EML 5709 Fall 2007
Homework Problems

Do not print out this page. Keep checking for changes. Homeworks should normally be online by the Saturday before they are due.

Explain all reasoning.

1 9/3

1. If the density of air at sea level is 1.225 kg/m^3, what is the average spacing of the molecules? The molecular mass of air is 28 g/mol. For what size of bodies would you expect major problems in trying to define continuum values of density, velocity, and pressure?

   If the free path length of air at sea level is 6.6 10^{-8} m, at what size would you expect that normal equations of motion (like the Euler and Navier-Stokes equations) become unusuable? When the body size becomes comparable to the mean molecular spacing or to the mean free path length?

   Repeat for 200 km height, where the number of molecules is 8 10^{15}/m^3 and the free path length 200 m. Can you define a continuum air velocity and density for the flow around a rocket? Can you use continuum equations?

2. Is a rain droplet in saturated air a Lagrangian region / material region / control mass? How about a droplet in dry air? Explain.

3. For ideal stagnation point flow, what is the relation between the position vector $\vec{r}$ and the acceleration vector $\vec{a}$? Graphically show, by drawing a pathline in the first quadrant and acceleration vectors at points on that line, that at the point where $x = y$, the acceleration is all centripetal acceleration. Also show graphically that for $x < y$ the acceleration has a tangential component that slows the fluid down, while for $x > y$, the fluid speeds up again.

4. For ideal stagnation point flow, find the pressure in terms of x and y from the Bernoulli law (ideal stagnation point flow is inviscid, steady, and all streamlines have the same stagnation pressure). Now verify that the force $\rho\vec{a}$ per unit volume equals $-\nabla p$ where ∇p is the pressure gradient $(\partial p/\partial x, \partial p/\partial y)$. It is true for any *inviscid* flow that minus the pressure *gradient* gives the net force per unit volume on the particles.

5. A velocity field is given by $\vec{v} = \widehat{\imath}\cos t + \widehat{\jmath}\sin t$. Is this a steady flow? Find the particle paths and draw a few of them. Find the streamlines and draw planes of streamlines at a couple of times. Find the streakline coming from a smoke generator at the origin that is turned on at time $t = 0$; sketch the streakline at time $t = \pi$.

6. If the surface temperature of a river is given by $T = 2x + 3y + ct$ and the surface water flows with a speed $\vec{v} = \widehat{\imath} - \widehat{\jmath}$, then what is c assuming that the water particles stay at the same temperature? (Hint: $\mathrm{D}T/\mathrm{D}t = 0$ if the water particles stay at the same temperature.)

   A boat is cornering through this river such that its position is given by $x_b = f_1(t)$, $y_b = f_2(t)$. What is the rate of change $\mathrm{d}T/\mathrm{d}t$ of the water temperature experienced by the boat in terms of the functions f_1 and f_2?

2 9/10

1. A steady stream of air enter a pipe with a diameter of 1” at a velocity of 20 m/s at a pressure of 1 bar. The pipe has a contraction in diameter to $\frac{1}{2}$”. What are the velocity and pressure after the contraction? Ignore viscous effects. This is a steady flow, $\partial\vec{v}/\partial t = 0$, so explain how it is possible for the fluid to change velocity in a steady flow.

2. Write the velocity derivative tensor for ideal stagnation point flow. From this tensor, decide whether or not ideal stagnation point flow is an incompressible flow.

3. Find the strain rate tensor for ideal stagnation point flow. Diagonalize it. What are the principal strain rates? What are the principal strain axes (i.e. the directions of $\widehat{\imath}'$, $\widehat{\jmath}'$, and $\widehat{k}'$)?

4. Continuing that flow, based on the strain rate tensor, sketch the deformation of an initially square particle (aligned with the principal strain axes), during a small time interval. Also sketch the deformation of an initially circular particle.

5. Laminar flow through a long pipe is called Poisseuille flow. The velocity profile in cylindrical coordinates r, θ and z, with z along the pipe axis, is

$$\vec{v} = \widehat{\imath}_z v_{\max} \left(1 - \frac{r^2}{r_0^2}\right)$$

where r_0 is the radius of the pipe and $v_{\max}$ the center line velocity. Determine whether this is an incompressible flow field by looking up the divergence in cylindrical coordinates in Appendix B. Also look up the velocity derivative tensor and use it to evaluate the strain rate tensor at $r = \frac{1}{2}r_0$. Compare your answer to Appendix C. What is the strain rate tensor on the axis? So, how do small fluid regions at the axis deform?

