Dependent variables

How about, say, the drag force D?

$\frac{D}{\rho U^2 \ell \, 1} = D^*$ will be the same for the two flows if Re is the same

$1 \rightarrow$ unit length in z-direction

Definition: $\boxed{\text{Drag coefficient} \quad c_D = \frac{D}{\frac{1}{2}\rho U^2 A}}$ ← i.e. some characteristic area.

For a sphere,

C_D ← $\frac{D}{\frac{1}{2}\rho U^2 \pi R^2}$

→ low Reynolds number range, very viscous → $D = 6\pi\mu R U$

↓ better nondimensional constant is here

$$\frac{D}{\mu U R} = 6\pi = \frac{1}{2}\pi c_D \frac{\rho U R}{\mu}$$

$Re = \frac{\rho U R}{\mu}$

For high Reynolds numbers, a typical force is $\underbrace{\rho U^2}_{\text{dynamic pressure}} \ell^2$

For low Reynolds number, a typical force is $\underbrace{\mu \frac{U}{\ell}}_{\text{viscous shear}} \ell^2 = \mu U \ell$

Other nondimensional parameters:

What if the cylinder is oscillating back and forward with frequency Ω and amplitude A?

$$\Omega^* = \boxed{\frac{\Omega l}{U} \text{ : "Strouhal" number}}$$

$$A^* = \frac{A}{l}$$

} these numbers now have to be the same for flows ① and ② too.

What if there is a free surface and so gravity is important?

$$\frac{\rho\vec{g}}{\rho U^2/l} = \frac{gl}{U^2}\underbrace{\frac{\vec{g}}{g}}_{\vec{g}/g} = \frac{gl}{U^2}\vec{g}^*$$

$$\boxed{Fr = \frac{U}{\sqrt{gl}} \text{ ; "Froude" number}}$$

~~Dependent variab~~

What if the flow is not incompressible?

For simplicity, assume Euler equations: leave out p_∞

$$\vec{r} = l\vec{r}^* \quad \vec{v} = U\vec{v}^* \quad p = \rho U^2 p^* \quad t = \frac{l}{U}t^*$$

$$\rho = \rho_\infty \rho^*$$

$$\frac{1}{\rho}\frac{D\rho}{Dt} + \operatorname{div}\vec{v} = 0 \Longrightarrow \frac{1}{\rho^*}\frac{D\rho^*}{Dt^*} + \nabla^*\vec{v}^* = 0$$

$$\rho\frac{D\vec{v}}{Dt} = -\nabla p \cancel{\pm\rho\vec{g}} \Longrightarrow \rho^*\frac{D\vec{v}^*}{Dt^*} = -\nabla^* p^*$$

$$\frac{Dp}{Dt} = a^2\frac{D\rho}{Dt} \Longrightarrow \frac{Dp^*}{Dt^*} = \frac{a_\infty^2}{U^2}a^{*2}\frac{D\rho^*}{Dt} \qquad a^* = \frac{a}{a_\infty}$$

$$\boxed{M = \frac{U}{a_\infty} \text{: "Mach" number}}$$ of the incoming flow

Hold on: for a generic substance, say, $\frac{a^2}{a_c^2} = \frac{a^2}{a_c^2}\left(\frac{p}{p_c}, \frac{\rho}{\rho_c}\right)$

where subscript c denotes critical quantities

How about a^*??

Assume, say, that

$$\frac{a^2}{a_c^2} = F\left(\frac{p}{p_c}, \frac{\rho}{\rho_c}\right)$$

where c means the value at the critical point

Then

$$a^{*2} = \frac{a^2}{a_\infty^2} = \frac{a_c^2}{a_\infty^2}\frac{a^2}{a_c^2} = a_c^{*2} F\left(\frac{p^*}{p_c^*}, \frac{\rho^*}{\rho_c^*}\right)$$

If F is arbitrary, $\frac{p_\infty}{p_c}$ and $\frac{\rho_\infty}{\rho_c}$ will need to be the same for flows ① and ② too!

Fortunately, an ideal gas has no critical point.
For an ideal gas,

$$a^2 = \gamma\frac{p}{\rho} \quad \text{so} \quad a^{*2} = \frac{\gamma\frac{p}{\rho}}{\gamma\frac{p_\infty}{\rho_\infty}} = \frac{p^*/p_\infty^*}{\rho^*/\rho_\infty^*}$$ no new functions

<u>Viscous compressible</u>:

Need energy equation

$$\boxed{Pr = \frac{\mu c_p}{k} : \text{Prandtl number}}$$