

## Chapter 8: Entropy

### Properties so far and their meanings

V: volume

M: mass

P: force per unit area on the containing walls

T: "kinetic energy of the molecules" / ~~temperature~~ <sup>T.C</sup> ~~temperature~~ <sup>usage</sup> to give up heat

U: internal energy

Derived:  $v, p, x, u, H, h$

### Definition of "entropy" S

The difference in entropy between two states (1) and (2) is

$$S_2 - S_1 \equiv \int_{\text{any reversible process}}^2 \frac{\delta Q}{T}$$



Units:  $\text{kJ/K}$

Note that only differences in entropy are defined by this. The state for which  $S=0$  must be found another way. But normally only differences in S are of importance.

What entropy is physically is not very important either. (It is a measure of the internal disorder of the substance)

$$\text{Isothermal reversible: } T(S_2 - S_1) = Q_2$$

Since adiabatic means no heat added/removed:

$$\text{reversible adiabatic processes are isentropic}$$

$$S_2 = S_1$$

Since the heat exchanges in a Carnot cycle are isothermal

and the compressor and turbine were reversible adiabatic.

Carnot is two isotherms and two isentropes

Differential form:

$$dS = \frac{\delta Q}{T} \text{ if reversible}$$

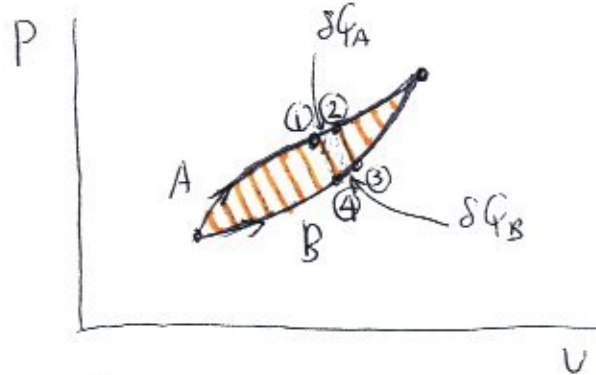
$$dS > \frac{\delta Q}{T} \text{ if irreversible (and } T \text{ is still meaningfully defined)}$$

For adiabatic systems ( $\delta Q=0$ ), entropy always increases  
Justification of the above claims

You can define whatever you want, but the claim that  $\int_{\text{①}}^{\text{②}} \frac{\delta Q}{T}$  is the same for all reversible processes from ① to ② must be justified. And so that  $dS > \frac{\delta Q}{T}$  for irreversible processes.

Let's compare two different processes, A and B, that lead from ① to ②:

heat pump ④③②①  
 need  $(\delta Q)_{\text{rev}} < (\delta Q)_{\text{irr}}$   
 so that  $W_{\text{in}}$  increases  
 → less efficient as a heat pump



— isentropes

heat pump ④③②①  
 $(\delta Q)_{\text{rev}} < (\delta Q)_{\text{irr}}$  or work in would reduce, more it

Also draw the isentropes. Now look at a piece ①②③④ of the integration paths picture. It is a Carnot cycle!

$$\text{so: } \frac{\delta Q_A}{\delta Q_B} = \frac{T_A}{T_B} \Rightarrow \frac{\delta Q_A}{T_A} = \frac{\delta Q_B}{T_B} \Rightarrow \int_A^{\text{②}} \frac{\delta Q}{T} = \int_B^{\text{②}} \frac{\delta Q}{T}$$

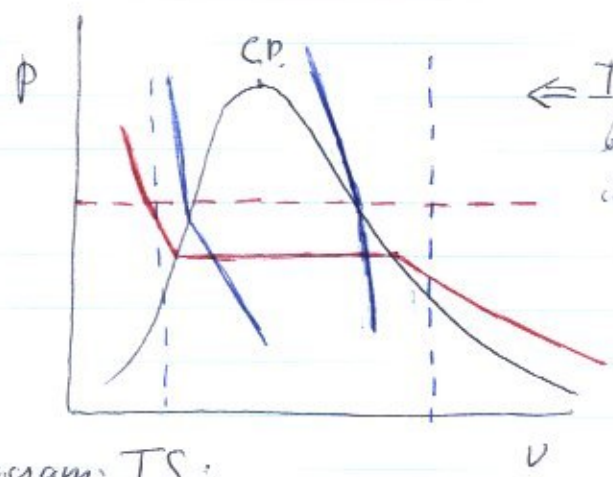
leave: B does not work since ③-④ is wrong way around

