EML 4930/5061 | **Analysis in M.E. II** | **03/17/11**
Closed book | *Van Dommelen* | *9:40-10:30 am*

Show all reasoning and intermediate results leading to your answer, or credit will be lost. One book of mathematical tables, such as Schaum's Mathematical Handbook, may be used, as well as a calculator and one handwritten letter-size single formula sheet.

1. A force field is given as

$$\vec{F} = \begin{pmatrix} 2x + y \\ e^z \cos(y) + x \\ e^z \sin(y) + 1 \end{pmatrix}$$

   (a) Find the divergence and curl of $\vec{F}$ (the curl will turn out to be zero).

   (b) What can you say about the flux integral $\int \vec{F} \cdot \vec{n} \, \mathrm{d}S$ over the entire surface of some volume?

   (c) Why must $\vec{F}$ be the gradient of some potential Φ? Now find Φ in a systematic way that leaves no doubt that the most general possible Φ has been found.

   (d) What can you say about the "work" integral $\int_{\mathrm{A}}^{\mathrm{B}} \vec{F} \cdot \mathrm{d}\vec{r}$ along any path going from a given point A to a given point B? What about the contour integral $\oint \vec{F} \cdot \mathrm{d}\vec{r}$ from a point back to that same point?

2. A curvilinear coordinate system (ξ, η, θ) satisfies

$$x = e^\xi \cos\eta \qquad y = e^\xi \sin\eta \qquad z = \theta^2$$

   Show that this coordinate system is orthogonal, then derive the metric indices h_ξ, h_η, and h_θ. Also derive $\partial\hat{\imath}_\xi/\partial\eta$ without using Cartesian components.

3. Simplify the PDE

$$2u_{xx} + 4u_{xy} - u_{yy} + 2u_x + u_y = 0$$

   by rotation of the coordinate system. Then reduce it further until its derivative terms are those of the wave equation for unit wave speed, with an additional nonderivative term.