~~Remainder of Problem 5-57 (3) in All Redux notes~~ ~~XII~~ XI

## 5.5.p 1st law as a rate equation

In any time interval $dt$: $\quad dE = \delta Q - \delta W$

Divide by $dt$: $\quad \dfrac{dE}{dt} = \dot{Q} - \dot{W}$

$\dfrac{dE}{dt}$ : Rate of change of energy of the system.

$$\frac{dE}{dt} = \frac{dU}{dt} + \frac{dKE}{dt} + \frac{dPE}{dt}, \text{ if any}$$

$\dot{Q} = \dfrac{\delta Q}{dt}$ : Rate of heat addition

$\dot{W} = \dfrac{\delta W}{dt}$ : Rate of work done by the system.

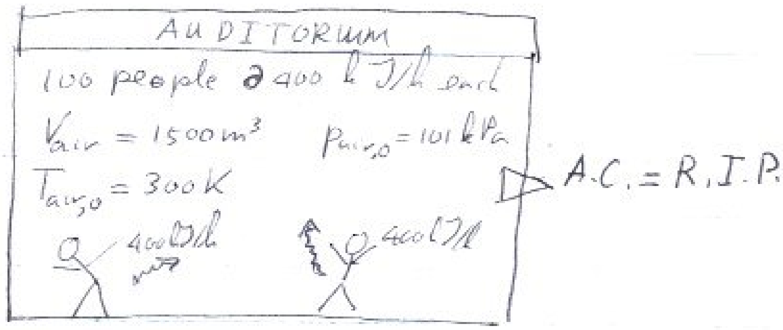
$= P\dfrac{dV}{dt}$ (= power produced by the system.)

Also $\dot{Q} = m c_p \dfrac{dT}{dt}$ isobar

$m c \dfrac{dT}{dt}$ isochor

~~VIII~~ X 2

p 5.110




Asked: air temperature change in degrees/minute.

Answer: 100 people dump $400 \frac{kJ}{h} * 100 = 40,000 \frac{kJ}{h}$ of heat into the air.

The 1st law says this can go into ~~energy~~ internal energy and work

$$\frac{dE_{air}}{dt} = \dot{Q} - \dot{W}_{air} = 40,000 \frac{kJ}{h}$$

but $\dot{W}_{air} = p\dot{V}_{air} = 0$ since $V$ is constant.

Also $$\frac{dE_{air}}{dt} = \frac{dU_{air}}{dt} + \cancel{\frac{d\frac{1}{2}mVel^2_{air}}{dt}}^{\approx 0} + \cancel{\frac{d\,mgZ_{air}}{dt}}^{\approx 0}$$

$$\frac{dU_{air}}{dt} = m c_v \frac{dT}{dt}$$ wanted

$$40,000 \frac{kJ}{h} = m c_v \frac{dT}{dt}$$

$p_{air}V_{air} = m_{air} R T_{air}$   $101 kPa \; 1500 m^3 = m \; 0.287 \frac{kJ}{kg K} \; 300 K$   from ~~A5~~ A-2(a)

$\rightarrow m_{air} = 1759.6 kg$

Should be good enough to use $c_v$ from ~~table A5~~ A-2(a) (at 250 K):
$c_{v0} = 0.717 kJ/kg K$

$$40,000 \frac{kJ}{h} \frac{1 h}{60 \text{ minute}} = 1759.6 kg \; 0.717 \frac{kJ}{kg K} \frac{dT_{air}}{dt}$$

$$\frac{dT_{air}}{dt} = 0.53 K/min = 0.53 °C/min$$

(If pressure is constant $40,000 \frac{kJ}{h} = 1759.6 \; c_p \frac{dT}{dt}$)

# Typical Control Mass Problem Chart

*(Not complete material coverage)*

**State ①**

Normally 3 variables are needed to fully determine it.

2 known intensive variables
(May get away with $T$ only in some approximations, eg, when using saturated values as an approximation for compressed liquids.)

Tables, eg B.1.1-B.1.4; $pv$ or $Tv$ diagrams; $v = v_f + x\,(v_g - v_f)$ and similar for $u$ and $h$.