If process A is irreversible however,  $|\delta Q_A|$  better be <sup>greater</sup> less than  $|\delta Q_{A,\text{rev}}|$  or a heat engine running ①②③④ would have an efficiency greater than Carnot (since relatively less heat is wasted)  
 $\Rightarrow \delta Q_A / T_A < dS_A$

S.P.3 Using tables, now including  $s$

Two phase :  $s = s_f + x(s_g - s_f)$

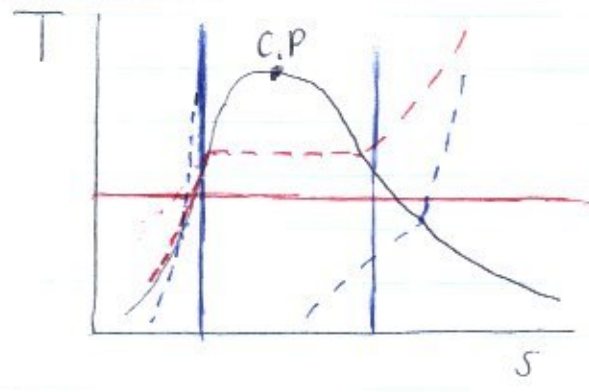
For compressed liquids, can use as an approximation the saturated liquid value at the same temperature if no better



⇐ Isotherms go down like isotherms but more steeply, and including in the two-phase region

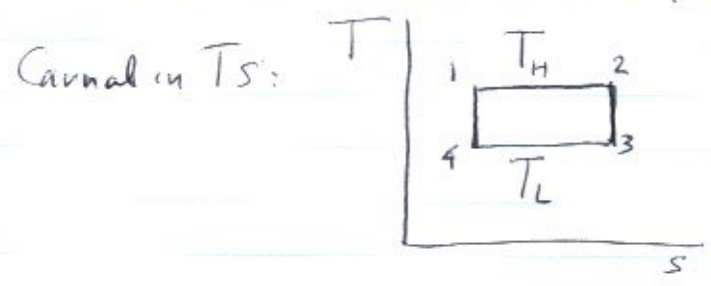
- Typical iso lines:
- isotherms
  - - - isobar
  - isotherms (2)
  - - - isochores (2)

New diagram:  $TS$ :



⇐ Isotherms similar to  $T$ -plane. Reversible adiabatic processes are vertical, isothermal ones horizontal.

Note: area below the curve in  $p$ - $v$  is work  
area below the curve in  $T$ - $s$  is reversible heat



P 2.26a:  $H_2O, 250^\circ C, 0.02 \text{ m}^3/\text{kg}$ . Asked  $s, T-s, (p-u)$

skip if needed

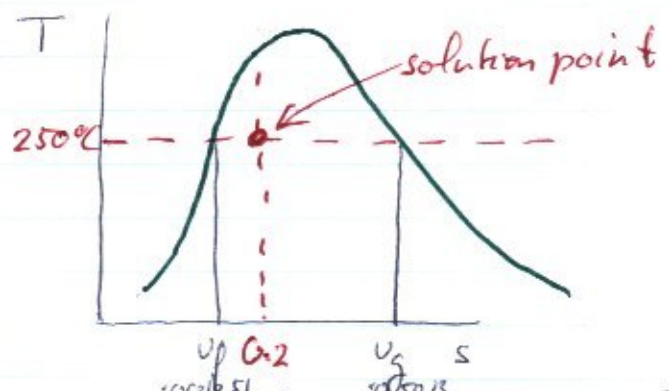
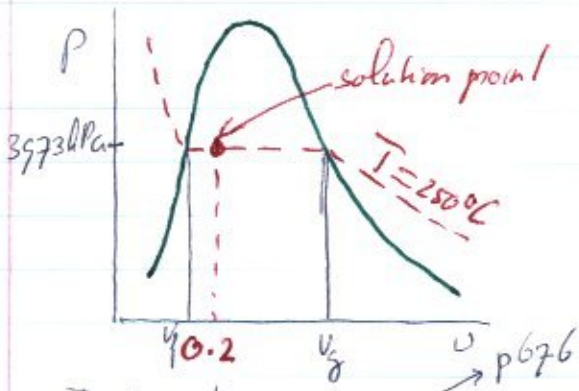
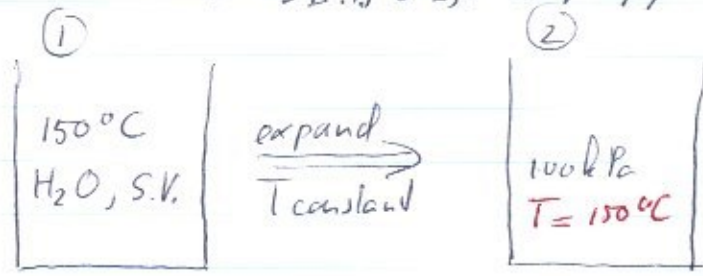


