed: p40, q82a, where second minus in **B** should be plus. Do it without finding the sides of the parallelogram.
7. 1st Ed: p33, q87, 2nd Ed: p41, q87.
8. 1st Ed: p33, q90, 2nd Ed: p41, q90a.

2 01/22 F

1. 1st Ed: p54, q47, 2nd Ed: p65, q47. (20 points)
2. 1st Ed: p78, q46, 2nd Ed: p91, q46.
3. 1st Ed: p78, q54, 2nd Ed: p92, q54.
4. 1st Ed: p78, q60, 2nd Ed: p92, q60.
5. 1st Ed: p78, q62, 2nd Ed: p92, q62.
6. 1st Ed: p79, q64, 2nd Ed: p92, q64.
7. 1st Ed: p79, q70, 2nd Ed: p92, q70.

3 01/29 F

1. 1st Ed: p80, q84, 2nd Ed: p93, q84.
2. 1st Ed: p80, q87, 2nd Ed: p93, q87. Include the divergence in the discussion.
3. 1st Ed: p80, q102, 2nd Ed: p94, q102.
4. 1st Ed: p81, q107, 2nd Ed: p94, q107. (20 points). You need to show that any solution of Maxwell's equations is given by scalar and vector potentials as shown. Hints: Recall that if the divergence of a vector is zero, the vector is the curl of some other vector $\vec{A}$. Also, you can certaintly *define* $\vec{E}_\phi$ by setting

$$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} + \vec{E}_\phi$$

but it is not automatic that $\vec{E}_\phi$ is the gradient of some scalar.

Next, show that you can still choose the divergence of $\vec{A}$ whatever you want. To show this, assume that you have some $\vec{A}_0$ that has the right curl, but the wrong divergence. Then show that if you define

$$\vec{A} = \vec{A}_0 + \nabla\psi$$

where the function ψ satisfies the Poisson equation

$$\nabla^2\psi = \text{correct-div}\, A - \text{div}\, \vec{A}_0$$

then A has both the right curl and the right divergence.

Use this freedom in the choice of the divergence of A to derive the asked equations for $\vec{A}$ and ϕ.

The final two asked equations are wave equations, whose solutions consist of waves propagating with speed c, the speed of light. Thus Maxwell concluded that light is electromagnetic waves.

For unknown reasons, the book has listed Maxwell's equations in inverse order.

5. 1st Ed: p102, q32, 2nd Ed: p122, q32.
6. 1st Ed: p103, q44, 2nd Ed: p123, q44. Do it without using Stokes.

4 02/05 F

1. 1st Ed: p103, q44, 2nd Ed: p123, q44. Do it with Stokes.
2. 1st Ed: p104, q62, 2nd Ed: p124, q62. (20 points) Do both directly and using the divergence theorem. Make sure to include the base of the cone. Use the Cartesian expression for $\vec{n}\,\mathrm{d}S$ to formulate the surface integral, then switch to polar to do it.
3. 1st Ed: p132, q50, 2nd Ed: p154, q50. Instead of $M_y = N_x$, show that the curl of the vector is zero and discuss Stokes after doing the integral directly.
4. 1st Ed: p133, q56, 2nd Ed: p155, q56.
5. Read through subsection 9.4 of QMFE[1] and write a half-page summary.
6. Derive $\vec{n}\,\mathrm{d}S$ in terms of $\mathrm{d}\theta$ and $\mathrm{d}\phi$, where (r, θ, ϕ) are spherical coordinates, assuming that the surface is given as $r = f(\theta, \phi)$. Use that:

$$\vec{r} = r\,\hat{\imath}_r \qquad \frac{\partial\,\hat{\imath}_r}{\partial\theta} = \hat{\imath}_\theta \qquad \frac{\partial\,\hat{\imath}_r}{\partial\phi} = \sin\theta\,\hat{\imath}_\phi$$

   as will be derived in class. Next generalize the result to the case that the surface is given by the implicit expression $F(r, \theta, \phi) = 0$. One way to do so is to find $\partial r/\partial\theta$ and $\partial r/\partial\phi$ from the total differential

$$\frac{\partial F}{\partial r}\mathrm{d}r + \frac{\partial F}{\partial\theta}\mathrm{d}\theta + \frac{\partial F}{\partial\phi}\mathrm{d}\phi = 0$$

   Express the vector part of the final expression in terms of vector calculus. Compare it to the Cartesian case in which x and y are the integration variables.
7. 1st Ed: p160, q38, 2nd Ed: p183, q38.