3 9/17

1. For the Poisseuille flow of the previous question, derive the principal strain rates and the principal strain directions.

2. If you put a cup of coffee at the center of a rotating turn table and wait, eventually, the coffee will be executing a "solid body rotation" in which the velocity field is, in cylindrical coordinates:

$$\vec{v} = \widehat{\imath}_\theta \Omega r$$

where Ω is the angular velocity of the turn table. Draw samples of the streamlines of this flow. Find the vorticity and the strain rate tensor for this flow, using the expressions in appendices B and C. Show that indeed the coffee moves as a solid body; i.e. the fluid particles do not deform, and that for a solid body motion like this, indeed the vorticity is twice the angular velocity.

3. An "ideal vortex flow" is described in cylindrical coordinates by

$$\vec{v} = \frac{C}{2\pi r}\widehat{\imath}_\theta$$

where C is some constant. Draw samples of the streamlines of this flow. In cylindrical coordinates, nabla is given by

$$\nabla = \widehat{\imath}_r \frac{\partial}{\partial r} + \widehat{\imath}_\theta \frac{1}{r}\frac{\partial}{\partial \theta} + \widehat{\imath}_z \frac{\partial}{\partial z}$$

Evaluate

$$\begin{vmatrix} \widehat{\imath}_r & \widehat{\imath}_\theta & \widehat{\imath}_z \\ \dfrac{\partial}{\partial r} & \dfrac{1}{r}\dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial z} \\ v_r & v_\theta & v_z \end{vmatrix}$$

for this flow. Compare the answer with the vorticity $\omega = \nabla \times \vec{v}$, for which you can find the correct expressions in appendix B. Explain why the determinant does not give the correct result for the vorticity.

4. The "circulation Γ" along a closed contour is defined as

$$\Gamma \equiv \oint \vec{v} \cdot \mathrm{d}\vec{r}$$

For both the solid body rotation of question 2, and the vortex flow of question 3, find the circulation along the unit circle in the x,y-plane. Next, *only for the vortex flow*, find the circulation along the closed curve consisting of the following segments:

(a) The part of the curve $y = \cosh(x)$, $z = 0$ from $x = 2$ to $x = -3$;

(b) vertically downward to the curve $y = -\cosh(\cosh(x))$, $z = 0$ at $x = -3$;

(c) following the curve $y = -\cosh(\cosh(x))$, $z = 0$ from $x = -3$ to the point $x = 0$, hence $y = -\cosh(1)$, $z = 0$;

(d) in a straight line along the z direction to the point $x = 0$, $y = -\cosh(1)$, $z = 3.5$;

(e) in a straight line from $x = 0$, $y = -\cosh(1)$, $z = 3.5$ to the starting point $x = 2$, $y = \cosh(2)$, $z = 0$.

Note that in cylindrical coordinates

$$\mathrm{d}\vec{r} = \widehat{\imath}_r \mathrm{d}r + \widehat{\imath}_\theta r \mathrm{d}\theta + \widehat{\imath}_z \mathrm{d}z$$

5. According to the Stokes theorem of Calculus III, you should have

$$\oint \vec{v} \cdot \mathrm{d}\vec{r} = \int \nabla \times \vec{v} \cdot \vec{n}\, \mathrm{d}S$$

where the second integral is over the inside of the contour. So instead of integrating the circulation Γ as you did in question 4, you could have integrated the component of vorticity normal to the circle over the inside of the circle. Show that if you do that integral using the vorticity that you found for solid body rotation in question 2, you do indeed get the same answer as you got in question 4. Fine. But now show that if you do the integral of the vorticity over the inside of the circle for the vortex flow of question 3, you do *not* get the same answer for the circulation as in question 4. Explain which value is correct. And why the other value is wrong.