Table B.1.1 @  $250^\circ C$ :  $v_f = 0.001251 \text{ m}^3/\text{kg}$ ,  $v_g = 0.05013 \text{ m}^3/\text{kg}$   
 Given  $v = 0.02$  in between these two  $\rightarrow$  two phase.  
 $0.02 = 0.001251 + x(0.05013 - 0.001251) \Rightarrow x = 0.3836$   
 $s = s_f + x s_{fg} = 2.7927 + 0.3836(3.2802) = 4.0509 \text{ kJ/kg-K}$   
 $\rightarrow$  B.1.1 @  $250^\circ C, p = 3977$

P 2.29a

skip if needed



Asked:  $u_2 - u_1, s_2 - s_1$

Table B.1.1 @  $150^\circ C$  (p.674, 675)  $u_1 = u_g = 2559.54 \text{ kJ/kg}$ ;  $s_1 = s_g = 6.8378 \text{ kJ/kg-K}$

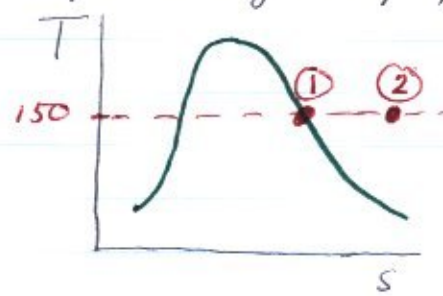
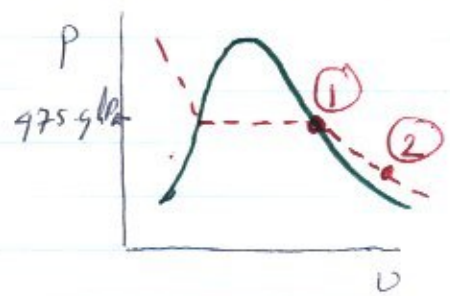


Table B.1.3 @  $100 \text{ kPa}, 150^\circ C$  (p.682)  $u_2 = 2502.75 \text{ kJ/kg}$ ,  $s_2 = 7.6133 \text{ kJ/kg-K}$

$u_2 - u_1 = 2502.75 - 2559.54 = -56.79 \text{ kJ/kg}$   
 $s_2 - s_1 = 7.6133 - 6.8378 = 0.7755 \text{ kJ/kg-K}$