[1] http://www.eng.fsu.edu/~dommelen/quantum/style_a/node86.html

5 02/12 F

1. Finish finding the derivatives of the unit vectors of the spherical coordinate system using the class formulae. Then finish 1st Ed p160 q47, 2nd Ed p183 q47, as started in class, by finding the acceleration. Note that the metric indices h_i for spherical coordinates are in mathematical handbooks. Also,

$$\frac{\partial \hat{\imath}_i}{\partial u_i} = \frac{1}{h_i}\frac{\partial h_i}{\partial u_i}\hat{\imath}_i - \sum_{j=1}^{3}\frac{1}{h_j}\frac{\partial h_i}{\partial u_j}\hat{\imath}_j \qquad \frac{\partial \hat{\imath}_i}{\partial u_j} = \frac{1}{h_i}\frac{\partial h_j}{\partial u_i}\hat{\imath}_j$$

2. Express the acceleration in terms of the spherical velocity components v_r, v_θ, v_ϕ and their first time derivatives, instead of time derivatives of position coordinates. Like $a_r = \dot{v}_r + \ldots$, etc. This is how you do it in fluid mechanics, where particle position coordinates are normally not used. (So, get rid of the position coordinates with dots on them in favor of the velocity components.)

3. The Laplace equation

$$\text{PDE:} \quad u_{xx} + u_{yy} = 0$$

where subscripts indicate derivatives, is an elliptic equation. Such a steady-state equation needs boundary conditions at all points of the boundary. For example, one properly posed problem on the unit square is

$$\text{BC:} \quad u(x,0) = 1 \quad u_y(x,1) = 0 \quad u(0,y) = 0 \quad u_x(1,y) = 0$$

Identify Ω, $\delta\Omega$, and the type of each boundary condition (Dirichlet, Neumann, Robin).

The wave equation

$$\text{PDE:} \quad u_{xx} - u_{yy} = 0$$

is an hyperbolic equation. The above boundary conditions are *not* properly posed for the wave equation. (as you will see in a later homework.) For the wave equation, one of the coordinates must be time-like, and must have initial conditions instead of boundary conditions. The following initial and boundary conditions are properly posed for the wave equation,

$$\text{BC and IC:} \quad u(x,0) = 1 \quad u_y(x,0) = 0 \quad u(0,y) = 0 \quad u_x(1,y) = 0 \qquad y \le 1$$

For each of the four, determine whether it is an IC or BC, and if so, what kind of BC. The time-like coordinate is not normally included in the domain Ω. Under those conditions, identify Ω and $\delta\Omega$.

Check that the following proposed solution satisfies the PDE and all BC/IC of both the Laplace and wave equation problems:

$$u = \begin{cases} 1 \text{ for } y < x \\ 0 \text{ for } y > x \end{cases}$$

However, it is a valid solution to only the wave equation. Explain for what qualitative reason is it not a valid solution to the Laplace equation.

4. The Laplace equation problem as written in the previous question does not have a simple solution. However, if you distort the domain into a quarter circle as in

$$\text{BC:} \quad u(x,0) = 1 \qquad u(0,y) = 0 \qquad \frac{\partial u}{\partial n} = 0 \text{ on } x^2 + y^2 = 1$$

then the solution is simple. Identify and draw Ω for the above problem. Identify the type of each boundary condition. Also draw Ω and the boundary conditions for the Laplace problem of the previous question, and then compare the two problems. Are they very similar?

Now verify, by checking PDE and boundary conditions, that the correct solution to the modified problem is simply

$$u = 1 - \frac{2\theta}{\pi} \qquad \theta = \arctan(y/x)$$

You may want to switch to a different coordinate system to do so.

Plot this valid solutions for the Laplace equation, as well as the valid solution for the wave equation of the previous question, on the circle $r = 0.5$ against θ. Comment on whether the change of a single sign between the wave equation and the Laplace equation makes any difference for the solution.

5. 4.18. You are to check that the given u_N satisfies the given PDE, the heat equation, and the given boundary conditions. The given solution is the one you will find using the so-called method of separation of variables, and normally $N = \infty$. What is the type of the boundary conditions? What can you say about the initial condition that is satisfied?