6. Following Friday's lecture, you would of course love to do some integrals of the form $\int \vec{v} \cdot \vec{n}\, \mathrm{d}S$ and $\int \vec{v} \cdot \vec{n} \vec{v}\, \mathrm{d}S$. Here is your chance. Do question 5.1(b) and 5.1(d) and explain their physical meaning. Take the surfaces S_I, S_{II}, S_{III}, and S_{IV} to be one unit length in the z-direction. (To figure out the correct direction of the normal vector $\vec{n}$ at a given surface point, note that the control volume in this case is the right half of the region in between two cylinders of radii r_0 and R_0 and of unit length in the z-direction. The vector $\vec{n}$ is a unit normal vector sticking *out* of this control volume.)

4 9/24

1. Unlike the ideal point vortex you analyzed in the previous homework, a true vortex diffuses out with time, and its velocity field is given by

$$\vec{v} = \widehat{\imath}_\theta \frac{C}{2\pi r} \left[1 - \exp\left(-\frac{r^2}{4\nu t}\right)\right]$$

Find the vorticity of this flow field. Also find the circulation along a circle of an arbitrary radius r. Then show that Stokes theorem *does* work for this flow. (The velocity is zero at $r = 0$; just apply l'Hopital.) Finally show that if the coefficient of viscosity is very small, the vorticity is only nonzero in some narrow spike near the origin, so that it looks almost like a an ideal vortex. (But the vorticity still integrates to Γ, despite the small radius of the region with appreciable vorticity.)

2. Integrate 5.1a, e, and f, and explain their physical meaning.

3. 5.14

4. 5.11

5. 5.12. As always, both mass and momentum conservation are needed.

5 10/1

1. Divide the fluid region outside a square cylinder into little finite elements of size $\Delta x \times \Delta y$. For a typical such element, write a finite element discretization for the continuity equation. Just like the continuity equation done in class, your final equation should *only* involve pressures, densities, and velocities at the center points of the finite volumes.

2. Write a finite element discretization for the x-momentum equation for a little finite element in polar coordinates. Just like the continuity equation done in class, your final equation should *only* involve pressures, densities, and velocities at the center points of the finite volumes.

3. 5.2 Make sure to write the full equations before assuming a radial flow. Make that a neat graph, and include the streamlines.

4. 5.3. This is two-dimensional Poiseuille flow (in a duct instead of a pipe). T_{ij} is the book's notation for the complete surface stress including the pressure,

$$T_{ij} = -p\delta_{ij} + \tau_{ij}$$

where δ_{ij} is called the Kronecker delta or unit matrix. So the book is really saying the pressure is -5 and there is an additional viscous stress $\tau_{xy} = -2\mu v_0 y/h^2$. Watch it, the expression $n_j\tau_{ji}$ gives the stress components in the x, y, z-axis system.

5. 5.6. Z is the height h. The final sentence is to be shown by you based on the obtained result. Hints: take the curl of the equation and simplify. Formulae for nabla are in the vector analysis section of math handbooks. If there is a density gradient, then the density is not constant. And neither is the pressure. T_{ij} is the book's notation for the complete surface stress, so the book is saying there is no viscous stress. (That is self-evident anyway, since a still fluid cannot have a strain rate to create viscous forces.)

6 10/8 postponed to 10/10

1. 6.1. Use the appendices. Based on the results, discuss whether this is incompressible flow, and in what direction the viscous stresses on the surface are. Also state in which direction the inviscid stress on the surface is.

2. 6.2 Discuss your result in view of the fact, as stated in (6.1), that the Reynolds number must be small for Stokes flow to be valid.

3. 7.5. Use the appendices. You may assume that $\vec{v} = \vec{v}(r)$. with $v_z = 0$, and $p = p(r, \theta)$ in cylindrical coordinates. Do not assume that the radial velocity is zero, derive it. Do not assume the pressure is independent of θ, derive it. Ignore gravity as the question says. Note that p must have the same value at $\theta = 0$ and 2π. Answer for v_θ:

$$\frac{\Omega r_0^2 r_1}{r_1^2 - r_0^2}\left(\frac{r_1}{r} - \frac{r}{r_1}\right)$$

4. In 7.5, what is the power needed to keep the rod rotating, per unit axial length? What is the pressure difference between the surfaces of the pipe and the rod?