Ideal gas (applicable?) $pv = RT$ (4 forms) Tables A5-A8 for $u$, $h$.

Make do with the formulae for *differences* in intensive variables listed below under "Process"?

Remaining intensive variables $p, T, v, (x,)u, h$.

Amount of material

$$v = V/m \qquad u = U/m \qquad h = H/m$$

**Process**

**C1:** Type of process ($V$ constant, $p$ constant, $p$ linear in $V$, $pV^n$ constant, $T$ constant, ${}_1Q_2 = 0$?)
Adds info about ① or ② ?

**C2:** Mass: $$m_1\,(+m_{\text{added}}) = m_2$$
Adds info about ① or ② ?
Energy: $$E_2 - E_1 = {}_1Q_2 - {}_1W_2 \quad (E = U + KE? + PE?)$$
Adds info about ① or ② ?

**C3:** ${}_1W_2 = 0$ | $p_1(V_2 - V_1)$ | $\dfrac{p_1 + p_2}{2}(V_2 - V_1)$ | $\dfrac{p_2V_2 - p_1V_1}{1-n}$ | $p_1V_1 \ln\left(\dfrac{V_2}{V_1}\right)$ | other?

For ideal gases:

$$u_2 - u_1 = \int_1^2 C_v\,\mathrm{d}T \approx C_{v_{\text{ave}}}(T_2 - T_1) \qquad h_2 - h_1 = \int_1^2 C_p\,\mathrm{d}T \approx C_{p_{\text{ave}}}(T_2 - T_1)$$

For solids and compressed liquids, *by approximation*, best at constant pressure:

$${}_1Q_2 = m\int_1^2 C_{(p)}\,\mathrm{d}T \approx mC_{(p)_{\text{ave}}}(T_2 - T_1)$$

**State ②**

Same procedures as state ①

$\boxed{\text{Review Chapter 5}}$ $\boxed{E_2 - E_1 = {}_1Q_2 - {}_1W_2}$ $E = U + KE + PE$

$KE = \frac{1}{2} m Vel^2$

$PE = mgZ$

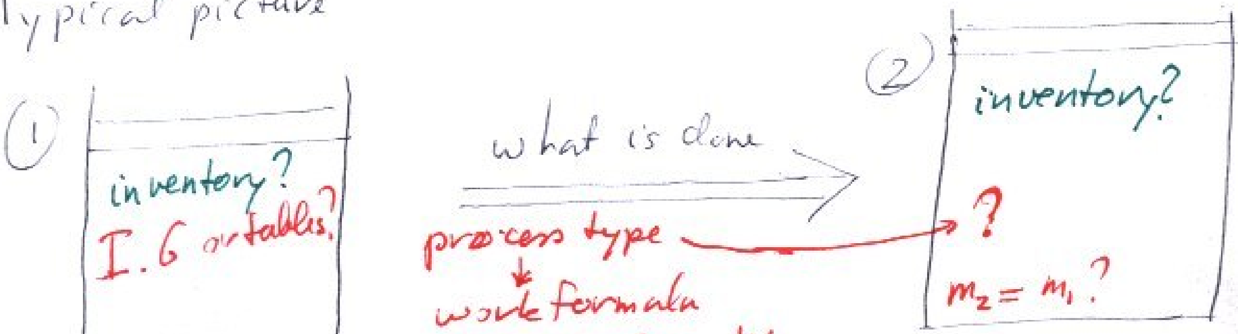
- know your tables, read headers
- use units
- have the correct formulae

$u$ much like $v$: $U = mu$ $\boxed{\begin{array}{l} u = u_f + x(u_g - u_f) \\ h = h_f + x(h_g - h_f) \end{array}}$ diagrams

$h$ " " " $H = mh$ "

Be systematic. $\underbrace{\text{3 variables}}_{}$ (independent) for your substance is ideal

Typical picture




$U_2 - U_1 = {}_1Q_2 - {}_1W_2$

$\boxed{\text{Insulated means } {}_1Q_2 = 0}$, $\boxed{\text{Isolated means } {}_1W_2 = 0 \text{ and } {}_1Q_2 = 0}$

pure saturated liquid $\rightarrow x = 0$

" " vapor $\rightarrow x = 1$

~~Table A.5~~

Multistep = 2 questions for the price of one.

