Fluid Mechanics
EML 5709
Homework

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Contents

1 HW 1

In this class,

- *Questions must be answered in order asked.*

- *Solutions must be neat.*
- *You must use the given symbols.*
- *You must show all reasoning.*
- *Copying is never allowed, even when working together.*

1. What is the key number that determines whether the continuum assumption is valid for a gas? Explain its definition in detail. So for a body moving through air at standard sea-level conditions, when is the continuum approximation valid?

2. Couette flow is the viscous flow in the gap between two horizontal plates, the top one of which is moving in the positive x-direction with some velocity U. If y is distance measured from the bottom plate at $x = 0$, for laminar flow the fluid velocity is given by
$$\vec{v} = \hat{\imath} u \qquad u = \frac{U}{h} y$$
where h is the distance between the plates. Sketch the plates and the fluid velocity vectors at some line of constant x (giving the velocity profile). For a simple two-dimensional unidirectional flow like this,
$$\tau_{yx} = \mu \frac{\partial u}{\partial y}$$
where the constant μ is called the kinematic viscosity of the fluid. Also, because the positive and negative z directions are equivalent in this problem, shear forces in the z-direction can only be zero, so
$$\tau_{yz} = \tau_{xz} = 0$$
At a point at a position 0.75 h above the bottom plate, the stress tensor is given by
$$\bar{\bar{\tau}} = \begin{pmatrix} -p & \mu U/h & 0 \\ \mu U/h & -p & 0 \\ 0 & 0 & -p \end{pmatrix}$$
Here p is the pressure (which is an inviscid effect). Fully explain every term in this stress tensor. Draw a little cube around the given point. Clearly show all stress components on the surfaces of this little cube, in terms of the quantities above, after drawing a magnified cube if needed.

3. Going back to the previous question, suppose there is a little area A going through the considered point above, parallel to the z-axis, but rotated 30° counterclockwise from the positive y-axis. (So take a small surface normal to $\hat{\imath}$ and then rotate it 30 degrees around the z-direction.) Find the stress force per unit area $\vec{R}$ acting on that area A. Then find the components of the stress force normal and tangential to area A. Comment on the tangential component of the pressure force and the normal component of the viscous stress force.

4. Going back to the second-last question, suppose you rotate the coordinate system xyz $45°$ counterclockwise around the z-direction to get an $x'y'z'$ coordinate system. Find the stress tensor in this rotated coordinate system. Is the $x'y'z'$ coordinate system the principal axis system? Draw again a little cube around the given point with the stresses on its surfaces. But this time show the cube and correct stress components aligned with the new coordinate system.

2 HW 2

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1. Two-dimensional ideal stagnation point flow is given by

$$\vec{v} = \hat{\imath}cx - \hat{\jmath}cy$$

   where c is some constant. Find the vorticity (in 3D). Find the strain rate tensor. Diagonalize it by rotating the axes suitably. What are the principal strain axes (i.e. the directions of $\hat{\imath}'$, $\hat{\jmath}'$, and $\hat{k}'$)? What are the principal strain rates? Neatly sketch the deformation of a small initially square particle (aligned with the principal strain axes), during a small time interval. Also sketch the deformation of a small initially circular particle. Also show the complete particle changes when you include the solid body rotation.

2. If you put a cup of coffee at the center of a rotating turn table and wait, eventually, the coffee will be executing a "solid body rotation" in which the velocity field is, in cylindrical coordinates:

$$\vec{v} = \hat{\imath}_\theta \Omega r$$

   where Ω is the angular velocity of the turn table, r the distance from the axis of rotation, and θ the angular position around the axis. Find the vorticity and the strain rate tensor for this flow, using the expressions in appendix B. *Do not guess.* (The book might different, bad, symbols for r and θ.) Show mathematically that indeed the coffee moves as a solid body, i.e. the fluid particles do not deform, and that for a solid body motion like this, indeed the vorticity is twice the angular velocity.

3. In Poiseuille flow (laminar flow through a pipe), the velocity field is in cylindrical coordinates given by

$$\vec{v} = \hat{\imath}_z v_{\max}\left(1 - \frac{r^2}{R^2}\right)$$

   where $v_{\max}$ is the velocity on the centerline of the pipe and R the pipe radius. Use Appendix B to find the strain rate tensor of this flow. *Do not guess.* Evaluate the strain rate tensor at $r = 0$, $\frac{1}{2}R$ and R. What can you say about the straining of small fluid particles on the axis?

4. Is the Poisseuille flow of the previous questions an incompressible flow?

5. Find the vorticity of the Poisseuille flow of the previous questions. Do the fluid particles on the axis rotate?

6. For the Poisseuille flow of the previous questions, given that the principal directions of the strain rate tensor are everywhere

$$\hat{\imath}' = \frac{1}{\sqrt{2}}\left(\hat{\imath}_r + \hat{\imath}_z\right) \qquad \hat{\jmath}' = \frac{1}{\sqrt{2}}\left(\hat{\imath}_r - \hat{\imath}_z\right) \qquad \hat{k}' = \hat{\imath}_\theta$$

find the principal strain rates. Sketch the deformation of a fluid particle at an arbitrary radius r. In particular, in the r,z-plane, neatly sketch a particle that was spherical at time t at time $t + \mathrm{d}t$. What happens in the θ direction with the particle?

3 HW 3

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1. For the Poisseuille flow of the previous homework, find the viscous stress tensor and the total stress tensor at an arbitrary radial position r from the axis. Assume a Newtonian fluid. Show the stresses acting on a small volume $\mathrm{d}r\mathrm{d}\theta\mathrm{d}z$ at an arbitrary radius r graphically in the r,z-plane. (Describe the stresses on its two surfaces normal to $\hat{\imath}_\theta$ only in words.) Include the pipe in the picture.

