Matlab Homework 7c

The same requirements as for homework 3c apply.

1. Given the matrices

$$A = \begin{pmatrix} -2 & 2 \\ 0 & 1 \\ 14 & 2 \\ 6 & 8 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 4 \\ 2 & 1 \\ 14 & 16 \\ 1 & 25 \end{pmatrix}$$

   find,

   - the unit matrix that A can be pre-multiplied by (i.e. as IA), and demonstrating that this does not change A;
   - the unit matrix that A can be post-multiplied by (i.e. as AI), and demonstrating that this does not change A;
   - the zero matrix that can be added to A, and demonstrating that this does not change A;
   - the square zero matrix that A can be pre-multiplied by, demonstrating that this produces a zero matrix the size of A;
   - the square zero matrix that A can be post-multiplied by, demonstrating that this produces a zero matrix the size of A;
   - a zero row vector that A can be pre-multiplied with, demonstrating that this produces a zero row vector with the same number of columns as A.
   - a zero column vector that A can be post-multiplied with, demonstrating that this produces a zero column vector with the same number of rows as A.

2. Reconsider the power line between two poles. In addition to sagging down under its own weight, it can vibrate like a string. In this case we will use 8 points along the entire line from pole to pole, and number them from 0 to 7. But since the line is attached to the poles at points 0 and 1, the amplitudes of vibration a_0 and a_7 of these points are zero and can be eliminated. The amplitudes of vibration a_i, $i = 1, 2, \ldots, 6$ for the six interior points then satisfy the equations:

$$\begin{aligned} -2a_1 + a_2 &= -\omega^2 a_1 && (1) \\ a_1 - 2a_2 + a_3 &= -\omega^2 a_2 && (2) \\ a_2 - 2a_3 + a_4 &= -\omega^2 a_3 && (3) \\ a_3 - 2a_4 + a_5 &= -\omega^2 a_4 && (4) \\ a_4 - 2a_5 + a_6 &= -\omega^2 a_5 && (5) \\ a_5 - 2a_6 &= -\omega^2 a_6 && (6) \\ & && (7) \end{aligned}$$

   for a given frequency of vibration ω. Formulate this as an eigenvalue problem for an appropriate matrix A, vector $\vec{v}$, and eigenvalue λ. Then solve that eigenvalue problem to find the 6 frequencies of vibration. Print out the frequencies in the generic format

```
Frequency N: *.123
```

where N ranges from 1 to 6.

Note incidentally that the eigenvalues better be negative real numbers! A symmetric matrix that has all eigenvalues negative is called *negative definite*.

3. Check whether the matrix in the eigenvalue problem above is symmetric. If it is, check whether the eigenvectors are really length 1 and mutually orthogonal as they should be.

4. For each individual frequency of vibration, the coefficients of the corresponding eigenvector give the amount that the power line deflects vertically at the six points at the moment that the deflections from equilibrium are greatest. So plotting the values of an eigenvector against the position along the line (in percent) shows the "mode shape" of vibration at that frequency. It shows you how the power line looks (except for sag) when it is vibrating at that single frequency alone. There is a different mode shape for each frequency. All 6 are to be plotted.

   However, note that the mode shape includes the starting point 0 (at 0%) and the end point 7 (at 100%). And the eigenvectors as you got them have only the interior 6 points. So you must add the end points. To do so, first define an 8 by 6 matrix, called `modes`, of zeros using the appropriate Matlab function. Then replace the middle 6 rows of that matrix with the eigenvectors that Matlab gave you. To do so use START:END specifications that specify the center 6 by 6 part of the matrix `modes`. (Leave START and END out where possible.)

   Now the full mode shapes, including end points, are given by the columns of matrix `modes`. Take them out individually and call them `mode1` up to `mode6`.

5. Plot all six mode shapes together in a single plot with appropriate title axes labels, and legend. However, 8 plot points from 0 to 100% are not enough to plot a good-looking mode shape. So use `spline` to first interpolate 100 points between 0 and 100% for each of the 6 mode shapes. Format the individual legend entries as, for example,

   ```
   ['omega1 = ' num2str(omega(1))]
   ```

   Use a y-axis from -5 to 5. Add appropriate axes labels and a suitable title.

   Note: the mode shapes for the larger values of ω will be inaccurate. To get these mode shapes more accurate, we would need more interior points.