P0.30d R-134a  $50^\circ\text{C}$   $s = 1.9 \text{ kJ/kg-K}$   
 Asked:  $p, x$

Skip  
if needed

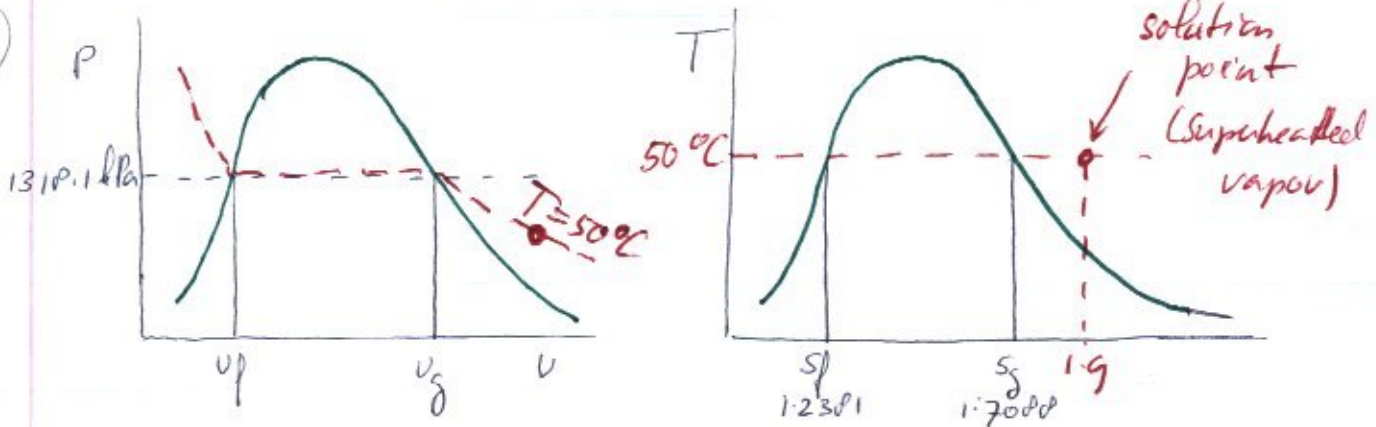


Table B.5.1 @  $50^\circ\text{C}$  ( $p_{700}, 709$ ):  $p_s = 1310.1$   $s_f = 1.2301$   $s_g = 1.7000 \frac{\text{kJ}}{\text{kg-K}}$   
 Since  $s = 1.9 > 1.7000$ : superheated vapor

Table B.5.2 @  $50^\circ\text{C}$ , interpolated between  $p = 200 \text{ kPa}$  ( $s = 1.9117 \frac{\text{kJ}}{\text{kg-K}}$ )  
 and  $p = 300 \text{ kPa}$  ( $s = 1.8755 \frac{\text{kJ}}{\text{kg-K}}$ ) ( $p_{710}, 711$ ):

$$p = 200 + \frac{1.9 - 1.9117}{1.8755 - 1.9117} (300 - 200) \text{ kPa} = \underline{\underline{232.3 \text{ kPa}}}$$

x undefined

Note that  $p-v$  diagram is useless here (I don't know either  $p$  nor  $v$ )

3/28/8

Chapter 5 type problems (S & 4): Expressions for heat added  
~~signs and solids entropy.~~

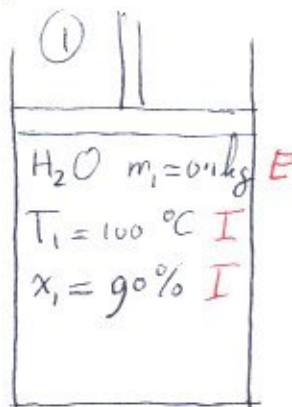
I may now have formulae for the heat added, instead of, or in addition to, the chapter 4 work formulae:

$$S_2 - S_1 = m(s_2 - s_1) = \int_1^2 \frac{dQ}{T} \text{ if reversible}$$

reversible adiabatic = <del>isenthalpic</del> = isentropic; $S_2 = S_1$	(and $Q_2 = 0$ )
reversible isothermal $mT(s_2 - s_1) = {}_1Q_2$	(and $T_2 = T_1$ )

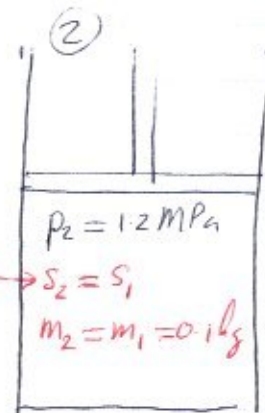
reversible adiabatic (isentropic)	${}_1Q_2 = 0$	$S_2 = S_1$
reversible isothermal	${}_1Q_2 = mT(s_2 - s_1)$	$T_2 = T_1$

p 0.431



compress insulated } isentropic  
 (reversible) }  $Q_2 = 0$

$W_2 = ?$   
 $u_2 - u_1 = Q_2 - W_2$   
 no work formula, but had formula:  $Q_2 = 0$