Compressed liquids: may have to use saturated values at right T and wrong P if no compressed liquid table

$\boxed{u, h, c_v, c_p = u, h, c_v, c_p (T)}$

A2(a), A2(b) A2(c), ~~A17-A25~~

Table A.5 / A.6 / A7.1 or A.8

better

I.G $\boxed{\begin{array}{l} u_2 - u_1 \approx c_{v\,ave}(T_2 - T_1) \\ h_2 - h_1 \approx c_{p\,ave}(T_2 - T_1) \end{array}}$

Solids, simple liquids: $\boxed{{}_1Q_2 = m c_p (T_2 - T_1)}$

rate form Eq. $\boxed{\dot{U} = \dot{Q} - \dot{W} \quad \dot{W} = P\dot{V}}$

## Compressed liquids if you have saturated, but not liquid tables

Use the saturated values at the right temperature.

This works best if you do it for $u$ and $v$, instead of $h$: $h = u + pv \approx u_{SAT} + p v_{SAT}$

Numerical example:

$H_2O$ @ 30,000 kPa and 100 °C

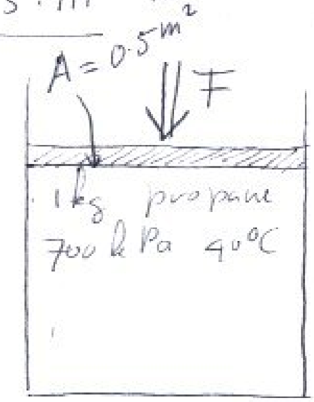
| | | |
|---|---|---|
| $v_{SAT} = 0.001044$ | $v_{exact} = 0.001029$ | 1% error: good |
| $u_{SAT} = 419$ | $u_{exact} = 411$ | 2% error: good |
| $h_{SAT} = 419$ | $h_{exact} = 442$ | 5% error: OK |
| $u_{SAT} + pv_{SAT} = 450$ | " " | 2% error: good |

2/16/06 end → but also did A6 in p.5.97

Nal dom

X 5

P 5.111 (nal dom 06)



$pV^{-2} = p_1 V_1^{-2} = p_2 V_2^{-2} = C$

$n = -2$

heat

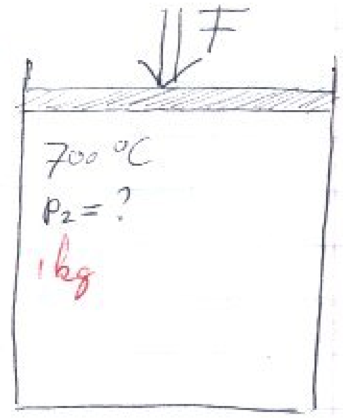
$F \div V^2$

${}_1W_2 = ?$

${}_1Q_2 = ?$

$U_2 - U_1 = {}_1Q_2 - {}_1W_2$

${}_1W_2 = \frac{p_2 V_2 - p_1 V_1}{1-n}$



Answer: Translate terms: $F \div V^2 \Rightarrow F = \text{const}\ V^2$

Now $F = pA$, so $p = \frac{F}{A} = \frac{\text{const}}{A} V^2 = C V^2$

(C another constant). So $pV^{-2} = C$ : polytropic with $n = -2$.

Propane: no "B" tables, but $p_c \approx 42$ bar; $T_c \approx$ ~~370~~ K: marginally an ideal gas. Not in table A.8, but is in table A.6 : $U_2 - U_1 = m c_{v_{ave}} (T_2 - T_1)$

"substance SUV"

A3(B), A17–A25

A2(c)

from A5

Find $V_1$: $p_1 V_1 = m R T_1$ $\quad 700\ \text{kPa}\ V_1 = 1\ \text{kg}\ 0.1886 \frac{\text{kJ}}{\text{kg K}} (40+273)\text{K}$

$\Rightarrow V_1 = 0.08433\ m^3$

Find $p_2$ or $V_2$.