2. Repeat the previous question, but now do it in principal axes.

3. You may have noticed that if a stream of water exits a faucet, immediately after it exits, it contracts. The radius of the stream rapidly decreases. The stream is "thinner" below the faucet exit than the faucet exit. This effect has nothing to do with gravity, and everything with viscosity. Your task is to explain why this happens and find out by what factor the stream gets thinner under idealized conditions. Ignore gravity. Use a control volume that is a circular cylinder of finite length. Take one end of the cylinder to be the circular exit area of the faucet. That is your surface 1. The other circular end is surface 2. Since the stream gets thinner, the stream will only occupy the center part of surface 2. There is no mass flow through the outer ring. (The density of air is assumed zero, and its viscosity too, but it has a pressure.) Take the curved surface of the cylinder to be surface 3. Sketch the flowfield and control volume. Assuming that at surface 1, the velocity is our beloved axial Poiseuille flow

$$\vec{v} = \hat{\imath}_z v_{\max}\left(1 - \frac{r^2}{R^2}\right)$$

   write the mass and momentum outflows $\int \rho\vec{v}\cdot\vec{n}\,\mathrm{d}A$ and $\int \rho\vec{v}\cdot\vec{n}\vec{v}\,\mathrm{d}A$ for this surface. (Recall that $\mathrm{d}A = r\mathrm{d}r\mathrm{d}\theta$ in polar coordinates in a plane of constant z.) Also write these integrals for surface 3. For surface 2, assume that at this position, viscous effects have smoothed out the initial parabolic velocity profile to a uniform one, in which all particles now move at the same velocity V_e. As already noted the area A_e is less than the cross sectional area of the cylinder. At this time, V_e and A_e are still unknowns. Write the mass and momentum outflows through surface 2 in terms of these unknowns.

4. Continuing the previous question, write the z-components of surface force integrals for surfaces 1, 2, and 3. You can assume that $\tau_{zz}^{\text{viscous}}$ is negligible on surfaces 1 and 2, but other viscous stresses like say $\tau_{zr}^{\text{viscous}}$ are *not* negligible on surface 1. You can assume the pressure is atmospheric on surface 1. What do you think of surface 2? Show all reasoning.

5. If you now write the equations of mass conservation and momentum conservation for the control volume, you get two equations that you can use to find expressions for V_{e} and A_{e}. Use the expression for A_{e} to find the contraction factor of the stream. Notes: For a real faucet, the stream is probably turbulent, and the velocity profile at the faucet exit would be flatter than parabolic. This should reduce the contraction, because there is less change in velocity needed to create the uniform velocity profile. And gravity will also thin the stream, but not just at the exit but also further down.

6. In a piping system, a stream of water enters an elbow with a pressure of 325 kPa and a velocity of 5 m/s. The entrance area has a diameter of 3 cm. The elbow bends the stream around over 120 degrees and the water then exits at the ambient pressure of 100 kPa through an area one fifth of the entrance area. Use mass and momentum conservation to find the force required to keep the elbow into place. Hints: You may want to include the elbow itself in the control volume. (If you do not, do not forget to account for the pressure force that the atmospheric pressure exerts on the outer surface of the elbow.) Either way, you may want to use the concept of "gauge,"or "gage," pressure to simplify the force integrals over the weird surfaces of your control volume. (See your undergraduate thermo or fluids book.)

4 HW 4

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1. Write the momentum equation in the x-direction for the same cylindrical coordinates finite volume as used in class. Approximate the time derivative of the x-momentum inside the control volume. Angle θ is the angle between $\vec{r}$ and the positive x-axis, positive counterclockwise. Remember that the unknowns are the *cylindrical* velocity components v_r and v_θ. The x-component of velocity v_x is not, and must be rewritten in terms of the chosen unknowns.

2. Continuing the previous question, approximate the integral giving the net flow of x-momentum out of the control volume. Use similar techniques as in class.

3. Continuing the previous question, approximate the x-components of the surface and gravity forces. Assume that the stress tensor *in cylindrical coordinates* at the finite volume center points can be computed. (This would be done using the formulae as found in Appendix B.) Note that the x-component of the force is needed. I think it is easier to take x-components of the cylindrical-coordinate forces than to transform the cylindrical tensor into a Cartesian one.

4. A cylinder of radius R is surrounded by a fluid. The cylinder is rotating with angular velocity Ω. (a) If the fluid is inviscid, what is the fluid velocity boundary condition at the surface of the cylinder, in terms of cylindrical coordinates? (b) Same question, but viscous fluid. (c) Is it possible for the fluid outside the cylinder to be at rest?

5. Consider the following velocity field:

$$u = C(x^3 - 3xy^2) \qquad v = C(y^3 - 3x^2y)$$

   where C is a "constant" that only depends on time. This flow field applies for $y > 0$; at $y = 0$ the flow meets a stationary solid surface along the x-axis. Based on the boundary condition, would this be a viscous flow, an inviscid one, or an impossible one (that enters the solid wall)? Now write the complete three-dimensional viscous stress tensor, assuming a Newtonian fluid.

6. Continuing the previous question, assume that the fluid has a constant density ρ and viscosity μ and that the pressure is given as

$$-p = \tfrac{1}{2}\rho C^2(x^3 - 3xy^2)^2 + \tfrac{1}{2}\rho C^2(y^3 - 3x^2y)^2 + \tfrac{1}{4}\rho \dot{C}(x^4 - 6x^2y^2 + y^4) + \rho g y$$

   Note the leading minus sign. Also, gravity g is in the minus y direction and $\dot{C}$ means the time derivative of C. Write out the conservative continuity and x- and y momentum equations given in class. Then plug in the given velocity and pressure and the stress tensor and so show that the equations of *viscous* incompressible flow are satisfied. Note: I think it is quickest not to multiply out products but just use the product rule of differentiation.

5 HW 5

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1. In the upper half plane $y \geq 0$, consider the two-dimensional flow

$$u = cx \qquad v = -cy \qquad p + \rho g y = p_0 - \tfrac{1}{2}\rho u^2 - \tfrac{1}{2}\rho v^2$$

where c and p_0 are constants. Find and neatly draw the streamlines, particle paths and streaklines of this flow. Put flow direction arrows on the lines. What is the name for the shape of the streamlines?

2. Find the particle acceleration vector field for the flow of the previous question. Very neatly draw a few acceleration vectors in your graph. Does density times acceleration equal the pressure force per unit volume plus the viscous force per unit volume, plus the downward gravity force per unit volume, assuming a Newtonian fluid?

3. Consider the velocity field

$$\vec{v} = \frac{t}{r}\hat{\imath}_r + \frac{1}{r}\hat{\imath}_\theta$$

This represents an ideal circulatory flow around an expanding cylinder of radius t. (So the fluid is restricted to $r > t$). Find and draw the streamlines and particle paths of this flow.

4. For the flow of the previous question, find the expression for the streakline at some time t if the moving smoke generator at some earlier time τ was at position $r_g = 2\tau$, $\theta_g = 0$. There is no requirement to draw the curve. But be sure to eliminate τ to get a relation between r and θ only.

