Matlab Homework 5c

The same requirements as for homework 3c apply.

1. We continue the problem of the previous homework, where we measured a function f that, "unknown to us," is exactly equal to $f_{\text{exact}}(t) = \sin(t)$. Use the same 9 measured data including the random errors as before. Integrate the fitted quintic numerically from 0 to $\pi/2$. Compare with the exact value. Also integrate the spline interpolate numerically and compare.

2. Your city is being attacked by a ship. You want to shoot cannon balls at it, but the ship may be too far away. Your cannon can shoot the cannon ball out at an initial total velocity V_0 of 100 m/s, and the acceleration of gravity g at your city happens to be 9.81 m/s^2. You know from basic physics that ignoring air resistance, your cannon ball will travel farthest when you shoot the ball out at 45 degrees from the horizontal. It will then travel 1,019 m in 14.42 seconds to a ship at the same height as the cannon. You would like to know how this changes in the presence of air resistance.

   To do so, the first step is identify the equations of motion of a canon ball in air, but without significant spin. They are:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = u \tag{1}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = v \tag{2}$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = (-F_{\text{air}}u/V)/m \tag{3}$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = (-F_{\text{gravity}} - F_{\text{air}}v/V)/m \tag{4}$$

   where x is the horizontal distance traveled from the canon, y the height above the canon, u the velocity component in the x-direction and v the one in the y-direction. Also m is the mass of the canon ball, which has radius r and is made of iron with a density ρ_{iron} equal to 7272 kg/m^3. Also

$$V = \sqrt{u^2 + v^2} \qquad F_{\text{gravity}} = mg \qquad F_{\text{air}} = C_d \pi r^2 \tfrac{1}{2} \rho_{\text{air}} V^2$$

   where the drag coefficient C_d can be taken to be 0.5 and the density of air ρ_{air} to be 1.225 kg/m^3.

   Write a well-documented Matlab function cannonBall that takes as inputs the time t from firing the cannon, a vector consisting of values of the four unknowns (x, y, u, v), and the cannon ball radius r, and that outputs the corresponding values of the derivatives of the unknowns in a column vector.

3. Use the function of the previous question to find the motion of a canon ball of 10 cm radius in 14.42 seconds using the data as described above. Plot the path of the cannon ball as y versus x (not against time). To do so neatly, you will want to take the x and y values out of `unknownValues`. You can do that as

```
xValues=unknownValues(:,1);
yValues=unknownValues(:,2);
```

(If you have to know, the second number is the column number, and the first is the row number; the bare colon means all row numbers from 1 to end.) Next, if you just plot `yValues` against `xValues`, it looks pretty poor, because the points are too far apart. So create 100 plot positions `xPlot` from zero to `xValues(end)`, then interpolate a spline through `xValues` and `yValues` to get the corresponding y values `yPlot`, and plot that. Add an appropriate title and axis labels, of course. (Note: instead of using the spline, you could force `ode45` to create more points. You do that by changing the vector that provides the initial and final times in the `ode45` call. If you replace that vector with a vector consisting of 100 times from the initial to the final times, `ode45` will compute the solution at those times.)