5. 7.9. You can assume that the film thickness is so small that the curvature of the pipe wall can be ignored. In that case, it becomes 2D steady flow along a flat wall of spanwise length $2\pi r_0$ in the z-direction. Take the x-axis downwards. Assume $v = 0$ (vertical streamlines), $u = u(x, y)$ and $w = 0$ (two-dimensional flow), and that $p = p(x, y, z)$. Everything else must be derived; derive both pressure and velocity field. Do not ignore gravity. For the boundary conditions at the free surface, assume that the liquid meets air of zero density and constant pressure p_a there. Also write appropriate boundary conditions where the fluid meets the cylinder surface.

7 10/15

1. 7.6. Do not ignore gravity, but assume the pipe is horizontal. Do not use the effective pressure. Careful, the gravity vector is *not* constant in polar coordinates. Do not ignore the pressure gradients: assume the pressure can be any function $p = p(r, \theta, z, t)$ and derive anything else. Merely assume that the pressure distribution at the end of the pipe and rod combination is the same as the one at the start. For the velocity assume $v_r = v_\theta = 0$ and $v_z = v_z(r, z)$. Anything else must be derived. Give both velocity and pressure field.

2. For the case of question 7.6, what is the force required to pull the rod through the axis, per unit length? In 7.9, (previous homework), what is the net downward shear force on the pipe? Does the simple answer surprise you? Why not?

3. 7.1a Assume only that the velocity only depends on r, $\vec{v} = \vec{v}(r)$, that $p = p(r, \theta, z, t)$ is arbitrary, and that the pipe is horizontal. Use the effective pressure. Show that two velocity components must be zero. (You should be able to show that the effective pressure is independent of θ from the appropriate momentum equation by noting that p at $\theta = 2\pi$ must be the same as at $\theta = 0$; otherwise just assume it is. Also note that the velocity can obviously not be infinitely large on the pipe centerline.)

4. 7.1b Continuing the previous question, derive the velocity and pressure fields.

5. 7.4. Argue your answer. In what terms would you ballpark the answer? What is the importance of the pressure level? Of the flow velocity? What are the relevant values involved? What are the most important uncertainties? You might want to think of what the right answer for the head loss would be if there is no flow.

8 10/22

1. (A small part of 7.17 with $n = 1$.) Assume that an infinite flat plate normal to $\widehat{\jmath}$ accelerates from rest, so that its velocity is given by $u_p = \dot{U} t \widehat{\imath}$ where $\dot{U}$ is a constant. There is a viscous Newtonian fluid above the plate. Assuming only that $\vec{v} = \vec{v}(y, t)$, $w = 0$, and that the effective pressure far above the plate is constant, derive a partial differential equation and boundary conditions for the flow velocity of the viscous fluid. List them in the plane of the independent variables.

2. (A small part of 7.17 with $n = 1$.) Assuming that the velocity profile is similar, derive that

$$f - \frac{\dot{\delta} t}{\delta} \eta f' = \frac{\nu t}{\delta^2} f''$$

   where $f(\eta)$ is the similar velocity profile and $\delta(t)$ is the boundary layer thickness used to get similarity. By examining the above equation at the plate, where $\eta = 0$, show that within a constant, δ must be the same as in Stokes' second problem. Take it the same, then write the final equation for the similar profile f.

3. (A small part of 7.17 with $n = 1$.) Differentiate the equation for f twice with respect to η, and so show that $g = f''$ satisfies the equation

$$g'' + 2\eta g' = 0$$

   This equation is the same as the one for f in Stokes' second problem, and was solved in class. The general solution was

$$g(\eta) = C_1 \int_{\bar{\eta}=\eta}^{\infty} e^{-\bar{\eta}^2} \, \mathrm{d}\bar{\eta} + C_2$$

   Explain why C_2 must be zero. Explain why then f' can be found as

$$f'(\eta) = -\int_{\bar{\eta}=\eta}^{\infty} g(\bar{\eta}) \, \mathrm{d}\bar{\eta} = -C_1 \int_{\bar{\eta}=\eta}^{\infty} \int_{\bar{\bar{\eta}}=\bar{\eta}}^{\infty} e^{-\bar{\bar{\eta}}^2} \, \mathrm{d}\bar{\bar{\eta}} \, \mathrm{d}\bar{\eta}$$

