Vector Matrix notation:

$$\vec{R} = \overline{\overline{\tau}}^T \vec{n}$$

where superscript T means "transpose",
i.e. ~~swap~~ turn the rows into columns or vice versa.

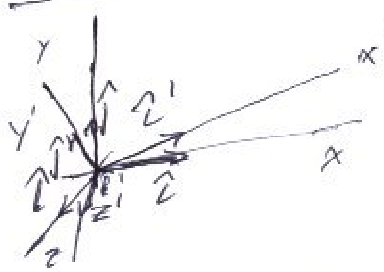
Note: Normally $\overline{\overline{\tau}}$ is "symmetric", meaning that $\overline{\overline{\tau}}^T = \overline{\overline{\tau}}$.
(Exception if there are external moments on the fluid elements that do not disappear in the limit of zero size.)

If symmetric, $\tau_{12} = \tau_{21}$, $\tau_{31} = \tau_{13}$ $\tau_{23} = \tau_{32}$
$\tau_{xy} = \tau_{yx}$ → in 2D, often written as simply τ

Index notation: $R_i = \sum \tau_{ij}^T n_j$ "sum over neighboring indices" $= \tau_{ij}^T n_j$

~~end~~ Changes under rotation of coordinate system

$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} \quad \text{old representation}$$



$$\vec{r}\,' = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} r_1' \\ r_2' \\ r_3' \end{pmatrix} \quad \text{new representation}$$

Transformation in vector matrix notation

$$\vec{r} = C\vec{r}\,' \qquad \vec{r}\,' = C^{-1}\vec{r} = C^T\vec{r}$$

for coordinate rotations

$C = (\hat{\imath}' \; \hat{\jmath}' \; \hat{k}')$: new basis vectors written in terms of old ones

$$= \begin{pmatrix} \hat{\imath}'\cdot\hat{\imath} & \hat{\jmath}'\cdot\hat{\imath} & \hat{k}'\cdot\hat{\imath} \\ \hat{\imath}'\cdot\hat{\jmath} & \hat{\jmath}'\cdot\hat{\jmath} & \hat{k}'\cdot\hat{\jmath} \\ \hat{\imath}'\cdot\hat{k} & \hat{\jmath}'\cdot\hat{k} & \hat{k}'\cdot\hat{k} \end{pmatrix}$$

where $\hat{\imath}'\cdot\hat{\imath} = \cos\theta_{\hat{\imath}'\hat{\imath}}$
$\hat{\imath}'\cdot\hat{\jmath} = \cos\theta_{\hat{\imath}'\hat{\jmath}}$ etc.

~~For tensors~~ 2nd: For matrices

$$\overline{\overline{\tau}}\,' = C^T \overline{\overline{\tau}}\, C \qquad \text{(compute row column)}$$

$$\overline{\overline{\tau}} = C\, \overline{\overline{\tau}}\,' C^T$$

($\overline{\overline{\tau}}$, $\overline{\overline{\tau}}$ understood)

Index notation $\tau_{ij} = c_{ik}\, \tau_{kl}\, c_{jl}$

Note Cartesian tensors: form of transformation the same for index l as for k.

Diagonalization theorem

~~For~~ You can turn any symmetric real matrix into a diagonal one if you rotate the axis system just right.

$$\overline{\overline{\tau}} = \begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{pmatrix} \xrightarrow[\text{right}]{\text{rotate just}} \overline{\overline{\tau}}' = \begin{pmatrix} \tau_1 & 0 & 0 \\ 0 & \tau_2 & 0 \\ 0 & 0 & \tau_3 \end{pmatrix}$$

τ_1, τ_2, τ_3 are called <u>the principal stresses</u>

Note: the ~~new~~ needed new basis vectors $\hat{i}', \hat{j}'$, and $\hat{k}'$ are ~~"eigenvalues~~ "eigenvectors" of $\overline{\overline{\tau}}$, i.e.

$$\overline{\overline{\tau}}\,\hat{i}' = \tau_1 \hat{i}' \quad \overline{\overline{\tau}}\,\hat{j}' = \tau_2 \hat{j}' \quad \overline{\overline{\tau}}\,\hat{k}' = \tau_3 \hat{k}'$$

$$C = (\hat{i}' \; \hat{j}' \; \hat{k}')$$

3 <u>Deformation of small fluid particles</u>

In a constant velocity field, the fluid is not deformed