I have $p_2 V_2^{-2} = p_1 V_1^{-2}$ but know neither $p_2$ nor $V_2$

I also have $p_2 V_2 = mRT_2$ " " " " " "

It is a 2 equations in 2 unknowns problem!

$p_2 V_2^{-2} = 700\ \text{kPa} / (0.08433\ m^3)^2 = 98431\ \text{kPa}/m^6$

$p_2 V_2 = 1\ \text{kg}\ 0.1886 \frac{\text{kJ}}{\text{kg K}} (700+273)\text{K} = 183.5\ \text{kJ}$

The trick to solve is to "solve" one of the equations for one of the unknowns and then to plug that into the other equation:

$$p_2 V_2 = 183.5\,kJ \Longrightarrow p_2 = \frac{183.5\,kJ}{V_2} \xrightarrow[\text{other eq}]{\text{plug in}}$$

$$\frac{183.5\,kJ}{V_2} V_2^{-2} = 98431\,kPa/m6 \ \cancel{98431}$$

$$\frac{183.5\,kJ}{V_2^{3}} = \frac{98431\,kJ}{m9}$$

$$V_2^3 = \frac{183.5}{98431}\,m9 \qquad V_3 = \left[\frac{183.5}{98431}\right]^{\frac{1}{3}} m^3$$

$$V_2 = 0.123\,m^3$$

Now we can find $p_2$

$$p_2 = \frac{183.5\,kJ}{V_2} = \underline{\underline{1491\,kPa}}$$

~~Now use 1st law~~ $\cancel{U_2 - U_1 = {}_1W_2}$

Find work now $\cancel{{}_1W_2 = \frac{p_2V_2 - p_1V_1}{1-n} = 1491\,kPa\,0.123\,m^3 - 700\,kPa}$

$${}_1W_2 = \frac{p_2V_2 - p_1V_1}{1-n} = \frac{1491\,kPa\ 0.123\,m^3 - 700\,kPa\ 0.08433\,m^3}{1-(-2)}$$

$$= \underline{\underline{41.45\,kJ}}$$

Find heat now from 1st law

$$U_2 - U_1 = {}_1Q_2 - {}_1W_2 \qquad U_2 - U_1 = mc_{v\,ave}(T_2 - T_1)$$

Use ~~table~~ A3(c) ~~A6~~ p 654 to compute $c_{v\,ave}$

$$T_{ave} = \frac{700+40}{2} = 370°C = 643\,K = 0.643\,kK$$

$$c_{p0} = -0.096 + 6.95\ 0.633 - 3.6\ 0.633^2 + 0.73\ 0.633^3$$
$$= 3.046\,kJ/kg\,K$$

over.

$$\bar{c}_{p_{ave}} = -4.04 + 30.48\,10^{-2}\,643 - 15.72\,10^{-5}\,643^2 + 31.74\,10^{-9}\,643^3 = 135.39\ \frac{kJ}{kmol\,K}$$

X 7

$$c_{p_{ave}} = \frac{\bar{c}_p}{M} = \frac{135.39\ kJ/kmol\,K}{44.097\ kg/kmol} = 3.07\ \frac{kJ}{kg\,K}$$

not 1.4909 like A2(a) says!

$$c_{v_{ave}} = c_{p_{ave}} - R = (\overset{3.07}{3.046} - \overset{.1885}{0.1886})\ \frac{kJ}{kg\,K} = \overset{2.882}{2.857}\ \frac{kJ}{kg\,K}$$

$$U_2 - U_1 = 1\,kg\ \ 2.857\ \frac{kJ}{kg\,K}\ (700-40)K = 1886\ kJ$$

(Using cv from A.5 would have given 983 kJ!!)

$${}_1Q_2 = U_2 - U_1 + {}_1W_2 = 1886\ kJ + 41.45\ kJ = \underline{1927\ kJ}$$