

Common approximations for solids and liquids

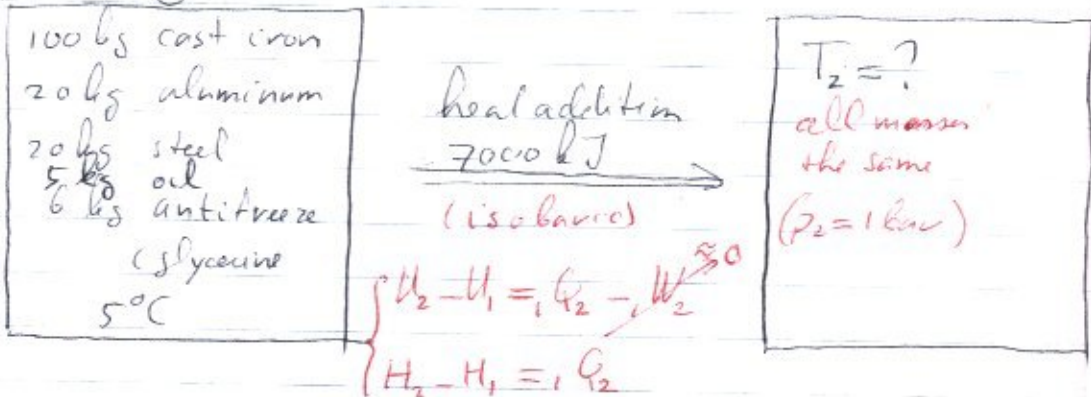
$$C_p \approx \text{constant}$$

$$u_2 \approx m C_{(p)} (T_2 - T_1) \quad \text{used even if } p \text{ is not really constant}$$

[Note: when v is constant, it is a better approximation to say ~~$u_2 - u_1 \approx m C_{(p)} (T_2 - T_1)$~~ than $h_2 - h_1$.
Numerical example on page next page]

P5.01 Engine

(1)



Answer: detailed no tables for H_2 and H_1 .

So use instead $Q_2 = (\text{sum over all masses}) (m C_p (T_2 - T_1))$

Table A.3 p 657

Table A.4 p 657

use if V is given

Substance	ρ lb/m ³	C_p Btu/lb R
Aluminium		0.90
Iron, cast		0.42
Iron, 304 St Steel		0.46

Substance	C_p Btu/lb R
Glycerine	2.92
Oil, engine	1.9

$$\begin{aligned}
 7000 \text{ J} &= (m_{\text{iron}} C_{p,\text{iron}} + m_{\text{al}} C_{p,\text{al}} + m_{\text{steel}} C_{p,\text{steel}} + m_{\text{oil}} C_{p,\text{oil}} + m_{\text{gly}} C_{p,\text{gly}}) (T_2 - T_1) \\
 &= (100 \cdot 0.42 + 20 \cdot 0.90 + 20 \cdot 0.46 + 5 \cdot 1.9 + 6 \cdot 2.92) (T_2 - T_1) \\
 7000 \text{ J} &= 93.22 \frac{\text{J}}{\text{K}} (T_2 - T_1) \quad T_2 - T_1 = \frac{7000 \text{ J}}{93.22 \frac{\text{J}}{\text{K}}} = 75 \text{ K} = 75^\circ\text{C} \\
 T_2 &= T_1 + 75^\circ\text{C} = 5^\circ\text{C} + 75^\circ\text{C} = 80^\circ\text{C}
 \end{aligned}$$

+ mol $C_{p,\text{al}}$
 $1.8 \text{ Btu/lb} \cdot 1 \text{ lb/l}$
 $2.32 \text{ Btu/lb} \cdot 1 \text{ lb/l}$
 temperature difference

S 5.6 & 7

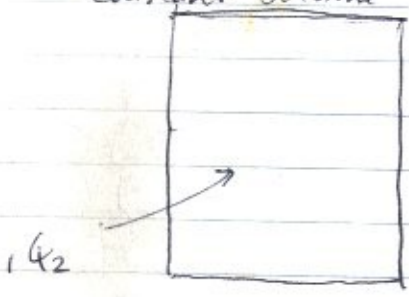
How about u for ideal gasses? This is usually ~~not~~ done through specific heat concepts. Similarly for solids and liquids.