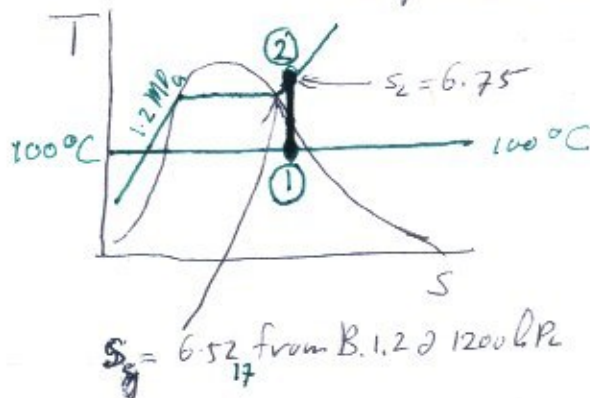
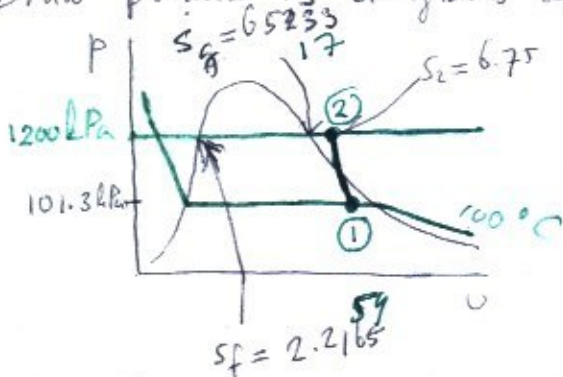


Asked  $P, T, s, Q_2, W_2$   
 Answer:

Find relevant quantities for state ① in B.1.1 (saturated water) @  $100^\circ\text{C}$   
 (p 674, 675):

$p_1 = 101.3 \text{ kPa}$   
 $u_1 = 410.61 + 0.9 \cdot 2087.5 = 2297.73 \text{ kJ/kg}$   
 $s_1 = 1.3068 + 0.9 \cdot 6.0470 = 6.7500 \text{ kJ/kg K} = s_2$

Draw  $p-v$  and  $T-s$  diagrams of the process, and find phase 2 in them:



Find  $u_2$  from table B.1.3 @  $1200 \text{ kPa}$  and  $s = 6.75$ : must interpolate between  $200^\circ\text{C}$  ( $s = s_1 = 6.5909 \text{ kJ/kg K}$  and  $u = u_1 = 2612.74 \text{ kJ/kg}$ ) and  $250^\circ\text{C}$  ( $s = s_2 = 6.8293 \text{ kJ/kg K}$  and  $u = u_2 = 2704.20 \text{ kJ/kg}$ ):

$$u_2 = 2612.74 + \frac{6.7500 - 6.5909}{6.8293 - 6.5909} (2704.20 - 2612.74) = 2673.52 \text{ kJ/kg}$$

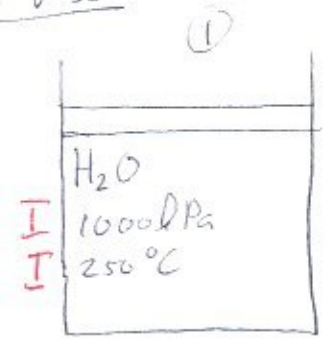
1st law:  $u_2 - u_1 = \overset{\text{insulated}}{\cancel{q_2}} - W_2$        $u_1 = mu_1$      $u_2 = mu_2$