Draw the region of integration in the $\bar{\eta},\bar{\bar{\eta}}$-plane. Use the picture to change the order of integration in the multiple integral and integrate $\bar{\eta}$ out. Show that

$$f'(\eta) = C_1 \left[\eta \int_{\bar{\bar{\eta}}=\eta}^{\infty} e^{-\bar{\bar{\eta}}^2} \, \mathrm{d}\bar{\bar{\eta}} - \frac{1}{2} e^{-\eta^2} \right]$$

Integrate once more to find $f(\eta)$. Apply the boundary condition to find C_1.

4. Do bathtub vortices have opposite spin in the southern hemisphere as they have in the northern one? Derive some ballpark number for the exit speed of a bathtub vortex at the north pole and one at the south pole, assuming the bath water is initially at rest compared to the earth. What do you conclude about the starting question?

5. A Boeng 747 has a maximum take-off weight of about 400,000 kg and take-off speed of about 75 m/s. The wing span is 65 m. Estimated the circulation in the trailing vortices, and from that, ballpark the typical circulatory velocities around the trailing vortices. Compare to the typical take-off speed of a Cessna 52, 50 mph.

9 10/29

1. Find boundary conditions for the streamfunction for transverse ideal flow around a circular cylinder. The velocity far away from the cylinder is $U\hat{\imath}$ and the radius of the cylinder is r_0.

2. Following similar lines as in class, but watching the new boundary conditions, solve the equation for the streamfunction around the circular cylinder. Before continuing, check your results for the radial and tangential velocity components at the surface of the cylinder against the one from the velocity potential solution obtained in class. Is the velocity at the top and bottom points $2U$? Are the stagnation points correct?

3. Find the pressure on the surface of the cylinder.

4. Integrate the pressure forces over the surface of the cylinder to get the net force on the cylinder.

5. Now add to the above velocity field the velocity field of an ideal vortex,

$$\vec{v} = \frac{\Gamma}{2\pi r} \hat{\imath}_\theta$$

Check whether the correct flow boundary conditions are still satisfied at the surface of the cylinder and far from the cylinder. Integrate the pressure again, and compare the forces to D'Alembert and Kutta-Joukowski.

10 11/5

1. Sketch streamlines for the potential flow

$$F = z^{2/3}$$

Explain why such a flow might be relevant to flow about a corner. What is the (effective) pressure on the positive x-axis? Comment on what happens to the pressure when $x = 0$.

2. Find the polar velocity components and pressure of the source flow

$$F = \frac{Q}{2\pi} \ln z$$

where Q is a constant. Show that this is a valid solution of the Navier Stokes equations for the flow outside a cylindrical balloon whose radius R is expanding according to the relationship

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{Q}{2\pi R}$$

Then show that this means that the cross sectional area of the balloon is linearly increasing with time.

3. Show that the potential flow
$$F = \frac{Q}{2\pi} \ln z$$
where Q is not a constant but equal to $2\pi t$ is an *exact* solution of the viscous Navier-Stokes equation for flow around a balloon whose radius expands as $R = t$. Then find the pressure, using the correct Bernoulli equation for an *unsteady* potential flow. Comment on the pressure far from the balloon.

4. In the familiar potential flow around a cylinder, the Uz term produces the incoming uniform flow and the Ur_0^2/z term produces the flow induced by the cylinder. That means that if the *fluid* is at rest at infinity and it is the cylinder that moves, the potential is given by
$$F = -\dot{x}_0 \frac{r_0^2}{z - x_0}$$
where $x_0(t)$ is the position of the center of the cylinder on the x-axis. Find the time derivative $\partial F/\partial t$ and the spatial derivative $W = \partial F/\partial z$. Watch it: both x_0 and $\dot{x}_0$ in F depend on time. Now evaluate these derivatives on the surface of the cylinder where $z - x_0 = r_0 e^{i\theta}$. Then find $\partial\phi/\partial t$ as the real part of $\partial F/\partial t$. Also find the square magnitude of the velocity as $W\bar{W}$, where $\bar{W}$ is the complex conjugate of W. Use this to find the pressure on the surface of the cylinder. Answer:
$$p_{\text{eff}} = p_\infty - \frac{1}{2}\rho\dot{x}_0^2 + \rho r_0 \ddot{x}_0 \cos\theta + \rho\dot{x}_0^2 \cos 2\theta$$

5. From the pressure of the previous question, find the force on the cylinder. Show that it implies that to accelerate the cylinder, in addition to the force required to accelerate the cylinder itself, there will be an additional force as if an additional mass equal to an amount of water with the volume of the cylinder also must be accelerated. Explain why an apparent mass effect must be there on behalf of the second law of thermodynamics.

