S9.3 Work formulae for a C.V.

Work formulae for a C.M $\quad {}_1w_2 = \int_1^2 P\,dv$

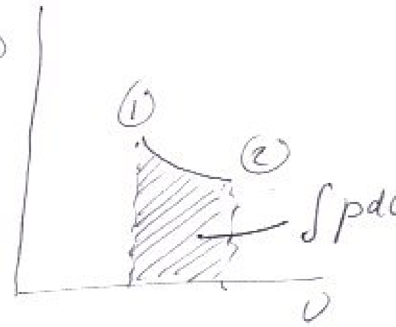
$${}_1w_2 = 0 \quad \text{isochoric}$$

$${}_1w_2 = P(v_2 - v_1) \quad \text{isobaric}$$

$${}_1w_2 = \frac{P_2 + P_1}{2}(v_2 - v_1) \quad P \text{ linear in } v$$

$${}_1w_2 = Pv \ln \frac{v_2}{v_1} \quad \text{polytropic } n = 1$$

$${}_1w_2 = \frac{P_2 v_2 - P_1 v_1}{1-n} \quad \text{polytropic } n \neq 1$$



Work for a C.V.? ~~Assume SEE + reversible~~ Assume SEE

1st law $\quad q + h_1 + \tfrac{1}{2}Vel_1^2 + gZ_1 = w + h_2 + \tfrac{1}{2}Vel_2^2 + gZ_2$

$$w = q + h_1 - h_2 + \underbrace{\tfrac{1}{2}Vel_1^2 - \tfrac{1}{2}Vel_2^2 + gZ_1 - gZ_2}_{\text{often negligible}}$$

2nd law: assume reversible

$$dq = T\,ds = dh - v\,dP \quad \text{(Gibbs / thermo property relations)}$$

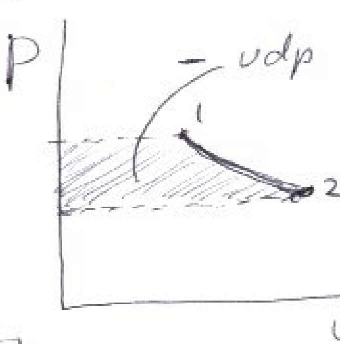
$$q = \int_1^2 dq = h_2 - h_1 - \int_1^2 v\,dp$$

plug in

$$\boxed{w = \underbrace{-\int_1^2 v\,dp}_{\text{turbine}} + \overbrace{\underbrace{\tfrac{1}{2}Vel_1^2 - \tfrac{1}{2}Vel_2^2}_{\text{net kinetic energy in}}}^{\text{windturbine}} + \overbrace{\underbrace{gZ_1 - gZ_2}_{\text{net pot energy in}}}^{\text{waterwheel}}}$$

if SEE + reversible

often ignored



$$-\int_1^2 v\,dp = 0 \quad \text{if isobaric}$$

$$-\int_1^2 v\,dp = v(p_1 - p_2) \quad \text{if isochoric}$$

$$-\int_1^2 v\,dp = \frac{n(P_2 v_2 - P_1 v_1)}{1-n} \quad \text{polytropic } n \neq 1$$

$$-\int_1^2 v\,dp = -Pv \ln \frac{P_2}{P_1} \quad \text{polytropic } n = 1$$

$Pv \ln \frac{v_2}{v_1}$? yes

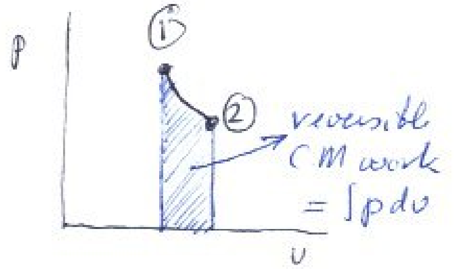
Turbine: pressure decreases Waterwheel: work from potential energy

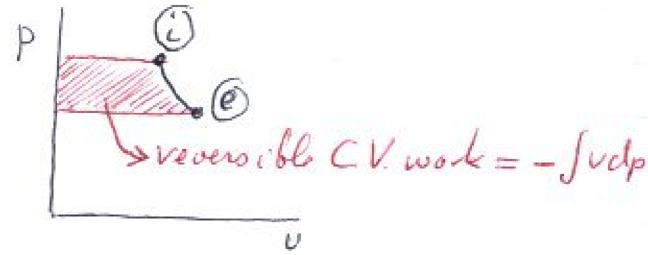
Sg.3 Work done by the substance inside a C.V.