5. Write the equation that applies for hydrostatics of a Newtonian fluid, in terms of h. Now take the curl of the equation (i.e. premultiply by $\nabla\times$). Simplify the expressions using the formulae for ∇ in the vector analysis section of your mathematical handbook. This should show you that $\nabla(\rho g)$ is parallel to $\nabla(h)$ where h is the height above sea-level. Explain why that means that ρg only varies with h, i.e. the density must be everywhere the same in a plane of constant height. In your explanation, use a coordinate system with its z-axis upward, so that $z = h$, and write out the gradients.

6 HW 6

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1. In the Euler equations given in your notes, ignore gravity. Now use the energy equation to convert the $\mathrm{D}\rho/\mathrm{D}t$ derivative in the continuity equation by a $\mathrm{D}p/\mathrm{D}t$ one. That achieves that continuity and x-momentum involve derivatives with respect to only p and $\vec{v}$. Write out these equations for the special case of one-dimensional, unsteady, inviscid, compressible flow (nonlinear acoustics in a pipe). Now add ρa^2 times your continuity equation to $\pm a$ times your momentum equation and show that the result is of the form:
$$\left[\frac{\partial p}{\partial t} + (u \pm a)\frac{\partial p}{\partial x}\right] \pm \rho a \left[\frac{\partial u}{\partial t} + (u \pm a)\frac{\partial u}{\partial x}\right] = 0$$
Argue from your knowledge of calculus that the terms in the square brackets are derivatives $\mathrm{d}p/\mathrm{d}t$ and $\mathrm{d}p/\mathrm{d}t$, not along particle paths, but along the paths of sound waves moving left and right. The above two equations (one for each sign) are called the "compatibility equations."

2. Continuing the previous question, for normal acoustics the variations in u and p are small enough that the u in $u \pm a$ can be ignored, and that the coefficients a and ρ can be assumed to be constants. Write the equations under these conditions. Now you know from calculus (shifting a function to the right) that a pressure wave moving to the right without changing shape is given by $p = f_1(x - at)$ (where function f_1 describes the shape of the pressure wave). Similarly a wave in the velocity field moving in the same direction takes the form $u = f_2(x - at)$. Show that to satisfy both compatibility equations, the shapes of the pressure and velocity waves must be related as $f_1' = \rho a f_2'$. (The energy equation then gives the perturbation in density as the final of the three unknowns in a one-dimensional compressible flow.)

3. Write the nondimensionalized incompressible Navier-Stokes equations out fully in terms of the scaled Cartesian velocity components (u^*, v^*, w^*) and the scaled pressure p^*. Include the continuity equation too!

4. A fan makes a lot of aerodynamic noise. Assume that the power P of the emitted acoustic noise and the volumetric flow rate Q produced by the fan depend on the fan diameter D, its frequency Ω, the air density ρ, and the air speed of sound a. Use the Buckingham Π theorem to find simplified expressions for the acoustic power and the volumetric flow rate. As selected parameters, use the two parameters that are beyond your control as a fan designer, and the fan diameter.

5. Assuming that the volumetric flow rate is also a given, what can you say about the likelyhood of improving the noise generated by the fan by messing around with fan diameter and frequency? Hints: consider the acoustic power per unit flow rate. And think about the physical interpretation of the relevant nondimensional parameter(s).

6. Using the formulae in the scanned class notes, and corresponding notations, find the stress tensor for Stokes flow around a sphere, in spherical coordinates.

7. Find the drag force on the sphere by finding the stresses on the surface of the sphere, taking z-components of them, and then integrating over the surface.

7 HW 7

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1. A rotating cylindrical axis of radius r_0 is enclosed within a stationary concentric pipe of inner radius r_1. The angular velocity of the axis is Ω_0. There is fluid in the gap between the axis and pipe. Find the fluid velocity. Make the following assumptions:

    1. "Incompressible" fluid.
    2. Newtonian fluid.
    3. The velocity only depends on r, not θ, z, or t.
    4. There is no velocity component in the axial direction.
    5. No gravity, (or alternatively, the pressure is the kinetic pressure).

    Show that given *only* the above assumptions, the pressure must be of the form

    $$p = p_0(r) + p_1(t) \qquad p_0(r) = \int_{r_1}^{r} \frac{\rho v_\theta^2}{\bar{r}} \,\mathrm{d}\bar{r}$$

    where $p_1(t)$ is the pressure on the pipe surface, and that v_θ must be of the form

    $$v_\theta = \Omega r + \frac{\Gamma}{2\pi r}$$

    where Ω and Γ are integration constants. The first term above is our beloved "solid body rotation" with angular velocity $\Omega\hat{\imath}_z$ around the z-axis, while the second term is called an "ideal vortex flow" with circulation Γ. *At every step that you make, list which one of the above assumptions, or earlier result, you are using.*

2. Now put in the boundary conditions for v_θ to find Ω and Γ. Then find the moment around the z-axis that the pipe exerts on the fluid. Also find the power that the axis loses due to friction from the fluid.

3. Show the changes that occur in your analysis above if you do not use the kinetic pressure, but include g_r, g_θ, and g_z explicitly in the equations. Assume that the axis of the pipe is slanting downwards by an angle α. Take the $\theta = 0$ plane to be the plane sticking vertically upward from the axis. Warning: g_r and g_θ are *not* constants, but depend on θ. (I think it may be easiest first to define a Cartesian coordinate system where x is inside the mentioned vertical plane, normal to z, and y is sideways. Note that the height then does not depend on the sideways coordinate y. Figure out the height h in terms of x and z by looking in the x,z plane and convert that to cylindrical. Then you can find the gravity vector components by taking a gradient of $-gh$.) Show that the final result is the kinetic pressure as expected from your earlier solution. Do you now appreciate kinetic pressure?

4. A water pipe of radius r_0 is sticking straight up. Water is coming out of the top of the pipe and runs down the outer surface of this pipe as a thin sheet of water. You are to find the flow field in the sheet sufficiently far below the top end. Assume that the sheet is sufficiently thin compared to the radius of the pipe that you can approximate it as a sheet along a *flat* pipe surface, with z the coordinate along the perimeter of the pipe, y the distance from the pipe surface, and x the downward coordinate. So x, y, z can be approximated as Cartesian coordinates. Next make the following assumptions:

    1. "Incompressible" fluid.
    2. Newtonian fluid.
    3. The streamlines well below the top of the pipe go straight down. (What does that mean?).
    4. The velocity field is steady and independent of z, i.e. $\vec{v} = \vec{v}(x, y)$.
    5. The velocity field is steady, i.e. $\vec{v} = \vec{v}(x, y, z)$.
    6. The air exerts a constant atmospheric pressure p_a on the free water surface.
    7. However, the shear stress that it exerts on the surface may safely be ignored.