6. The solution to the wave equation problem of question 3 that you would find using separation of variables is:

$$u = \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{4}{\pi n} \sin(\tfrac{1}{2}n\pi x) \cos(\tfrac{1}{2}n\pi y)$$

   Verify the PDE, BC, and IC for this solution. For the first IC, you will want to look up the sum you get in the "Fourier series" section of a mathematical handbook, with maybe a rescaled x-coordinate.

   Comment whether it would be easy to see the simple form of the solution from merely looking at the above sum. To make understanding the solution easier, use the fact that

$$\sin\alpha\cos\beta = \tfrac{1}{2}\sin(\alpha+\beta) + \tfrac{1}{2}\sin(\alpha-\beta)$$

   Describe the two component solutions you get this way in physical terms.

7. The solution to the Laplace equation problem of question 3 that you would find using separation of variables is:

$$u = \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{4}{\pi n \cosh(\frac{1}{2}n\pi)} \sin(\tfrac{1}{2}n\pi x) \cosh(\tfrac{1}{2}n\pi(1-y))$$

   Verify the PDE and BC for this solution.

   Next, shed some light on the question why this solution is smooth for $y > 0$, while the previous solution of the wave equation has a jump for all y. To do so, first argue that a *finite* sum of sines of the form $\sin\frac{1}{2}n\pi x$ is necessarily a smooth function of x with no jumps or whatever other singularities.

   Then show that the coefficients of the sines go to zero much faster for $n \to \infty$ for the solution of the Laplace equation than for the one of the wave equation. In particular show that for any positive power p,

$$\lim_{n\to\infty} n^p \frac{4}{\pi n} \frac{\cosh(\frac{1}{2}n\pi(1-y))}{\cosh(\frac{1}{2}n\pi)} = 0 \quad \text{if} \quad 0 < y \le 1$$

   while

$$\lim_{n\to\infty} n^p \frac{4}{\pi n} \cos(\tfrac{1}{2}n\pi y)$$

   does not exist for $p > 1$.

   With such a fast decay of the coefficients of the Laplace equation solution, the sum is almost finite and singularities are not possible.

8. 2.19b, h. Show a picture of the different regions.

6 02/19 F

1. 2.20.

2. 2.21, in three spatial dimensions, and time as appropriate.

3. 2.23. Reduce to canonical form by rotating the coordinate system. (Not using characteristic coordinates as the book does.) What is the angle the coordinate system must be rotated over?

4. 2.24.
5. By identifying d' show that the PDE of the previous question may be reduced to

$$u_{\xi_1\xi_1} + 3u_{\xi_2\xi_2} + 4u_{\xi_3\xi_3} \pm \frac{16}{\sqrt{6}}u_{\xi_1} \pm \frac{8}{\sqrt{2}}u_{\xi_2} \pm \frac{20}{\sqrt{3}}u_{\xi_3} = 0$$

where the signs you will end up with depend on how you choose the sign of your eigenvectors. Multiply the equation you got by 6 and then rescale the independent variables to get an equation of the form

$$u_{\xi\xi} + u_{\eta\eta} + u_{\theta\theta} \pm 16u_\xi \pm 8u_\eta \pm 10\sqrt{2}u_\theta = 0$$

6. Get rid of the first derivatives in the obtained equation by defining a new unknown $v = u/e^{\alpha\xi+\beta\eta+\gamma\theta}$ where α, β, and γ are constants to be found from the condition that the first order derivatives disappear. Write and name the final equation.
7. 4.20. A number T is rational if it can be written as the ratio of a pair of integers, e.g. $1.5 = 3/2 = 6/4 = 9/6 = \ldots$. It is irrational if it cannot, like $\sqrt{2}$. Near any rational number, irrational numbers can be found infinitely closely nearby, and vice versa. For example, the value π to one billion digits, as found on the internet, is the rational number $31415927\ldots/10000000\ldots$; π itself is not rational. The wave equation problem when $T = \pi$ has no nonzero solutions, but when $T = \pi$ to 1 billion digits has infinitely many of them. Obviously, in physics it is impossible to determine the final time to infinitely many digits, so there is no physically meaningful solution to the stated problem.

