EGN 5456 | **Intro to Computational Mechanics** | **10/13/05**
Closed book | *Van Dommelen* | *10:15-11:30 am*

Show all reasoning and intermediate results leading to your answer, or credit will be lost. One book of mathematical tables, such as Schaum's Mathematical Handbook, may be used, as well as a calculator and one handwritten letter-size single formula sheet.

1. Clasify the following partial differential equation:

$$u_{xx} + 5u_{xy} + u_{yy} = 0$$

   Based on your result, state yes or no on the following questions: would this equation

   (a) allow nonsmooth solutions?

   (b) have a boundary condition at every point of the boundary of the region in the x, y-plane in which the PDE is to be solved?

   (c) allow wave propagation?

   (d) be typically solved numerically using a marching procedure?

2. The partial differential equation

$$u_t = 2u + 4u_{xx}$$

   is to be solved in the infinite x-range $-\infty < x < \infty$, and all times t starting from $t = 0$ until a final time $t = T$. Find $u(x, t)$ for the initial condition:

$$u(x, 0) = u_n e^{\mathrm{i}nx}$$

   where $\mathrm{i} = \sqrt{-1}$ and n and u_n are constants. Based on your result, is the initial value problem properly posed? If so, why?

3. Consider a mesh with x-values: $x_0, x_1, x_2, \ldots, x_{j-1}, x_j, x_{j+1}, \ldots, x_J$ and corresponding mesh point values of some function $u(x)$: $u_0, u_1, u_2, \ldots, u_{j-1}, u_j, u_{j+1}, \ldots, u_J$. The x-values are spaced a constant distance Δx apart. Show using Taylor series expansion that

$$\frac{u_{j+1} - 2u_j + u_{j-1}}{(\Delta x)^2}$$

   approximates the second order derivative of $u(x)$ at point x_j to second order accuracy. Verify your answer using the operator form of the above expression.