Specific heats relate heat added to temperature change

$$\delta Q \equiv m C dT \quad C \text{ specific heat.}$$

Unfortunately, the value of C depends on what is being held constant. Two important cases exist

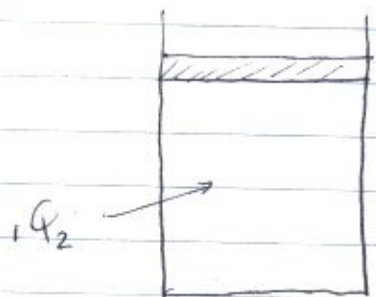
constant volume



$${}_1Q_2 = \int_1^2 m C_v dT = U_2 - U_1 \quad \text{if } V = \text{constant}$$

C_v : specific heat at constant volume

constant pressure:



$${}_1Q_2 = \int_1^2 m C_p dT = H_2 - H_1 \quad \text{if } p = \text{constant}$$

H is the enthalpy:
$$\begin{cases} H \equiv U + pV \\ h \equiv u + pv \end{cases}$$

C_p constant pressure specific heat

Why the pV ? Some of the heat goes into work done by the substance, which is $p(V_2 - V_1) = p_2 V_2 - p_1 V_1$

(We could have done 5.49 using H and saved a bit of work)

SKIP

IX 3a

Common approximation for solids and liquids

~~For p is constant (e.g. $p = p_{atm}$), $Q_2 \approx m C (T_2 - T_1) + v (h_2 - h_1)$~~

~~This assumes that $C = C_p = \text{constant}$~~

~~Numerical justifications:~~

~~(Eg, water at 500 kPa: C_p at $20^\circ\text{C} = \left(\frac{\partial h}{\partial T}\right)_p \approx \frac{\Delta h}{\Delta T} = \frac{167.50 - 0.51}{40 - 0.01} \approx 4.17$~~

~~C_p at $100^\circ\text{C} \approx \frac{503.5 - 335.24}{120 - 10} = 4.21$~~

~~Difference is half a percent)~~

In addition, the approximation, $Q_2 \approx m C (T_2 - T_1)$ is also often made for liquids and solids even if p is not constant.

(Numerical example: constant volume change from

State 1: $T_1 = 40^\circ\text{C}$ $p_1 = 500 \text{ kPa}$ $v_1 = 0.001000 \text{ m}^3/\text{kg}$

$u_1 = 167.47$ $h_1 = 167.50$ $Q_1 \approx \frac{251.51 - 84.41}{60 - 20} = 4.170$

State 2: $T_2 = 60^\circ\text{C}$ $p_2 = 20,000 \text{ kPa}$ $v_2 = 0.001000 \text{ m}^3/\text{kg} = v_1$

$u_2 = 247.66$ $h_2 = 267.02$ $C_2 \approx \frac{350.70 - 105.14}{60 - 40} = 4.141$

Exact heat added/unit mass = $u_2 - u_1 = 80.2 \text{ kJ/kg}$ since $v = \text{constant}$

Approximation $\frac{C_2 + C_1}{2} (T_2 - T_1) = 83.2 \text{ kJ/kg}$ (about 4% error)

Note that it is not a very good approximation to claim that $C(T_2 - T_1) \approx h_2 - h_1$, as the book does: $h_2 - h_1 = 99.5$ has a 24% error!