   For nonzero solutions, try $u = \sin(n\pi x)\sin(n\pi t)$. Show that this satisfies the wave equation and the boundary conditions at $x = 0$ and $x = 1$ and the initial condition at $t = 0$. See when it satisfies the end condition at $t = T$.

   The wave equation needs two initial conditions at $t = 0$, not one condition at $t = 0$ and one at $t = T$.
8. Show that the Laplace equation

$$u_{xx} + u_{yy} = 0 \qquad 0 \le x \le 1 \quad 0 \le y \le T$$

with the same boundary conditions, (replacing t by y), does not have the same problem. Hint: assume solutions of the form $u = \sin(n\pi x)f(y)$ and plug into the Laplace equation and $u(x, 0) = 0$ to figure out what $f(y)$ is. (You may want to recall the graphs of the hyperbolic functions.)

7 02/26 F

1. 2.22b,g. Draw the characteristics very neatly in the xy-plane,
2. 2.28d. First find a particular solution. Next convert the remaining homogeneous problem to characteristic coordinates. Show that the homogeneous solution satisfies

$$2u_{h,\xi\eta} = u_{h,\eta}$$

Solve this ODE to find $u_{h,\eta}$, then integrate $u_{h,\eta}$ with respect to η to find u_h and u. Write the total solution in terms of x and y.

3. 2.28f. In this case, leave the inhomogeneous term in there, don't try to find a particular solution for the original PDE. Transform the full problem to characteristic coordinates. Show that the solution satisfies

$$4u_{\xi\eta} - 2u_\xi \pm e^\eta = 0$$

where $\pm$ indicates the sign of xy, or

$$4\xi\eta u_{\xi\eta} - 2\xi u_\xi + \eta = 0$$

or

$$4\xi\eta u_{\xi\eta} + 2\xi u_\xi - \frac{1}{\eta} = 0$$

or equivalent, depending on exactly how you define the characteristic coordinates. Solve this ODE for u_ξ, then integrate with respect to ξ to find u. Write the solution in terms of x and y.

4. 2.28c. Show that the equation may be simplified to

$$u_{\xi\xi} = 0$$

Solve this equation and write the solution in terms of x and y.

5. 2.28b. Reduce to canonical form by solving the characteristic equation. In 2.23 you diagonalized essentially the same equation by rotating the coordinate system; and you could then have stretched the coordinates to reduce it to the Laplace equation. Are the coordinates that you find now equivalent to those? In particular, are the lines of constant ξ and η orthogonal like in 2.23? If not, how come that more than one linear coordinate transformation can turn the equation into the Laplace equation?

6. 2.28k. Reduce the PDE to the form

$$u_\eta = \left(e^{-\xi} + \frac{1}{\eta}\right) u_{\xi\xi}$$

Now discuss the properly posedness for the initial value problem, recalling from the class notes that the backward heat equation is not properly posed. In particular, given an interval $\xi_1 \le \xi \le \xi_2$, with an initial condition at some value of η_0 and boundary conditions at ξ_1 and ξ_2, can the PDE be numerically solved to find u at large η? If η_0 is positive? If η_0 is a small negative number? If η_0 is a large negative number?

7. Show that the Laplace equation

$$u_{xx} + u_{yy} = 0 \qquad 0 \le x \le 1 \quad 0 \le y \le T$$

is improperly posed for the initial/boundary value problem

$$\text{BC:} \quad u(0,y) = u(1,y) = 0 \qquad \text{IC:} \quad u(x,0) = u_0(x), u_y(x,0) = 0$$

because the solution at $y = T$ can be arbitrarily much larger than the given initial condition $u_0(x)$. To do so, assume that $u_0(x) = \varepsilon \sin(n\pi x)$, where ε is a small number. The solution is of the form $u = \sin(n\pi x) f(y)$ where $f(y)$ can be found from substitution into the Laplace equation and initial conditions. Show that the solution at $y = T$ can be any amount of times larger than ε, the magnitude of the initial condition. For example, show that the solution at $y = T$ can be a billion times larger than the initial condition at $y = 0$.

8. Repeat the argument to show that the wave equation does not have a problem with the above initial value problem.

8 03/05 F

1. 3.38. This does *not* require solution of the problem using the Poisson integral formula. You can just examine what symmetry properties the solution $u(x, y)$ should have to figure out the value at the origin. You might want to draw some isotherms to guide your thoughts. However, feel free to check your result against the Poisson integral.

