Matlab Homework 7*a*

In the online book:

- Do the "Challenge Activities" of: 5.6
- Do the "Participation Activities" of: 8.1-6,9; 18,1-3,8

Matlab Homework 7*b*

The same general requirements as for homework 4b apply. And you must study the posted lesson(s) and have done the online book part above before you can ask a TA or the instructor for help.

1. Consider a power line between two poles. It sags down under its own weight. Let h be the height of the powerline above the ground. The heights h at 6 points distributed along the first half of the power line satisfy the (approximate) equations

$$\begin{aligned} h_1 &= H & (1) \\ h_1 - 2h_2 + h_3 &= \rho/(n-1)^2 & (2) \\ h_2 - 2h_3 + h_4 &= \rho/(n-1)^2 & (3) \\ h_3 - 2h_4 + h_5 &= \rho/(n-1)^2 & (4) \\ h_4 - 2h_5 + h_6 &= \rho/(n-1)^2 & (5) \\ h_5 - h_6 &= 0 & (6) \end{aligned}$$

   Here H is the scaled height of the power line at the pole; assume it to be 1 (that is how it was scaled). Also ρ is the scaled mass of the power line per unit length; take this to be 1 too. Finally, n is the number of points, 6 in this example. (So $n-1$ is the number of segments between successive points.)

   Write this system of 6 equations in 6 unknown heights in matrix-vector form `AOrg*xOrg=bOrg`. In particular, create matrix `AOrg` and right hand side vector `bOrg`.

   Check whether the equations have a meaningful solution; use `disp` to discuss this. In particular, estimate the relative error in the solution that will be caused by the fact that floating point numbers in Matlab are only stored to about 16 significant digits.

   If you conclude that a reasonably or highly accurate solution will be found, find it.

   Then plot the height against the horizontal position of the points. (The points are located at 6 equally spaced x-positions from 0 to 50% of the distance between the poles.) Plot the points as black circles connected by straight lines. Provide appropriate axes labels and title, and extend the horizontal axes for no more than the 50%. For horizontal axis label, use "Position, percent". Does it look roughly like half a sagging power line?

2. Repeat the previous question when we are using $n = 11$ points along the half power lines. For 11 points, the power line get divided into $n - 1 = 10$ segments.

$$\begin{aligned}
h_1 &= H & (7)\\
h_1 - 2h_2 + h_3 &= \rho/(n-1)^2 & (8)\\
h_2 - 2h_3 + h_4 &= \rho/(n-1)^2 & (9)\\
\vdots &= \vdots & (10)\\
h_9 - 2h_{10} + h_{11} &= \rho/(n-1)^2 & (11)\\
h_{10} - h_{11} &= 0 & (12)
\end{aligned}$$

This time, write the system as `AMod*xMod=bMod`. But rather than type out the big vector `bMod`, first define a variable n and give it value 11. Next see what matrix is produced by `ones(n,1)`. (But do not include this in your submitted solution.) You should then see that

```
bMod=rho/(n-1)^2*ones(n,1)
```

will give you most components in `bMod` correct. Correct the remaining two incorrect components without messing up the already correct components.

Similarly, you should see that you can create most of `AMod` by taking a sum of the form

```
AMod=C1*diag(ones(1,n))+C2*diag(ones(1,n-1),1)+C3*diag(ones(1,n-1),-1)
```

By printing out the `diag(...)` matrices, it should be obvious what you need to take the constants `C1`, `C2`, and `C3` to get most of matrix `AMod` right. Then correct the first and last row of that matrix without messing with the already correct components. Do not include the print-outs in your solution, just show what the final `AMod` and `bMod` are.

Plot again. You should see an improvement in the shape of the power line. Do you?

3. Repeat the first question when the first equation $h_1 = H$ is replaced by $h_1 - h_2 = 0$. Do not create a new matrix and right-hand side vector from scratch. Copy over `Aorg` into `ABad` and `bOrg` into `bBad`, and then just change the components that are different.

   If you find that the new system has no meaningful solution, say so, but try to solve anyway and comment on what you get using `disp`. If you get some numbers but they are no good, explain why.

4. Given the matrices

$$A = \begin{pmatrix} -2 & 2 \\ 0 & 1 \\ 14 & 2 \\ 6 & 8 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 4 \\ 2 & 1 \\ 14 & 16 \\ 1 & 25 \end{pmatrix}$$

   find, if it exists (else note that it does not using `disp`),

   - $-5A + 3B$;
   - A^{T};
   - AB, BA, $A^{\mathrm{T}}B$, and BA^{T} (note that the latter two are not equal);
   - the non-matrix (elementwise) products AB, BA, $A^{\mathrm{T}}B$, and BA^{T};