    Use *only* the above assumptions. *At every step that you make, list which one of the above assumptions, or earlier result, you are using.* Do not forget that since there is a free surface, you cannot use the kinetic pressure in this problem. You will need to include gravity explicitly.

5. Based on your solution above, answer a few physical questions:

    1. What is the vertical force per unit span in z that the pipe surface exerts on the water?
    2. Explain why that force has this simple value in physical terms. Use a suitable control volume to do so.
    3. What is the volumetric flow rate Q?
    4. Suppose you increase the flow rate coming out of the top of the pipe by a factor 8. What happens to the maximum velocity in the sheet below? What happens to the sheet thickness? Which of the two changes most to accommodate the larger volumetric flow rate?
    5. Form the most meaningful Froude number to describe the sheet flow, following the ideas of, say, pipe flow, and relate it to other nondimensional numbers based on your solution. In particular, consider a Reynolds number based on Q and r_0 as something your Froude number might depend upon.

8 HW 8

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1. Find the expressions for $\hat{\imath}_r$, $\hat{\imath}_\theta$, and $\hat{\imath}_\phi$ in spherical coordinates in your notes. Now answer the following questions:

    1. What is the x-component of $\hat{\imath}_r$? What is the z-component of $\hat{\imath}_\theta$.

    2. Write the (vector) flow velocity $\vec{v}$ in terms of the spherical velocity components v_r, v_θ, and v_ϕ and the unit vectors $\hat{\imath}_r$, $\hat{\imath}_\theta$, and $\hat{\imath}_\phi$. From that find the expression for the x-component of velocity v_x in spherical coordinates. Do that *by taking x-components of the vectors, not by making up nonexisting x-components of the scalars.* Do the same for the z-component of velocity v_z. Finally, write the expression for the linear momentum in the z-direction in a volume of fluid in terms of the spherical coordinates r, θ, and ϕ and spherical velocity components v_r, v_θ, and v_ϕ.

    3. Consider a spherical surface of radius ℓ around the origin. Find the unit vector $\vec{n}$ normal to that surface in terms of the spherical unit vectors $\hat{\imath}_r$, $\hat{\imath}_\theta$, and $\hat{\imath}_\phi$. Also find the spherical coordinates expression for the area $\mathrm{d}A$ of a little element $\mathrm{d}\theta\mathrm{d}\phi$ of this spherical surface. Find $\vec{v}\cdot\vec{n}$ in terms of v_r, v_θ, and v_ϕ. Find the z-component of $\vec{v}\cdot\vec{n}\vec{v}$ in terms of v_r, v_θ, and v_ϕ. Do that *by taking z-components of vectors, not by making up nonexisting z-components of scalars.*

    4. Continuing with the spherical surface, find the z-component of vector $\vec{n}$. Find the z-component of $-p\vec{n}$. Find the expression for the z-component of the integral $\int -p\vec{n}\mathrm{d}A$ over the spherical surface.

    5. Continuing with the spherical surface, in terms of the viscous stress tensor

    $$\bar{\bar{\tau}} = \begin{pmatrix} \tau_{rr} & \tau_{r\theta} & \tau_{r\phi} \\ \tau_{\theta r} & \tau_{\theta\theta} & \tau_{\theta\phi} \\ \tau_{\phi r} & \tau_{\phi\theta} & \tau_{\phi\phi} \end{pmatrix}$$

    find the force per unit spherical surface area $\bar{\bar{\tau}}^{\mathrm{T}}\vec{n}$ as a column of spherical stress components. Combine the components into an actual vector using $\hat{\imath}_r$, $\hat{\imath}_\theta$, and $\hat{\imath}_\phi$. Find the z-component of that vector. Find the expression for the z-component of the integral $\int \bar{\bar{\tau}}\vec{n}\mathrm{d}A$ over the spherical surface.

    6. Now reconsider the Stokes flow around a sphere done in a previous homework set, 6.6. Take as control *volume* the sphere $r \le \ell$ with as *surface* the spherical surface $r = \ell$. Write the integral z-momentum equation for this control volume. Ignore gravity. But besides the usual surface force

integrals, you should also include the external force needed to prevent the sphere from being blown away. This external force is, of course, equal to the drag force D that the fluid exerts on the sphere.

7. Substitute in the Stokes flow velocity components and pressure and viscous stresses of the earlier homework. Assume that ℓ is much larger than the radius of the sphere R, so that you can ignore the $O(1/\ell^4)$ terms in the viscous stresses. If the stresses in your earlier homework solution were not correct, get the correct ones from the posted homework.

8. Do the integrals and take the limit that $R/\ell \to 0$. Do you get the correct drag force of the sphere?

2. In an earlier homework set, 7.1, you solved a *viscous* flow around a rotating axis. This flow has circular streamlines. For this viscous flow, how much of the centripetal acceleration of the fluid particles comes from the pressure force per unit volume and how much from the viscous force per unit volume?

3. In an earlier homework set, 5.2, you examined "ideal stagnation point flow." Have another look at that solution (the correct solution is on the web). Was the pressure given by the Bernoulli law, even though the flow was assumed to be viscous? So, was the viscous stress zero? What was it? Was the viscous force per unit volume zero? The same things happen for any incompressible flow that is "irrotational," i.e. for which the vorticity is zero. Such flows are also called "ideal" flows or "potential" flows. Now consider another irrotational flow that you looked at in an earlier homework, 4.6. This flow was unsteady. Was the viscous stress tensor zero? What was it? Was the viscous force per unit area zero? If so, then apparently the pressure was given by an extended Bernoulli law that applies to *unsteady* flow.

4. Derive the major head loss and friction factor for laminar pipe flow. Explain all reasoning. The solution for the pressure distribution in pipe flow can be found in almost any fluids book. Sketch a Moody diagram to show how the friction changes when the flow in the pipe becomes turbulent.