2. 3.39. Again this does *not* require solution of the problem using the Poisson integral formula. You should be able to find the complete solution $u(x, y)$ by mere inspection. However, feel free to check your result against the Poisson integral; in that case, first write the boundary values in the form

$$f(\phi) = A + B\cos(\phi - \theta) + C\sin(\phi - \theta)$$

The integrals for A and B can be found in a table of definite integrals.

3. 3.41. This is similar to the Laplace version discussed earlier in class. Describe the reason that there is no solution physically, considering it as a heat conduction problem in a circular plate.

4. 3.44. This is mostly the uniqueness proof given in class, which can also be found in solved problems 3.14-3.16. However, here you will want to write out the two parts of the surface integral separately since the boundary conditions are a mixture of the two cases 3.14 and 3.15 (with $c = 0$).

5. Show that the following Laplace equation problem has a unique solution, $u = 0$:

$$\text{PDE: } \nabla^2 u = 0 \qquad \text{BC: } u(0,y) = u_y(x,0) = u_y(x,1) = u(1,y) + u_x(1,y) = 0$$

   This is essentially the uniqueness proof given in class, which can also be found in solved problems 3.14-3.16. However, you will want to write the four parts of the surface integral out separately since the boundary conditions are a mixture of the three cases 3.14-3.16.

6. Show that the following Laplace equation problem has infinitely many solutions beyond $u = 0$:

$$\text{PDE: } \nabla^2 u = 0 \qquad \text{BC: } u(0,y) = u_y(x,0) = u_y(x,1) = u(1,y) - u_x(1,y) = 0$$

   Hint: Guess a very simple nonzero solution and check that it satisfies all boundary conditions and that its second order derivatives are zero. Since the equations are linear, any arbitrary multiple of this solution is also a solution. Verify whether or not the uniqueness proof of the previous section conflicts with the nonunique solution of this problem. Why would a slight difference in one boundary condition make a difference?

9 03/19 F

1. Find the Green's function in three-dimensional unbounded space R^3. Use either the method of section 2.1 or 2.2 of the web page example[2] as you prefer.

10 03/26 F

1. See whether any terms must be changed in expression (5) in the notes on elliptic equations[3] in the three dimensional case. Then determine how (6) differs from the two-dimensional case.

2. 3.30. In (a), use the property mentioned in class that the minimum of a harmonic function must occur on the boundary. In (b), try $1 - y$ in the domain Ω given by $y \geq 0$. In (c), consider the functions $s = v - u$ and $t = w - v$.

3. 4.19. Plane wave solutions are solutions that take the form (2) in solved problem 4.12, with $\vec{\alpha}$ a constant vector and μ a constant. This sort of solutions are a multi-dimensional generalization of the $f(x - at)$ moving "wave" solution of the one-dimensional wave equation. In fact, if you take $\vec{\alpha}$ to be a unit vector, it gives the oblique direction of propagation of the wave and μ gives the wave propagation speed. However, in this case you will see that the function F cannot be an arbitrary function unless $b = 0$. You may want to do the case $b = 0$ separately. And also split up the cases for μ.

11 04/02 F

1. 5.25. Also: (c) Assume that

$$f(x) = e^{-|x-2|}$$

   In a single very neat plot, draw $u(x,1)$, $u(x,2)$, and $u(x,3)$ versus x.

2. 5.26b. Ignore the hint. Simplify your answer as much as possible. Draw the characteristic in the x,y-plane.

3. 5.27(a). Include a sketch of the characteristic lines. Is the solution you get valid everywhere?

[2] `http://www.eng.fsu.edu/~dommelen/courses/aim2/10/topics/pdes/elliptic/`
[3] `http://www.eng.fsu.edu/~dommelen/courses/aim2/10/topics/pdes/elliptic/`

4. 5.27(b). Do not try to use an initial condition written in terms of two different, related, variables. Get rid of either x or y in the condition. Then call the argument of your undetermined function γ and rewrite its expression in terms of γ. Include a sketch of the characteristic lines.

5. 5.29

12 04/09 F

1. In 7.27, acoustics in a pipe with closed ends, assume $\ell = 1$, $a = 1$, $f(x) = x$, and $g(x) = 1$. Graphically identify the extensions $F(x)$ and $G(x)$ of the given $f(x)$ and $g(x)$ to all x that allow the solution u to be written in terms of the infinite pipe D'Alembert solution. (F and G may have been called $\bar{f}$ and $\bar{g}$ in class.)