$$\boxed{\begin{array}{c}\underline{\text{If}}\text{ reversible and SEE}\\ -\int_i^e v\,dp = w + \tfrac{1}{2}Vel_e^2 - \tfrac{1}{2}Vel_i^2 + gZ_e - gZ_i\end{array}}$$

- $-\int_i^e v\,dp$: work done by the substance inside (i.e. not on the boundary of the C.V.
- w: work coming out of the C.V
- $\tfrac{1}{2}Vel_e^2 - \tfrac{1}{2}Vel_i^2$: net kinetic energy coming out of the C.V
- $gZ_e - gZ_i$: net potential energy coming out of the C.V.

Net kinetic and net potential energy: Can often be ignored





Proof: Gibbs: $\int_i^e T\,ds = \int_i^e \delta q = \int_i^e dh - \int_i^e v\,dp$ so

$q = h_e - h_i - \int_i^e v\,dp$: Plug in 1st law. QED.

skip for now

$$\boxed{\begin{array}{ll}-\int_i^e v\,dp = 0 & \text{if constant pressure (isobaric)}\\ -\int_i^e v\,dp = v(p_i - p_e) & \text{if incompressible } (v = \text{constant})\\ -\int_i^e v\,dp = \dfrac{n(p_e v_e - p_i v_i)}{1-n} & \text{if polytropic, } n \neq 1\\ -\int_i^e v\,dp = -pv \ln \dfrac{p_e}{p_i} & \text{if polytropic, } n = 1\end{array}}$$

Pg 77

$\dot{W}_{in} = 3\,kW = -\dot{W}$

liquid H_2O (ambient)
100 kPa, 25°C →
0.5 kg/s

pump reversible adiabatic

$\cancel{\dot{Q}} + \dot{m}(h_i + \cancel{\tfrac{1}{2}Vel_i^2 + gz_i})$
$= \dot{W} + \dot{m}(h_e + \cancel{\tfrac{1}{2}Vel_e^2 + gz_e})$

$w = \dot{W}/\dot{m} = -\int_i^e v\,dp = -v(p_e - p_i)$

if we assume incompressible

→ 0.5 kg/s
$p_e = ?$
$T_e = ?$
$\begin{pmatrix} s_e = s_i \\ h_e = \cdots \end{pmatrix}$
from 1st law
NOT

Answer: work formulae

$$w = \dot{W}/\dot{m} = -v(p_e - p_i)$$

table A4 [A-3a]: $v = \frac{1}{\rho} = \frac{1}{997}\,\frac{m^3}{kg}$ $\quad c_p = 4.18\,\frac{kJ}{kg\,K}$

$$-3\,kW / 0.5\,kg/s = -\frac{1}{997}\,\frac{m^3}{kg}\,(p_e - 100\,kPa)$$

$$\Rightarrow p_e = \underline{6{,}082\,kPa}$$

1st law $\quad \dot{m} h_i = \dot{W} + \dot{m} h_e$

$$3\,kW = \dot{m}(h_e - h_i) = \dot{m}\,c_{p\,ave}(T_e - T_i)$$

$$= 0.5\,kg/s\;4.18\,\frac{kJ}{kg\,K}\,(T_e - 25\,°C)$$

$$\rightarrow T_e = \underline{26.43\,°C}$$

Right answer: The result for ~~T_e~~ the temperature change $T_e - T_i$ is not accurate. For this small temperature difference, the effect of the large pressure difference on h cannot be ignored. Better:

1st law: $h_e = h_i - \frac{\dot{W}}{\dot{m}}$ Table B.1.1 for $h_i = u_i + p_i v_i = 104.56\,\frac{kJ}{kg}$ [A-4 h_f: 104.83] (B.1.1 @ 25°C)

$$h_e = h_i - \frac{\dot{W}}{\dot{m}} = 104.56\,\frac{kJ}{kg} + \frac{3\,kW}{0.5\,kg/s} = 110.56 \quad [110.83]$$

Table B.1.4 [A-7] @ 5 MPa and 110.56 [110.83] kJ/kg interpolated: $T_e = \underline{25.36}$ [25.33]