5. Consider the below graph for the minor head losses due to sudden changes in pipe diameter:

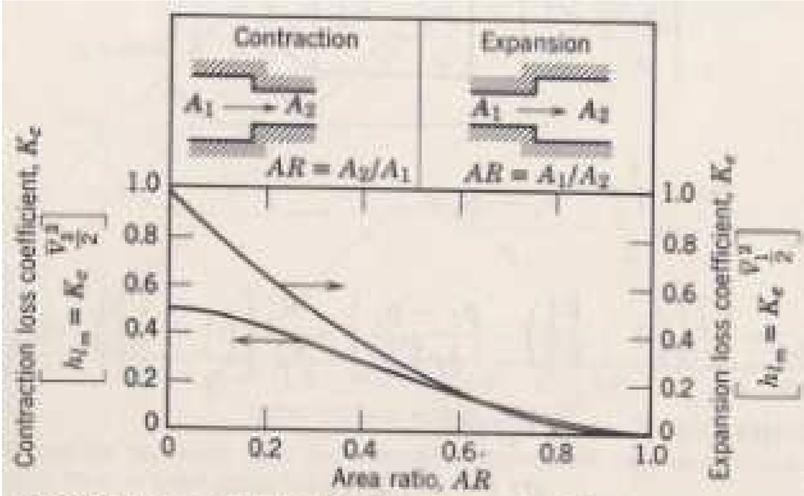

Fig. 8.16 Loss coefficients for flow through sudden area changes (Ref. 5).

Discuss the following issues as well as possible from the sort of flow you would expect.

(a) How come the head loss becomes zero for an area ratio equal to 1? Does that not violate thermodynamics? There is always some viscous friction, surely?

(b) Why would the head loss be exactly one for a large expansion? Coincidence?

(c) Why would the head loss be less than one if the expansion is less? If the expansion is less, is not the pipe wall in the expanded pipe closer to the flow, so should the friction with the wall not be more??

(d) Why is there a head loss for a sudden contraction? The mechanism cannot be the same as for the sudden expansion, surely? Or can it?

9 HW 9

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1. A stationary Newtonian fluid occupies the half space above a horizontal doubly-infinite plate. Take the y coordinate of your Cartesian coordinate system to be normally up wards from the plate. At time $t = 0$, the plate starts accelerating into the positive x direction with a constant *acceleration* $\dot{U}$, so that its velocity is $\dot{U}t$. Assuming that u does not depend on x and z because the plate is infinite in both directions, that w remains zero because of symmetry, and that the (kinetic) pressure at infinite y is a constant, simplify the Navier-Stokes to give equations for the kinetic pressure and velocity fields. Solve the equation for the pressure field. Reduce the equation for the velocity field to a single partial differential and find its initial and boundary conditions. Then use dimensional analysis to argue that the solution must have a similarity form. Derive the ordinary differential equation that the nondimensional velocity profile $f(y/\sqrt{4\nu t})$ must satisfy for this flow.

10 HW 10

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1. Differentiate the equation obtained in the previous question twice; if your equation is correct, f'' should satisfy the same equation as f_{Stokes} for Stokes' first problem. So f'' too should be a combination of a multiple of the complementary error function and a constant. Then two integrations should produce your desired function f, if you take account of the boundary conditions. Your task is now to find a simple expression for f in terms of the (complementary) error function and elementary functions. Note: you might want to google "repeated integrals of the error function." Note: Another way of doing this is to change the order of integration in the double integrals you encounter. For example, to find f' from f'', the integral of erfc can be written:

$$g(\eta) = \int_{\eta_1=\eta}^{\infty} \operatorname{erfc}(\eta_1)\,\mathrm{d}\eta_1 = \int_{\eta_1=\eta}^{\infty} \left(\int_{\eta_2=\eta_1}^{\infty} \frac{2}{\sqrt{\pi}} e^{-\eta_2^2}\,\mathrm{d}\eta_2 \right) \mathrm{d}\eta_1$$

   the second equality from the definition of the complementary error function. If you change the order of integration to integrate η_1 *first*, you can do the first integral since the integrand does not depend on η_1. To figure out the new limits of integration, simply draw the original region of integration in the η_1, η_2 plane and look at it. This trick will give you f' in terms of a single integral. Repeat the trick to find f from f' and identify in terms of erfc. Write the velocity field.

2. For an ideal point vortex at the origin, the velocity field is given in cylindrical coordinates r, θ, z by

$$\vec{v} = \frac{\Gamma}{2\pi r} \hat{\imath}_\theta$$

   Show that the vorticity $\vec{\omega} = \nabla \times \vec{v}$ of this flow is everywhere zero. Now sketch a contour (closed curve) C that loops *once* around the vortex at the origin, in the counter-clockwise direction. In fluid mechanics, (for *any* flow, not just this one), the "circulation" Γ of a contour is *defined* as

$$\bar{\Gamma} = \oint_C \vec{v} \cdot \mathrm{d}\vec{r}$$

   Here the integration starts from an arbitrary point on the contour and loops back to that point in the counter-clocwise direction. Evaluate the circulation of your contour around the vortex. *Do not take a circle as contour C; take a square or a triangle or an arbitrary curve.* Of course you know that in polar coordinates an infinitesimal change $\mathrm{d}\vec{r}$ in position is given by

$$\mathrm{d}\vec{r} = \hat{\imath}_r \mathrm{d}r + \hat{\imath}_\theta r \mathrm{d}\theta$$

(If not, you better also figure out what it is in spherical.) You should find that Γ has a nonzero value for your contour.

3. So far so good. But the Stokes theorem of Calculus III says

$$\oint \vec{v} \cdot \mathrm{d}\vec{r} = \int_A \nabla \times \vec{v} \cdot \vec{n} \, \mathrm{d}A$$

where A is an area bounded by contour C. You just showed that the left hand side in this equation is *not* zero, but that the right hand side is because $\nabla \times \vec{v}$ is. Something is horribly wrong???! To figure out what is going on, instead of using an ideal vortex, use the Oseen vortex from your notes. To simplify this, now take your contour C to be (the perimeter of) a circle around the origin in the x, y-plane, and take area A to be the inside of that circle in the x, y-plane. Do both the contour integral and the area integral. In this case, they should indeed be equal. Now in the limit $t \downarrow 0^+$, the Oseen vortex becomes an ideal vortex. So if you look at a very small time, you should be able to figure out what goes wrong for the ideal vortex with the Stokes theorem. You might want to plot the vorticity versus r for a few times that become smaller and smaller. Based on that, explain what goes wrong for $t \downarrow 0^+$. Is the area integral of the ideal vortex really zero?

