Matlab Homework 8*a*

In the online book:

- Do the "Challenge Activities" of:
- Do the "Participation Activities" of: 11.1-4; 12.1,3,4,

Matlab Homework 8*b*

The same general requirements as for homework 4b apply. And you must study the posted lesson(s) and have done the online book part above before you can ask a TA or the instructor for help.

1. Given the matrix

$$A = \begin{pmatrix} -2 & 2 \\ 0 & 1 \\ 14 & 2 \\ 6 & 8 \end{pmatrix}$$

   find,

   - the unit matrix that A can be pre-multiplied by (i.e. as IA), and demonstrating that this does not change A;
   - the unit matrix that A can be post-multiplied by (i.e. as AI), and demonstrating that this does not change A;
   - the zero matrix that can be added to A, and demonstrating that this does not change A;
   - the square zero matrix that A can be pre-multiplied by, demonstrating that this produces a zero matrix the size of A;
   - the square zero matrix that A can be post-multiplied by, demonstrating that this produces a zero matrix the size of A;
   - a zero row vector that A can be pre-multiplied with, demonstrating that this produces a zero row vector with the same number of columns as A.
   - a zero column vector that A can be post-multiplied with, demonstrating that this produces a zero column vector with the same number of rows as A.

2. Reconsider the power line between two poles. In addition to sagging down under its own weight, it can vibrate like a string. In this case we will use 8 points along the entire line from pole to pole, and number them from 0 to 7. But since the line is attached to the poles at points 0 and 7, the amplitudes of vibration a_0 and a_7 of these points are zero and can be eliminated. That leaves $n = 6$ unknown amplitudes of vibration $a_1, a_2, \ldots, a_6$ for the six

interior points. These amplitudes then satisfy the equations:

$$2a_1 - a_2 = \frac{\omega^2}{(n+1)^2} a_1 \tag{1}$$

$$-a_1 + 2a_2 - a_3 = \frac{\omega^2}{(n+1)^2} a_2 \tag{2}$$

$$-a_2 + 2a_3 - a_4 = \frac{\omega^2}{(n+1)^2} a_3 \tag{3}$$

$$-a_3 + 2a_4 - a_5 = \frac{\omega^2}{(n+1)^2} a_4 \tag{4}$$

$$-a_4 + 2a_5 - a_6 = \frac{\omega^2}{(n+1)^2} a_5 \tag{5}$$

$$-a_5 + 2a_6 = \frac{\omega^2}{(n+1)^2} a_6 \tag{6}$$

for a scaled frequency of vibration ω.

Formulate this as an eigenvalue problem for an appropriate matrix A, eigenvector $\vec{v}$ equal to $(a_1, a_2, \ldots, a_6)^{\mathrm{T}}$, and eigenvalue λ. What would matrix A be? And what would the eigenvalue be, in terms of the quantities above?

Matrix A should be symmetric; check that it is. (Note further that the eigenvalues better be positive real numbers! Or you would get complex frequencies. A symmetric matrix that has all eigenvalues positive is called *positive definite*.)

Now solve the eigenvalue problem to find the 6 eigenvectors, as a matrix `E`, and the corresponding eigenvalues, as a matrix `Lambda`.

From the first three eigenvalues in `Lambda`, compute and print out the corresponding frequencies `omega1`, `omega2`, and `omega3`, in the generic format

```
Frequency I: *.123
```

where I ranges from 1 to 3. No data numbers in FORMATSTRING allowed.

Note: the frequencies and eigenvectors for the larger values of I than 3 will be very inaccurate. To get these more accurate, you would need more interior points.

3. The eigenvectors describe the "mode shapes." In other words, for each eigenvalue, the corresponding eigenvector gives the amplitudes of vibration of the 6 interior points when the cable is vibrating at that frequency.

   However, it is ugly that the eigenvectors do not include the two end points. While the end points have zero amplitude, still to get a complete picture of the mode shapes, you should include the end points. So take the first 3 eigenvectors out of the matrix of eigenvectors and put them into a matrix `modes` that includes the zero end point values.

   To do so, first create an 8 by 3 matrix `modes` of zeros using the appropriate Matlab function. Then replace the middle 6 rows of that zero matrix with the first three eigenvectors that Matlab gave you. To do so use START:END specifications that specify the center-left 6 by 3 part of the matrix `modes`, and the first three eigenvectors in `E`. Leave out START and END where possible.

Next take the three individual mode shapes out of `modes`, calling them `mode1Vals`, `mode2Vals`, and `mode3Vals`.

Next check whether `mode1Vals`, `mode2Vals`, and `mode3Vals`, considered as 8-dimensional vectors, are really length 1 and mutually orthogonal as they should be.

4. Plotting the values of each of the mode shapes `mode1Vals`, `mode2Vals`, and `mode3Vals` against the position of the points along the cable (in percent) will show the mode shape of vibration at that frequency graphically. In other words, it shows you how the power line looks (except for sag) when it is vibrating at that single frequency alone.

   So plot these three mode shapes versus the x-values of the 8 points, where x ranges from 0 to 100% along the length of the power cable. (In other words, the array of x-values `xVals` consists of $n + 2 = 8$ equally spaced percent values from 0 to 100. It should be a column vector, not a row vector.)

   However, before plotting the mode shapes, multiply each by $\sqrt{n}$. This corrects for Matlab scaling the eigenvectors to length one, which is not really desired here.

5. You should have found in the previous question that using the 8 computed amplitudes from 0 to 100% is not enough points to plot good-looking mode shapes. So define an array `xPlot` consisting of 100 equally spaced x-values from 0 to 100%. Then for each of the three mode shapes use `spline` to interpolate appropriate values of the mode shape at those plot points, call them `mode1Plot`, etcetera. Format the individual legend entries as, for example,

   ```
   ['omega1 = ' num2str(omega1)]
   ```

   where I ranges from 1 to 3. Make sure each mode shape can be distinguished from the others using the legend.

6. Recompute and replot the 3 mode shapes, but now define a variable `n` to represent the number of internal points. Set `n` to 13, but your code should work correctly for whatever number of internal points is specified in `n`. So write everything in terms of `n`. For example, the eigenvalue problem becomes

$$+2a_1 - a_2 = \frac{\omega^2}{(n+1)^2} a_1 \tag{7}$$

$$-a_1 + 2a_2 - a_3 = \frac{\omega^2}{(n+1)^2} a_2 \tag{8}$$

$$-a_2 + 2a_3 - a_4 = \frac{\omega^2}{(n+1)^2} a_3 \tag{9}$$

$$\vdots = \vdots \tag{10}$$

$$-a_{n-2} + 2a_{n-1} - a_n = \frac{\omega^2}{(n+1)^2} a_{n-1} \tag{11}$$

$$-a_{n-1} + 2a_n = \frac{\omega^2}{(n+1)^2} a_n \tag{12}$$

   Form matrix A for general n as

   ```
   A=C1*diag(ones(1,n))+C2*diag(ones(1,n-1),1)+C3*diag(ones(1,n-1),-1)
   ```

   for appropriately chosen values of the constants.