(Reason: $dQ = du + p dv$ where $p dv$ is small for liquids and solids and u depends on temperature, but hardly on pressure)

Better: $h_2 - h_1 \approx C(T_2 - T_1) + v(p_2 - p_1)$

5.7 Ideal gases

For ideal gases

$$\begin{aligned} c_p - c_v &= R \\ \bar{c}_p - \bar{c}_v &= \bar{R} \quad \text{mole basis} \end{aligned}$$

⇓
only one of c_p and c_v is needed

Also: $u = u(T)$, $h = h(T)$, $c_v = c_{v0}(T)$, $c_p = c_{p0}(T)$
only depend on temperature

Note: the zeros in c_{v0} and c_{p0} merely indicate they are the values corresponding to zero or low pressure states of the real gases.

Note: all of the above can be proved from the 2nd law of thermo
 Different methods: must use most accurate only

Table A-5^{A2} gives c_{p0} and c_{v0} at 25°C, 100 kPa

$$\begin{aligned} u_2 - u_1 &\approx c_{v0} (T_2 - T_1) \\ h_2 - h_1 &\approx c_{p0} (T_2 - T_1) \end{aligned} \left. \begin{array}{l} \text{may be OK,} \\ \text{as long as } T_2 \text{ and } T_1 \text{ are real} \\ \text{or noble gases} \\ \text{too far from room temperature} \end{array} \right\}$$

Table A-2 (b) gives c_p at various temperatures

Table A-6 gives c_{p0} approximated as a cubic in T (which is the value of T in K). Note that $c_{v0} = c_{p0} - R$ and $\bar{c}_p = \bar{c}_p / M$

$$u_2 - u_1 = \int_1^2 c_{v0} 1000 \, dT \approx c_{v0, \text{avg}} (T_2 - T_1)$$

$$h_2 - h_1 = \int_1^2 c_{p0} 1000 \, dT \approx c_{p0, \text{avg}} (T_2 - T_1)$$

A-17

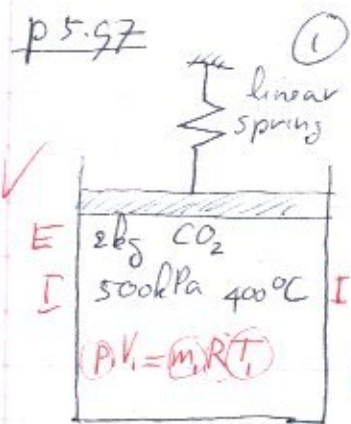
Table A-7.1 T, u, h, \dots for air (pressure = any, not just 1 bar)

Table A-9 T, u, h, \dots for other ideal gases

MUST USE THE MOST ACCURATE DATA

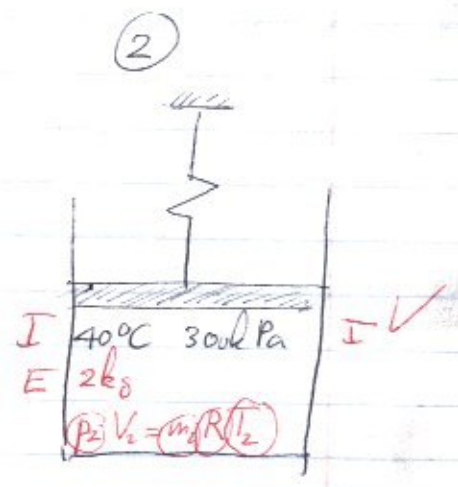
A-17-A-25 / A-26 / A-2c / A-2(a)

tables / A-7.1 / A-6 / A-6 approx / A-5 Example 5.7 compares the approaches



p linear in V
coal
→

$Q_2 = ?$
 $U_2 - U_1 = Q_2 - W_2$
 $W_2 = \frac{p_2 + p_1}{2} (V_2 - V_1)$



Answer: No "B" tables for CO₂: presumably an ideal gas.
 → Check for presence in A. Yes! page 663. Must be used
 indeed 500 kPa \ll $p_c = 7400$ kPa
 A-17-A-25 A-20