4. Do bathtub vortices have opposite spin in the southern hemisphere as they have in the northern one? Derive some ballpark number for the exit speed and angular velocity of a bathtub vortex at the north pole and one at the south pole, assuming that the bath water is initially at rest compared to the rotating earth. Use Kelvin's theorem. Note that the theorem applies to an inertial frame, not that of the rotating earth. So assume you look at the entire thing from a passing star ship. (But define the direction of rotation as the one someone on earth looking at the bathtub sees.) What do you conclude about the starting question? In particular, how do you explain the bathtub vortices that we observe?

5. Consider a two-dimensional cylindrical "balloon" of radius R surrounded by an incompressible fluid with an ideal vortex flow field. If we lower pressure inside the balloon, its radius decreases. Then there is also an ideal "sink" flow field proportional to $-\dot{R}$. The complete ideal flow field is then:

$$\vec{v} = \frac{\bar{\Gamma}}{2\pi r}\hat{\imath}_\theta + \frac{R\dot{R}}{r}\hat{\imath}_r$$

Now if this is a Newtonian fuid, over time viscous boundary layers would develop around the surface of the balloon that would propagate outwards and the rotational motion would slow down. But suppose we apply just enough of an axial moment on the balloon to keep it rotating with the ideal flow fluid velocity? Then we have a *viscous no-slip* flow around a body that is also an *ideal* one, i.e. an irrotational flow, i.e. one with zero vorticity. Sounds interesting?

   (a) Is the above flow indeed irrotational?

(b) Integrate the circulation along a circular fluid contour around the cylinder. What is it?

(c) Is the ring of fluid particles right at the expanding balloon surface a material contour? Why?

(d) Suppose R decreases in time like $R = R_0 t_0 / t$, what happens when time increases from t_0 to 10 t_0? In particular, what does the Kelvin theorem say about what velocity component of the fluid particles at the surface? And what about the other velocity component? What happens to the *angular* velocity Ω at which the cylinder must rotate?

(e) What is the moment per unit axial length that must be exerted on the cylinder to keep it rotating at the right speed? (Ignore the inertia of the balloon.) Does the moment become infinite when R tends to zero? Should it not take more and more effort to keep the flow rotating when total dissipation $\int \varepsilon \, \mathrm{d}V$ increases.

11 HW 11

In this class,

- *Questions must be answered in order asked.*
- *Solutions must be neat.*
- *You must use the given symbols.*
- *You must show all reasoning.*
- *Copying is never allowed, even when working together.*

1. A Boeng 747 has a maximum take-off weight of about 400,000 kg and take-off speed of about 75 m/s. The wing span is 65 m. Estimate the circulation around the wing from the Kutta-Joukowski relation. This same circulation is around the trailing wingtip vortices. From that, ballpark the typical circulatory velocities around the trailing vortices, assuming that they have maybe a diameter of a quarter of the span. Compare to the typical take-off speed of a Cessna 52, 50 mph.

2. Model the two trailing vortices of a plane as two-dimensional point vortices (three-dimensional line vortices). Take them to be a distance 2ℓ apart, and to be a height h above the ground. Take the ground as the x-axis, and take the y-axis to be the symmetry axis midway between the vortices. Now:

   (a) Identify the mirror vortices that represent the effect of the ground on the flow field. Make a picture of the x, y-plane with all vortices and their directions of circulation.

   (b) Find the velocity at an arbitrary point x on the ground due to all the vortices.

   (c) From that, apply the Bernoulli law to find the pressure changes that the vortices cause at the ground. Sketch this pressure against x for both h significantly greater than d and vice-versa.

   (d) Also find the velocity that the right-hand non-mirror vortex R experiences due to the other vortices. In particular find the Cartesian velocity components $u_{\rm R}$ and $v_{\rm R}$ in terms of Γ, h and ℓ.

3. Continuing the previous question, the right non-mirror vortex R moves with the velocity that the other vortices induce:

$$\frac{{\rm d}\ell}{{\rm d}t} = u_{\rm R} \qquad \frac{{\rm d}h}{{\rm d}t} = v_{\rm R}$$

   If you substitute in the found velocities and take a ratio to get rid of time, you get an expression for ${\rm d}h/{\rm d}\ell$. Integrate that expression using separation of variables to find the trajectory of the vortices with time. Accurately draw these trajectories in the x, y-plane, indicating any asymptotes. Do the vortices end up at the ground for infinite time, or do they stay a finite distance above it?

4. Describe transverse ideal (inviscid) flow around a circular cylinder of radius r_0 using a streamfunction approach. Write the partial differential equation to be satisfied by

the streamfunction in an appropriate coordinate system. Also write the boundary conditions at the cylinder surface, $r = r_0$, and the boundary condition at infinity, $r \to \infty$.

5. Find the streamfunction of the previous question. To do so, guess it to be a single separation of variables term of the form $f(r)g(\theta)$. Assume an appropriate form for $g(\theta)$ and then find $f(r)$. Note: the equation that you get for $f(r)$ should have two different solutions of the form r^n for some n.

12 HW 12

In this class,

- *Questions must be answered in order asked.*
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1. From your streamfunction solution of the previous question, find the pressure on the surface of the cylinder. Integrate the Cartesian components of the pressure force to find the lift and drag per unit span that the cylinder experiences. Disappointing results? Add an ideal vortex of strength $-\Gamma$ to the flow around the cylinder. This adds an additional velocity $-\hat{\imath}_\theta\Gamma/2\pi r$. (Note that this addition does not lead to violation of the boundary condition $v_r = 0$ on the cylinder surface.) Find the pressure and then the lift and drag forces. You should find the D'Alembert result that the drag is zero, and the Kutta-Joukowski result that the lift is $\rho U\Gamma$ per unit span. They apply for any finite body sitting in an incoming uniform flow if the flow is ideal (no vorticity in the fluid.)