11 11/14

1. Find the ideal flow about an ellipse whose horizontal axis length is $\frac{5}{3}$ its vertical axis. Is the velocity still $2U$ at the top of the ellipse like for a cylinder? If not, how big is it?

2. From the airfoil program page,[1], click on and read program airfoil.m. Save airfoil.m as type "all files" and run it in Matlab or octave 3.0 or higher. You will get plots of flow past a cylinder in the ζ plane (ζ_2 in your notes) and an unimpressive flush flow past an infinitely thin plate. So set the radius `r0` to a value a bit greater than one to give the airfoil thickness. Use screen capture or the *print* command to make a hardcopy of the airfoil plot. The airfoil does not seem to produce much lift. Set the angle of attack `alp` to a suitable value to fix that and plot. Darn, still no lift. Set the circulation `Gamma` to some nonzero value (the program will correct the value you put in.) Replot.

3. Plot the lines of constant pressure coefficient $C_p = (p - p_\infty)/\frac{1}{2}\rho U^2$ on the airfoil with `alpha` equal to 20 and `r0` equal to 1.1. Stay completely in complex variables to compute the pressure coefficient. You can use the `abs` function to get the magnitude of W after you have found it. Use the chain rule. Print out the modified program and the isobars.

4. Would it not be nice to have some camber? Change program airfoil.m to produce an airfoil with positive camber. You will need to correct one line in the program. Print out the modified program and the airfoil.

5. According to potential flow theory, what would be the lift per unit span of a flat-plate airfoil of chord 2 m moving at 100 m/s at sea level at an angle of attack of 10 degrees? What would be the drag? What would be the circulation around the airfoil?

[1] `http://www.eng.fsu.edu/~dommelen/courses/flm/progs/jou_air/`

6. Compute approximate values of the Reynolds number of the following flows:
   (a) your car, assuming it drives;
   (b) a passenger plane flying somewhat below the speed of sound (assume an aerodynamic chord of 30 ft);
   (c) flow in a 1 cm water pipe if it comes out of the faucet at .5 m/s,

   In the last example, how fast would it come out if the Reynolds number is 1? How fast at the transition from laminar to turbulent flow?

12 11/21

1. Using suitable neat graphics, show that the boundary layer variables for the boundary layer around a circular cylinder of radius r_0 in a cross flow with velocity at infinity equal to U and pressure at infinity p_∞ are given by:
$$x = r_0\theta \qquad y = r - r_0 \qquad u = v_\theta \qquad v = v_r$$

2. Write the appropriate equations for the unsteady boundary layer flow around a circular cylinder in terms of the boundary layer variables above. Assuming that the potential flow outside the boundary layer is steady and unseparated, give all boundary conditions to be satisfied. Make sure to write them in terms of boundary layer variables only. Solve the pressure field inside the boundary layer.

3. For the same flow, rewrite the boundary continuity equation in terms of the polar coordinates r, θ, v_r, v_θ, and p. Compare this with the exact continuity equation in polar coordinates and explain why the difference is small if the boundary layer is thin. Also write the boundary layer x and y momentum equations in terms of the polar coordinates. Are they different from the exact momentum equations? Which of the two is most simplified?