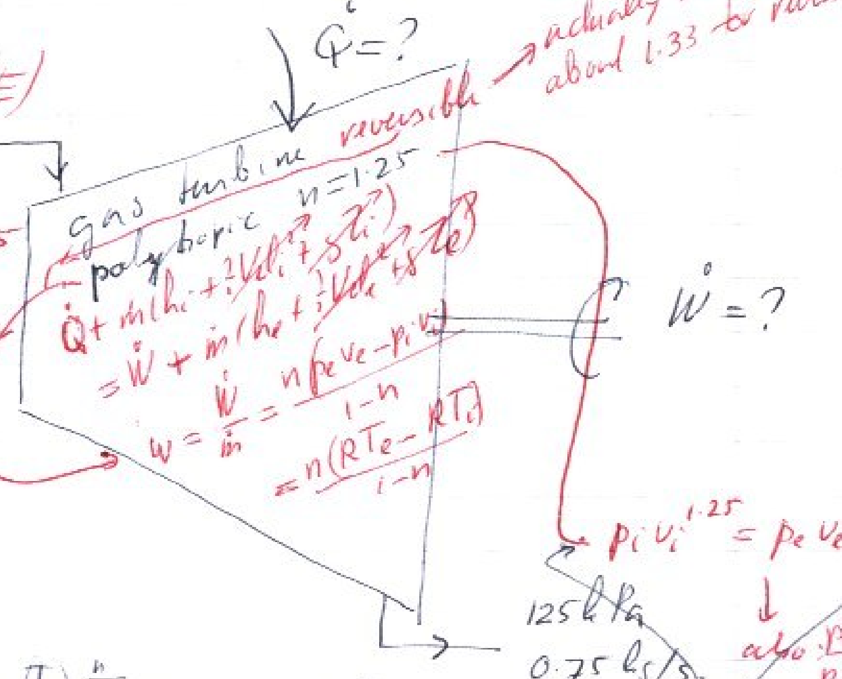
Table B.1.4 @ 6.082 MPa and 110.56 kJ/kg doubly interpolated: $T_e = \underline{25.11}$

Note: to do this "exactly": $h_e = 110.56$ kJ/kg
$s_e = s_i = .3673$ kJ/kg,K } nightmare to interpolate

P9.94

(I) air 0.75 kg/s (E)
1200 K 800 kPa (I)

$R = .287 \frac{kJ}{kg K}$ from A-5



$\dot Q + \dot m(h_i + \frac{1}{2}V_i^2 + g z_i) = \dot W + \dot m(h_e + \frac{1}{2}V_e^2 + g z_e)$

$w = \frac{\dot W}{\dot m} = \frac{n(p_e v_e - p_i v_i)}{1-n} = \frac{n(RT_e - RT_i)}{1-n}$

$p_i v_i^{1.25} = p_e v_e^{1.25}$

also $\frac{p_e}{p_i} = \left(\frac{T_e}{T_i}\right)^{\frac{n}{n-1}}$

Answer: Lets use $\frac{p_e}{p_i} = \left(\frac{T_e}{T_i}\right)^{\frac{n}{n-1}}$ to compute T_e, (instead of for v_e) just to be original:

$$\left(\frac{p_e}{p_i}\right)^{\frac{n-1}{n}} = \frac{T_e}{T_i} \qquad T_e = T_i\left(\frac{p_e}{p_i}\right)^{\frac{n-1}{n}} = 1200\,K\left(\frac{125\,kPa}{800\,kPa}\right)^{\frac{.25}{1.25}} = 827.94\,K$$

work formula

$$\dot W = \frac{\dot m\, nR(T_e - T_i)}{1-n} = \frac{.75\,\frac{kg}{s}\ 1.25 \cdot .287\,\frac{kJ}{kg K}(827.94 - 1200)K}{1-1.25}$$

$= \underline{400.5\ kW}$

1st law: A-17 @ 1200 K: $h_i = 1277.79\ \frac{kJ}{kg}$

@ 827.94 K: $h_e = 852.64\ \frac{kJ}{kg}$ (interpolated)

$\dot Q + \dot m h_i = \dot W + \dot m h_e$

$\dot Q = -.75\,\frac{kg}{s}\ 1277.81\,\frac{kJ}{kg} + 400.5\,kW + .75\,\frac{kg}{s}\ 852.53\,\frac{kJ}{kg}$

$= \underline{81.84}$ kW goes in ??