2. When the cylinder of the last question is accelerating however, D'Alembert no longer applies. The unsteady term in the potential Bernoulli law comes into play. You must be careful here, however: if you move along with the cylinder, your coordinate system is no longer an inertial one and the potential Bernoulli law as written in class does not apply. To keep it simple, assume that the cylinder is moving only along the x-axis of a true inertial coordinate system. Take its center point position x_0 to be some arbitrary function of time $x = x_0(t)$. Moving along with the cylinder, the fluid at infinity seems to move in the negative x-direction with speed $-\dot{x}_0$. So the complex velocity potential that you see when moving along with the cylinder is

$$F' = -\dot{x}_0\left(z' + \frac{a^2}{z'}\right) \qquad z' = z - x_0 = (x - x_0) + \mathrm{i}y$$

Here a is the radius of the cylinder and primes denote quantities perceived by someone moving along with the cylinder. But we need the flow in the inertial system to find the pressure. The $-\dot{x}_0 z'$ term is the *apparent* uniform flow velocity caused by the motion of the observer. An observer in the inertial coordinate system does not observe this term; the fluid at infinity is at rest compared to this observer, so in the inertial system

$$F = -\dot{x}_0\frac{a^2}{z'}$$

Noting that $\mathrm{d}/\mathrm{d}z$, keeping time constant, is the same as $\mathrm{d}/\mathrm{d}z'$, show then that for an inertial observer

$$W = \dot{x}_0\frac{a^2}{(x - x_0 + iy)^2} \qquad \phi = -\dot{x}_0\frac{a^2(x - x_0)}{(x - x_0)^2 + y^2}$$

the latter from first multiplying top and bottom of F with $\bar{z}'$. Now find the pressure. The expression for W allows you to find the kinetic energy. When differentiating ϕ with respect to time, make sure to differentiate every x_0 and $\dot{x}_0$, not just half of them. Next evaluate the pressure in particular on the surface, by noting that on the surface

$$x - x_0 = a\cos\theta' \qquad y = a\sin\theta'$$

There should be one additional term that you did not have for the nonaccelerating cylinder. Show that it produces an additional pressure force

$$F_x = -\rho\pi a^2 \ddot{x}_0$$

To balance this force of the fluid on the cylinder, you will have to exert the opposite force. So the force above represents a drag D. Of course, if the cylinder has mass m per unit span, to give it acceleration $\ddot{x}_0$ you *also* have to apply a force $F_x = m\ddot{x}_0$. The total force you must exert is therefore

$$F_x = (m + \rho\pi a^2)\ddot{x}_0$$

So apparently the surrounding fluid exerts an additional force on the cylinder that acts as if you have to accelerate an additional mass $\rho\pi a^2$. This additional mass is called the "added mass" or "apparent mass". It expresses the fact that in accelerating the cylinder, you must *also* do work to add kinetic energy to the fluid in its vicinity. For a circular cylinder, the apparent mass happens to be exactly that of a cylinder of fluid of that radius. In general however, the apparent mass is different from that of a body of fluid of the same shape.

3. Reconsider the two trailing vortices above the ground of the previous homework. In this case however, write down the total complex potential due to the four vortices first. Note: to find a source sitting at z_0 instead of at the origin, replace z by $z - z_0$. For one vortex, z_0 might be $\ell + \mathrm{i}h$. Differentiate the total potential with respect to z to find the total W. From that find the pressure using complex conjugates. Check that you get the same pressure as before. Also find the streamfunction of one vortex and its mirror. Evaluate at the ground, where $z = x$, and then show that the streamfunction is a constant, zero, at the ground as it should. Note, for any complex number $a = a_r + \mathrm{i}a_i = |a|e^{\mathrm{i}\alpha}$, where a_r, a_i, and α are complex numbers,

$$\ln(a) = \ln(|a|) + \mathrm{i}\alpha \qquad |a| = \sqrt{a_r^2 + a_i^2} \qquad \alpha = \arctan(a_i/a_r)$$

If a is a ratio b/c=$|b|e^{\mathrm{i}\beta}/|c|e^{\mathrm{i}\gamma}$, note that $|a| = |b|/|c|$ and $\alpha = \beta - \gamma$.

4. Consider a wall that for $x > 0$ is along the x-axis. A fluid is flowing in the *minus* x-direction along this wall. At the origin however, the wall bends upwards by 30 degrees, producing an inside corner of 150 degrees. Find the expression for the complex velocity potential of this flow. To find the sign of the constant, find the velocity at a single, easy point, and check its sign. As noted, the flow must be going in the negative x-direction. Find the streamfunction and from that, sketch the streamlines. Find the velocity, and

so show that the corner point is a stagnation point. Find the wall pressure and sketch its distribution with x. In a real viscous flow at high Reynolds number, a thin boundary layer along the wall upstream of the corner will be unable to withstand much of the adverse pressure gradient slowing it down. So the boundary layer will separate *before* it reaches the corner, and reattach to the wall downstream of it. Based on that, sketch how you think the *viscous* streamlines will look like.

Next assume that at the origin the wall bends downwards by 30 degrees, producing a 210 degree corner. Repeat the analysis and sketching. In this case you should find that there is infinitely large negative pressure at the corner. The boundary layer approaching the corner now finds things plain sailing until it reaches the corner. But right at the corner it is not going to go around it, as that would produce a very strong adverse pressure gradient. Instead the boundary layer just keeps going straight along the x-axis immediately behind the corner. That effectively eliminates the corner and its associated pressure gradient. This effect is why flows around airfoils with sharp trailing edges and sufficiently blunted leading edges satisfy the Kutta-Joukowski condition.

Finally, if the flow is unsteady (i.e. if the constant in your complex potential varies with time), how does that affect whether the ideal flow at the corner has stagnation or infinitely negative pressure?

13 HW 13

In this class,

- *Questions must be answered in order asked.*
- *Solutions must be neat.*
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1. Consider the combination of a uniform flow of velocity U in the x-direction with a source of strength q (volumetric flow rate per unit span) at the origin. (a) Find the complex conjugate velocity W and from that the stagnation point(s). (b) Find the equation for the streamline(s) passing through the stagnation point(s). Draw this and other streamlines. (c) If you "solidify" (replace by a solid material) the fluid coming out of the sink, you get the flow around a solid body. Shade this solid body in your graph. (d) What is the maximum cross sectional (in the y-direction) area of the body?

2. Write down the complex velocity potential for flow around a cylinder of radius 2 in a complex ζ plane if the velocity at infinity is $U\hat{\imath}$. What is the maximum velocity on the surface of the cylinder? Then use the Joukowski transformation to map the circle to an ellipse in a complex z-plane. What is the aspect ratio of the ellipse? What is the maximum velocity on the surface of the ellipse? (Use the chain rule of differentiation to find W.) What is the pressure coefficient $C_p = (p - p_\infty)/\frac{1}{2}\rho U^2$ at that point?