4. For the same flow, consider the proposed solution (from Stokes' second problem)
$$u = u_e(x)\mathrm{erf}\left(y/\sqrt{4\nu t}\right)$$
where u_e is the potential flow slip velocity immediately above the boundary layer. The solution above satisfies the equation
$$u_t = \nu u_{yy}$$
Find out what the velocity u_e must be. Also find the velocity v inside the boundary layer. Note:
$$\int_{\bar{z}=0}^{z} \mathrm{erf}(\bar{z})\,\mathrm{d}\bar{z} = z\,\mathrm{erf}(z) + \frac{1}{\sqrt{\pi}}e^{-\bar{z}^2} - \frac{1}{\sqrt{\pi}}$$

5. Define a suitable boundary layer thickness for the proposed solution of the previous question. How does it vary with x and t? Explain why the proposed solution is reasonable for very small times. Hint: Ask yourself, what happens to the magnitude of u_{yy} when $t \to 0$? Does the same happen to the magnitudes of u, v, u_x, u_y. and p_x? Argue that for larger times, the proposed solution is no longer good. Base yourself here on results like those found on the program web page[2] and links on that page like Shankar's thesis[3] and the Van Dommelen & Shen separation process[4] as well as what you know about the proposed solution, such as, say, its boundary layer thickness.

6. According to potential flow theory, what would be the lift per unit span of a flat-plate airfoil of chord 2 m moving at 30 m/s at sea level at an angle of attack of 10 degrees? What would be the viscous drag if you compute it as if the airfoil is a flat plate aligned with the flow with that chord and the flow is laminar? Only include the shear stress over the last 98% of the chord, since near the leading edge the shear stress will be much different from an aligned flat plate. What is the lift to drag ratio? Comment on the value. Use $\rho = 1.225$ kg/m^3 and $\nu = 14.5\ 10^{-6}$ m^2/s.

[2] http://www.eng.fsu.edu/~dommelen/courses/flm/progs/bl_flow
[3] http://www.eng.fsu.edu/~dommelen/papers/subram/style_a/node56.html
[4] http://www.eng.fsu.edu/~dommelen/research/ini2d/ini2dnum.html

13 12/1

1. Assume that a flow enters a two dimensional duct of constant area. If no boundary layers developed along the wall, the centerline velocity of the flow would stay constant. Assuming that a Blasius boundary layer develops along each wall, what is the correct expression for the centerline velocity?

2. Continuing the previous question. Approximate the Blasius velocity profile to be parabolic up to $\eta = 3$, and constant from there on. At what point along the duct would you estimate that developed flow starts based on that approximation? Sketch the velocity profile at this point, as well as at the start of the duct and at the point of the duct where the range $0 \leq \eta \leq 3$ corresponds to $\frac{1}{8}$ of the duct height accurately in a single graph. Remember the previous question while doing this!

3. Write down the vorticity for Stokes flow around a sphere. The velocity field was given in homework question 6.1, and a more extensive discussion is in section 21.8. Sketch some typical lines of constant vorticity, in particular $\omega = 0.25$, 0.5, 0.75 and 0.99 $\omega_{\max}$, where $\omega_{\max}$ is the maximum vorticity. Now compare this very low-Reynolds number vorticity field with that of high Reynolds number boundary flow. As the boundary layer solution, you can use the error function profile one of the previous homework, and assume that $\sqrt{4\nu t}$ is say one tenth of the radius. (The fact that it is a cylinder instead of a sphere makes no important difference here.) You can use the boundary layer approximation for the vorticity here.

4. Streamlines for very low Reynolds number, very *viscous* Stokes flow look superficially the same as those for high Reynolds number ideal *in*viscid flows: both are symmetric front/rear. But do they really look the same? Plot their streamlines reasonably accurately. They are given in sections 19.8 and 21.8. You could use some plotting package to plot them. Alternatively, you could figure out where the streamlines through the points $r = 1.25$, 1.5, 1.75, and 2 sphere radii from the center in the symmetry plane end up far upstream and downstream, and then sketch the streamlines as well as possible based on that info.

14 12/5

1. What is the expression for the expected thickness of a turbulent plane shear layer? What is the constant of proportionality? Does this seem in reasonable agreement with experiments?

2. What is the expression for the expected diameter of a turbulent jet? What is the constant of proportionality? Does this seem in reasonable agreement with experiments?

3. The book claims that two-dimensional and round turbulent jets grow in size at the same rate, but that their velocity decays at a different rate. Does that make sense? Why would one be the same and the other different?

4. Would the similarity arguments made in class for a turbulent jet hold for a jet in a coflowing stream (like a jet coming out of a jet engine of a plane in flight)? If not, why not?