

Show all reasoning and intermediate results leading to your answer, or credit will be lost. One book of mathematical tables, such as Schaum's Mathematical Handbook, may be used, as well as a calculator and one handwritten letter-size single formula sheet.

1. (20%) Solve

$$u_t + xu_x = u \text{ for } -\infty < x < \infty, t \geq 0 \quad u(x, 0) = \frac{1}{1+x^2}$$

2. (15%) Classify and solve the PDE

$$u_{xx} + 8u_{xy} + 7u_{yy} + u_x + u_y = 0$$

3. (50%) Solve the following problem for heat conduction in a bar with one end at zero temperature, and the other end insulated:

$$u_t = \kappa u_{xx} \text{ for } 0 \leq x \leq L, t \geq 0 \quad u(x, 0) = 1$$
$$u(0, t) = 0 \quad u_x(L, t) = 0$$

4. (15%) Solve the following problem:

$$u_t = \kappa u_{xx} \text{ for } 0 \leq x \leq L, t \geq 0 \quad u(x, 0) = x$$
$$u(0, t) = -1 \quad u_x(L, t) = 1$$

by relating its solution to the one of the previous question.