Let's do V_1 and V_2 first

$p_1 V_1 = m R T_1$ R from A-1 or A-2(a) $R = 0.1889 \frac{kJ}{kg \cdot K}$

500 kPa $V_1 = 2 \text{ kg} \cdot 0.1889 \frac{kJ}{kg \cdot K} (400 + 273) K \Rightarrow V_1 = 0.5007 \text{ m}^3$

300 kPa $V_2 = 2 \text{ kg} \cdot 0.1889 \frac{kJ}{kg \cdot K} (40 + 273) K \Rightarrow V_2 = 0.3542 \text{ m}^3$

Now we can do W_2

$W_2 = \frac{p_2 + p_1}{2} (V_2 - V_1) = \frac{500 \text{ kPa} + 300 \text{ kPa}}{2} (0.3542 - 0.5007) \text{ m}^3$
 $= -45.72 \text{ kJ}$

Need U_2 and U_1 , or rather, $U_2 - U_1$: I can use different methods:

Worst method: Read c_{v0} from table A-5, then use $U_2 - U_1 \approx m c_{v0} (T_2 - T_1)$
 $U_2 - U_1 \approx 2 \text{ kg} \cdot 0.657 \frac{kJ}{kg \cdot K} (40 - 400) K = -470.16 \text{ kJ}$
 Then, $Q_2 = U_2 - U_1 + W_2 = -470.16 - 45.72 \text{ kJ} = -515.9 \text{ kJ}$
 Major problem: $T_1 = 400^\circ C$ not at all close to $25^\circ C$
 value from table A-5 for c_v $27^\circ C$

Some what better method

Use again $U_2 - U_1 = m c_v (T_2 - T_1)$ but use the value of c_v at the average temperature of $\frac{400^\circ\text{C} + 40^\circ}{2} = 220^\circ\text{C}$ instead of at 25°C c_v in table A.5. The value of c_v at 220°C can be obtained from table A.6 (p.659):

$220^\circ = 493\text{K}$
 $A-2(B)$ interpolated
 $c_{v,ave} \approx 1.01 \frac{\text{kJ}}{\text{kg}}$

$$c_p = C_0 + C_1 \theta + C_2 \theta^2 + C_3 \theta^3$$

Gas	Formula	C_0	C_1	C_2	C_3
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Carbon dioxide	CO_2	0.45	1.67	-1.27	0.39
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$$\bar{T} = 220^\circ\text{C} \rightarrow \theta = \frac{220 + 273}{1000} \text{ kK} = 0.493 \text{ kK}$$

$$c_p = 0.45 + 1.67(0.493) - 1.27(0.493)^2 + 0.39(0.493)^3$$

$$= 1.01136 \frac{\text{kJ}}{\text{kg K}}$$

$$c_v = c_p - R = (1.01136 - 0.1889) \frac{\text{kJ}}{\text{kg K}} = 0.8225 \frac{\text{kJ}}{\text{kg K}}$$

$$U_2 - U_1 = 2 \text{ kg} \cdot 0.8225 \frac{\text{kJ}}{\text{kg K}} (40 - 400) \text{ K}$$

$$= -592.2 \text{ kJ}$$

$$Q_2 = -592.2 - 45.72 = \underline{\underline{-637.9 \text{ kJ}}}$$

(a bit different from 515 kJ!)

Still a bit better

Do not assume that c_v is constant. Instead integrate:

$$U_2 - U_1 = \int_1^2 m c_v dT = \int_1^2 m (c_p - R) dT =$$

$$= m \int_1^2 c_p dT - mR \int_1^2 dT =$$

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v dT = \frac{1}{M} \int_{T_1}^{T_2} \bar{c}_v dT = \frac{1}{M} \int_{T_1}^{T_2} (c_p - \bar{R}_u) dT$$