3. Write down the complex velocity potential for the combination of (a) a uniform flow of magnitude U in the positive x-direction, (b) a source of strength $q = C/\varepsilon$ at the origin, (c) a sink (negative source) of strength $q = C/\varepsilon$ at $z = \varepsilon$. Show that in the limit $\varepsilon \to 0$, you get the potential flow around a cylinder. Find the radius of that cylinder.

4. Consider the complex velocity potential

$$F = U\left(\zeta_0 + \frac{r_0^2}{\zeta_0}\right) + \frac{\mathrm{i}\Gamma}{2\pi}\ln\frac{\zeta_0}{r_0}$$

where U, r_0, and Γ are real positive constants. (a) What sort of flow is this? Sketch the streamlines for $\Gamma = 0$, $0 < \Gamma < 4\pi r_0 U$, and $4\pi r_0 U < \Gamma$ in separate complex ζ_0 planes. (b) Consider the conformal transformation $\zeta = \zeta_0 e^{\mathrm{i}\alpha}$. What does this transformation do? In particular sketch the streamlines of velocity potential F in separate complex ζ planes for α, say, 0.25 radians (about 15 degrees). (c) Find the value of Γ for which the complex conjugate velocity W at the "trailing edge" $\zeta = r_0$ is zero. (Rewrite F first in terms of ζ.) (d) What is the lift per unit length l of the cylinder for this circulation? (e) Assume now that $r_0 = 1$. In that case, the Joukowski transformation $z = \zeta + 1/\zeta$ maps the cylinder into a flat plate airfoil. Show that the lift coefficient

$$c_l = \frac{l}{\frac{1}{2}\rho U^2 c} = 2\pi \sin\alpha$$

for that airfoil, if c is the chord, i.e. the distance between trailing edge and leading edge of the flat plate in the z-plane.

5. Continuing the previous question, if we want a Joukowski airfoil instead of a flat plate, we can make the radius r_0 slightly bigger than 1. In that case, the trailing edge is of course no longer at $\zeta = 1$, but at $\zeta = r_0$. Then the usual Joukowski transformation no longer works correctly to produce a sharp trailing edge. But we can fix this by shifting the transformation by an amount s equal to the shift in trailing edge:

$$z = \zeta - s + \frac{1}{\zeta - s} \quad \text{where} \quad s = r_0 - 1$$

Noting this, download the matlab program airfoil.m[1] and read through it. Now use the program to make a nice picture of the streamlines around a cambered Joukowski airfoil at an angle of attack. Note: you may want to make use of an undocumented feature of the program. Explain what that feature is.

6. (a) Sketch the Joukowski airfoil of the previous question and then sketch and describe the boundary layer coordinates and velocity components that you would use in finding the boundary layer solution around the airfoil. (Do so at a nontrivial arbitrary point in the boundary layer to make the features clear.) (b) Do the same for the boundary layer around the ellipse, taking as the boundary layer starting point the front (upstream) stagnation point. What are the boundary conditions at the wall? What is the initial condition for $u(0, y)$ at the start of the boundary layer? What is the boundary condition for the boundary layer u when $y/\sqrt{\nu}$ becomes "infinite" at the top point of the ellipse? Suppose that the circle in the complex ζ-plane is given as $\zeta = 2e^{\mathrm{i}\phi}$. Then for any arbitrary ϕ, at the corresponding x-position on the ellipse, what is the boundary condition for the boundary layer velocity component u when $y/\sqrt{\nu}$ becomes "infinite?"

7. The Blasius solutions puts a semi-infinite plate along the positive x-axis in a uniform flow in the x-direction. Then it finds the boundary layer that develops at large Reynolds numbers along that plate. But consider the flow towards a sink at the origin of strength $q = 2\pi s$:

$$F = -s \ln z$$

We can put a semi-infinite plate along the positive x-axis in that flow instead. Find the boundary layer solution for that flow. In particular, first find the potential flow velocity on the surface of the plate. Then write the boundary layer equations. Write out the boundary condition $u \to u_e$ when $y/\sqrt{\nu}$ becomes "infinite;" in particular identify u_e. Now use dimensional analysis to find the form of the solution, noting that s, not some U, is the given constant in this problem. Put the obtained streamfunction expression in the boundary layer equations. Note, you may want to include a minus sign in your expression for the streamfunction, since the flow is now in the negative x-direction. Do not forget that unlike for Blasius, $\partial p/\partial x$ is not zero in this case.

(The obtained differential equation can be solved analytically without a computer, a rarity in boundary layer theory. Can you do this for extra credit? The key step is to define new variables $\alpha = f'$ and $\phi = f''$ and then write an equation for $\mathrm{d}\phi/\mathrm{d}\alpha$ in terms of α and ϕ, noting that $f''' = \mathrm{d}f''/\mathrm{d}\eta = (\mathrm{d}f''/\mathrm{d}f')(\mathrm{d}f'/\mathrm{d}\eta$.)

[1] `http://www.eng.fsu.edu/~dommelen/courses/flm/progs/jou_air/airfoil.m`

8. According to potential flow theory, what would be the lift per unit span of a flat-plate airfoil of chord 2 m moving at 30 m/s at sea level at an angle of attack of 10 degrees? What would be the viscous drag if you compute it as if the airfoil is a flat plate aligned with the flow with that chord and the flow is laminar? Only include the shear stress over the last 98% of the chord, since near the leading edge the shear stress will be much different from an aligned flat plate. What is the lift to drag ratio? Comment on the value. Use $\rho = 1.225$ kg/m^3 and $\nu = 14.5\ 10^{-6}$ m^2/s.

9. Assume that a flow enters a two dimensional duct of constant area. If no boundary layers developed along the wall, the centerline velocity of the flow would stay constant. Assuming that a Blasius boundary layer develops along each wall, what is the correct expression for the centerline velocity in the entrance part of the duct?

10. Continuing the previous question. Approximate the Blasius velocity profile to be parabolic up to $\eta = 3$, and constant from there on. Sketch the duct, including the lines that correspond to $\eta = 3$ and the lines that correspond to the displacement thickness. At what point along the duct would you estimate that developed flow starts based on the parabolic approximation? Sketch the velocity profile at this point, as well as at the start of the duct, and at the point of the duct where the range $0 \le \eta \le 3$ corresponds to $\frac{1}{8}$ of the duct height accurately in a single graph. Remember the previous question while doing this!