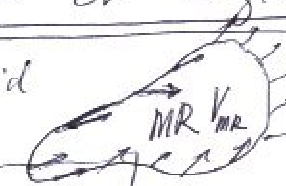
"Control mass" or "Material Region" or "Lagrangian System" Panton Kundu (9)

= some chunk of fluid (or other mass)



S or A_{MR} (imaginary) surface, moves with the fluid

mass: $\boxed{M_{MR} = \int_{MR} dm = \int_{V_{MR}} \rho \, dV}$ ρ = density = mass per unit volume

"mass conservation" "mass is constant" = Continuity equation $\quad \boxed{\frac{DM_{MR}}{Dt} = 0}$

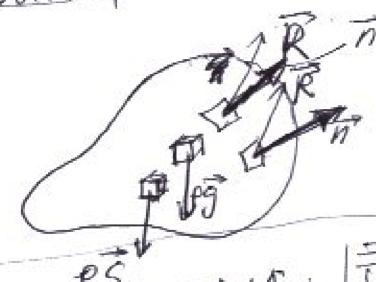
Convention in fluids: use "D" instead of "d" for Lagrangian derivatives

linear momentum $\quad \boxed{\vec{l}_{MR} = \int_{MR} \vec{v}\, dm = \int_{V_{MR}} \rho \vec{v}\, dV}$

momentum equation

$$\boxed{\frac{D\vec{l}_{MR}}{Dt} = \vec{F}_{net,\, external}}$$

momentum is ~~preserved~~ conserved if no external forces

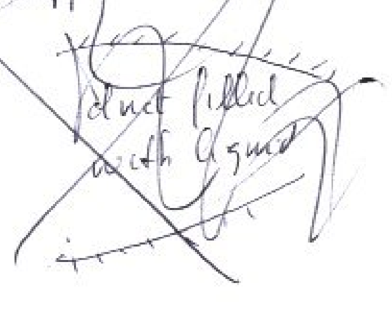


$\vec{F}_{net,ext} = \vec{F}_{\text{surface forces on surface } A_{MR} \text{ of } V_{MR}} + \vec{F}_{\text{body forces on interior fluid particles inside } V_{MR}}$

$$\boxed{\begin{aligned}\vec{F}_{\text{surface forces}} &= \int_{A_{MR}} \vec{R}\, dA = \int_{A_{MR}} \overline{\overline{\tau}}^T \vec{n}\, dA = -\int_{A_{MR}} p \vec{n}\, dA + \int_{A_{MR}} \overline{\overline{\tau}}_{\text{viscous}} \vec{n}\, dA \\ \vec{F}_{\text{gravity body force}} &= \int_{V_{MR}} \vec{g}\, dm = \int_{V_{MR}} \rho \vec{g}\, dV = \vec{g} M_{MR}\end{aligned}}$$

$$\frac{D \int_{MR} \rho \vec{v}\, dV}{Dt} = -\int_{S_{MR}} p \vec{n}\, dA + \int_{S_{MR}} \overline{\overline{\tau}}_{\text{viscous}} \vec{n}\, dA + M_{MR} \vec{g}$$

~~Simple applications:~~



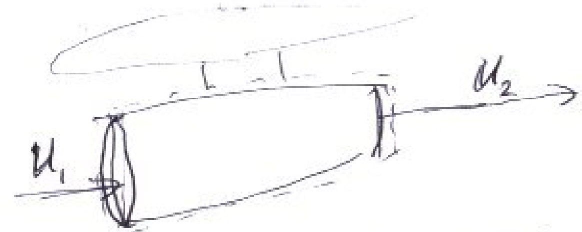
"Control Volume" or "Fixed Region" or "Arbitrary Region"

Not a material region

e.g. rocket

not a chunk of fluid → fluid is lost

e.g. jet engine



not a chunk of fluid: contains a different chunk of fluid at different times

$\boxed{\text{For a control volume, must correct for in and out flow of fluid from the volume}}$

For a ^simple^ duct situation

mass outflow per unit time

$$"\dot{m}_2" = \rho_2 |\vec{v}_2| A_2 = \rho_2 V_2 A_2 \quad \text{if } V_2 = |\vec{v}_2|$$

mass inflow per unit time

$$"\dot{m}_1" = \rho_1 |\vec{v}_1| A_1 = \rho_1 V_1 A_1$$

mass inside $M_{cv} = \int_{V_{cv}} \rho \, dV$

$$\frac{DM_{cv}}{Dt} = \text{inflow} - \text{outflow} = \dot{m}_1 - \dot{m}_2$$

If M_{cv} is constant: inflow = outflow

simple duct filled with fluid, incompressible fluid: $A_1 V_1 = A_2 V_2$

simple duct, steady flow: $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

Simple duct continuity equation