$$\begin{aligned} &= \frac{1}{M} \int_{T_1}^{T_2} (a - \bar{R}_u + bT + cT^2 + dT^3) dT \\ \text{A-26e)} &\Rightarrow \frac{1}{M} \left[(a - \bar{R}_u)(T_2 - T_1) + \frac{1}{2}b(T_2^2 - T_1^2) \right. \\ &\quad \left. + \frac{1}{3}c(T_2^3 - T_1^3) + \frac{1}{4}d(T_2^4 - T_1^4) \right] \end{aligned}$$

$$M = 44.01 \frac{\text{kg}}{\text{kmol}} \quad \bar{R} = 0.1009 \frac{\text{kJ}}{\text{kmol K}} \quad \bar{R}_u = 8.31447 \frac{\text{kJ}}{\text{kmol K}}$$

$$a = 22.26 \quad b = 5.981 \cdot 10^{-2} \quad c = 3.501 \cdot 10^{-5} \quad d = 7.469 \cdot 10^{-9}$$

$$T_1 = 400 + 273.15 \quad T_2 = 40 + 273.15$$

$$\rightarrow u_2 - u_1 = -290.9 \text{ kJ/kg}$$

$$u_2 - u_1 = -501.01 \text{ kJ}$$

$$i_{R_2} = -501.0 - 45.72 = -546.72$$

$$u_2 - u_1 = \frac{1}{M} \int_{400+273}^{40+273} q_v dT = \frac{\text{kmol}}{44.01 \text{ kg}} \int_{673}^{313} (22.26 + 5.9 \times 10^{-2} T - 3.501 \times 10^{-5} T^2 + 7.46 \times 10^{-8} T^3) dT$$

IX p, g

$$= m \int_1^2 (0.45 + 1.67\theta - 1.27\theta^2 + 0.35\theta^3) d\theta - m R (T_2 - T_1)$$

$$= 1000 m \left[0.45(\theta_2 - \theta_1) + \frac{1.67}{2}(\theta_2^2 - \theta_1^2) - \frac{1.27}{3}(\theta_2^3 - \theta_1^3) + \frac{0.35}{4}(\theta_2^4 - \theta_1^4) \right] - R m (T_2 - T_1)$$

$\theta_1 = \frac{400+273}{1000} = 0.673$ $\theta_2 = \frac{40+273}{1000} = 0.313$

$$= -586.0 \text{ kJ}$$

$$\Rightarrow Q_2 = -586 - 45.72 = \underline{-632.5 \text{ kJ}}$$

(a bit different from above, about 1%)

Correct method

Table A.8 p663:
A-20

Ideal gas properties, mass basis

Carbon Dioxide
R = 0.1889

T [K]	u [kJ/kg]	h [kJ/kg]
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A-20
T [K] u

300	7230
320	7526

$u_2 = 7230 + \frac{313.15-300}{320-300} (7526-7230)$
 $= 7323.24 \text{ kJ/kg}$

* kmol
44.01 kg
 $= \frac{7323.24}{44.01} = 166.4 \text{ kJ/kg}$

670	20070
700	20489

300 157.70
350 191.70

Interpolate to $T_2 = 40^\circ\text{C} = 313 \text{ K}$
 $u_2 = 157.7 + \frac{313-300}{350-300} (191.70 - 157.70)$
 $= 166.6 \text{ kJ/kg}$

650 437.71
700 483.97

Interpolate to $T_1 = 400^\circ\text{C} = 673 \text{ K}$
 $u_1 = 437.71 + \frac{673-650}{700-650} (483.97 - 437.71)$

$u_1 = 20070 + \frac{673.15-670}{700-670} (20489 - 20070)$
 $= 20265 \text{ kJ/kmol} = 459.1 \text{ kJ/kg}$

1X10

$$U_2 - U_1 = m(u_2 - u_1) = -5.85 \text{ kJ} - 631.1$$

$$Q_2 = -584.0 - \frac{48.72}{1000} = -630.5 \text{ kJ}$$

(previous method worked pretty well)

end 2/19/06

but skipped

table A6