2. Continuing the previous problem, in three separate graphs, draw $u(x, 0)$, $u(x, 0.25)$, and $u(x, 0.5)$. For the latter two graphs, also include the separate terms $\frac{1}{2}F(x - at)$, $\frac{1}{2}F(x + at)$, and $\int_{x-at}^{x+at} G(\xi)\,\mathrm{d}\xi$. Use raster paper or a plotting package. Use the D'Alembert solution only to plot, do not use a separation of variables solution. Comment on the boundary conditions. At which times are they satisfied? At which times are they not meaningful? Consider all times $0 \le t < \infty$ and do not approximate.

3. Using the D'Alembert solution of the previous problems, find $u(0.1, 3)$.

4. Write the *complete* (Sturm-Liouville) eigenvalue problem for the eigenfunctions of 7.27.

5. Find the eigenfunctions of that problem. Make very sure you do not miss one. Write a symbolic expression for the eigenfunctions in terms of an index, and identify all the values that that index takes.

13 04/16 F

1. Continuing the previous homework, write $f = x$ and $g = 1$ in terms of the eigenfunctions you found for the case $\ell = 1$. Be very careful with one particular eigenfunction. Note that sometimes you need to write a term in a sum or sequence out separately from the others.

2. Substitute $u(x,t) = \sum_n u_n(t)X_n(x)$ into the PDE to convert it into an ordinary differential for each separate coefficient $u_n(t)$. Solve the ODE. Be very careful with one particular case.

3. By writing the initial conditions in terms of the eigenfunctions, identify the integration constants. Write out a complete summary of the solution. Make sure to identify the values of your numbering index in each expression.

4. Using some programming language, evaluate the found solution at 101 equally spaced x-values from 0 to ℓ at time $t = 0.25$ and so plot u versus x at that time. Repeat for $t = 0.5$. Include at least 50 nonzero terms in the summations. Take $\ell = 1$ and $a = 1$. Compare with your (or the instructor's) D'Alembert solution. It should show good agreement. What happens if you only include 10 term in the summations?

   To help you get started, a Matlab program that plots the solution to problem 7.28 is provided as an example. You need both p7_28.m[4] and p7_28u.m[5]. This program is valid for the PDE and BC solved in class, with the additional data

$$a = \tfrac{1}{2}, \quad \ell = \tfrac{1}{2}\pi, \qquad f(x) = \tfrac{1}{2}\pi - x \Rightarrow f_n = \frac{1}{(2n-1)^2}, \qquad g(x) = 0 \Rightarrow g_n = 0.$$

   These may of course not apply for your problem.

   To run the program, enter matlab and type in `p7_28`. If you do not have matlab, a free replacement is octave. Or you can use some other programming and plotting facilities.

[4] `p7_28.m`
[5] `p7_28u.m`

5. Solve 7.26, by Laplace transforming the problem as given in time. This is a good way to practice back transform methods. Note that one factor in $\widehat{u}$ is a simpler function at a shifted value of coordinate s.

6. Solve 7.35 by Laplace transform in time. Clean up completely; only the given function may be in your answer, no Heaviside functions or other weird stuff. There is a minor error in the book's answer.

14 04/23 F

1. Refer to problem 7.19. Find a function $u_0(x,t)$ that satisfies the inhomogeneous boundary conditions. Define $v = u - u_0$. Find the PDE, BC and IC satisfied by v.

2. Find suitable eigenfunctions in terms of which v may be written, and that satisfy the homogeneous boundary conditions. Write the relevant known functions in terms of these eigenfunctions and give the expressions for their Fourier coefficients.

3. Solve for v using separation of variables in terms of integrals of the known functions $f(x)$, $g_0(t)$, and $g_1(t)$. Write the solution for u completely.

4. Assume that $f = 0$, $k = \ell = 1$, and that $u_x = t$ at both $x = 0$ and $x = \ell$. Work out the solution completely.

5. Plot the solution numerically at some relevant times. I suspect that for large times the solution is approximately
$$u = (x - \tfrac{1}{2})t + \tfrac{1}{6}(x - \tfrac{1}{2})^3 - \tfrac{1}{8}(x - \tfrac{1}{2})$$
Do your results agree?