$$0.1 \text{ kg} (2673.52 - 2257.73) \frac{\text{kJ}}{\text{kg}} = -W_2$$

$$W_2 = -37.62 \text{ kJ} \quad (\text{work done on the water})$$



p 0.53

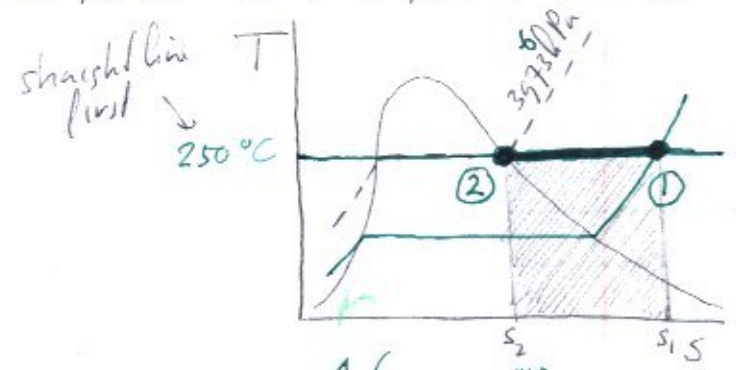
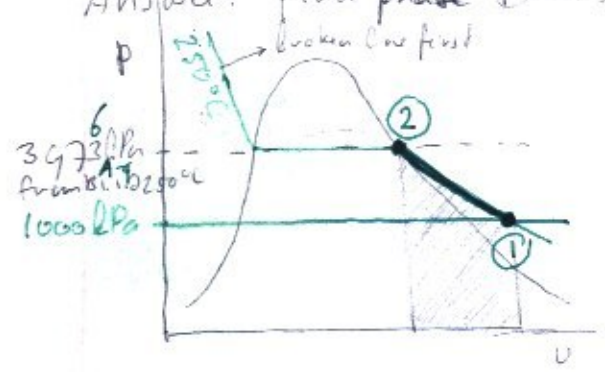


isothermal  
no work formula, but heat  
 $w_2 = ?$  ,  $q_2 = ?$   
 $u_2 - u_1 = q_2 - w_2 \rightarrow u_2 - u_1 = q_2 - w_2$   
 $q_2 = mT(s_2 - s_1) \rightarrow q_2 = T(s_2 - s_1)$



Asked: a)  $w_2$ ,  $q_2$ , and b) compare these with graphical estimates in the p-v and T-s diagrams

Answer: find state ① in either p-v or T-s. Also put in state ②



Either way, ① is superheated vapor. Table B.1.3 @ 1000 kPa, 250°C (p 6.03):  $s_1 = 6.5246 \text{ kJ/kg} \cdot \text{K}$ ,  $u_1 = 2709.31 \text{ kJ/kg}$ ,  $v_1 = 0.23260 \text{ m}^3/\text{kg}$

For ②, table B.1.1 @ 250°C:  $p_2 = 3973 \text{ kPa}$ ,  $s_2 = s_g = 6.0729 \text{ kJ/kg} \cdot \text{K}$   
 $u_2 = u_g = 2602.37 \text{ kJ/kg}$ ,  $v_2 = v_g = 0.05013 \text{ m}^3/\text{kg}$

2nd law:  $q_2 = T(s_2 - s_1) = (250 + 273) \text{ K} (6.0729 - 6.5246) \text{ kJ/kg} \cdot \text{K}$   
 $= -495.49 \text{ kJ/kg} = \text{exactly the shaded area in the T-s diagram (height T, width } s_2 - s_1)$

1st law:  $w_2 = q_2 - (u_2 - u_1) = -495.49 \text{ kJ/kg} - (2602.37 - 2709.31) \text{ kJ/kg}$   
 $= -337.50 \text{ kJ/kg} = \text{shaded area in p-v diagram}$

reached  
3/23/06  
(Dandy)

The exact work under the curve on the  $p-v$  diagram is  ${}_1w_2 = -337.9 \frac{\text{J}}{\text{kg}}$ .  
But we want to estimate it graphically, without using tables, for  $u$  and  $s$ , (i.e. without using the 1st law)

If we assume the line is straight, then  $p$  linear in  $v$ , so  
 ${}_1w_2 \approx \frac{p_1 + p_2}{2} (v_2 - v_1) = -453.9$  lousy! 34% error!

If we assume the line is hyperbolic, i.e.  $pv = \text{constant}$   
(which means we approximate it as an isothermal ideal gas):

$${}_1w_2 \approx p_1 v_1 \ln \frac{v_2}{v_1} = -357.17 \text{ better. } 6\% \text{ error}$$

Alternatively:

$${}_1w_2 \approx p_2 v_2 \ln \frac{v_2}{v_1} = -305.71 \quad \text{10\% error.}$$

If we average these results, or alternatively, use  $\frac{p_1 v_1 + p_2 v_2}{2} \ln \frac{v_2}{v_1}$

$${}_1w_2 \approx \frac{-357.17 - 305.71}{2} = -331.4 \text{ good! } 2\% \text{ error.}$$