HW#9

$\frac{\partial v}{\partial y} = 0 \rightarrow v = v(x, z, t) \quad v = 0$ at $y = 0$

$\frac{\partial p_{lin}}{\partial y} = 0 \rightarrow p = p(x, z, t) \quad p = p_\infty$ at $y = \infty$

~~$u = f(t, y, \nu$~~

$u = f(y, t, \nu, \dot{U})$

$\frac{u}{\dot{U}} = f(y, t, \nu)$

$\Pi_u \quad \Pi_y$

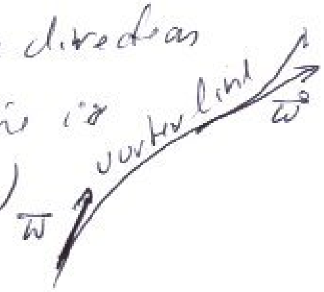
$\frac{u}{\dot{U}}$

$\downarrow \quad \downarrow$

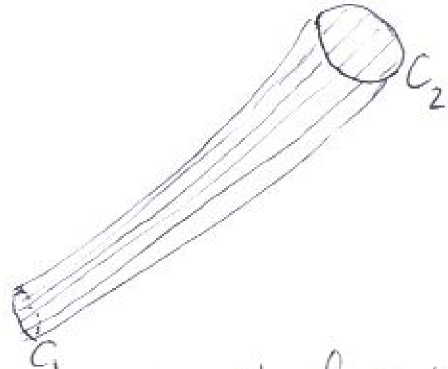
$\frac{u}{\dot{U}t} \quad \frac{y}{\sqrt{4\nu t}} \qquad u = \dot{U} t\, f\left(\frac{y}{\sqrt{4\nu t}}\right)$

Definitions:

A vortex line is a line everywhere in the direction of the vorticity vector. (Like a streamline is everywhere in the direction of the velocity)



A vortex tube is a tube made up out of vortex lines:



The strength of a vortex tube is the circulation around a contour, like C_1 or C_2 above.

Helmholtz theorems

~~Under the~~ If the Kelvin theorem applies

1) Vortex tubes move with the fluid. (Because a fluid contour inside the tube surface retains zero circulation)
2) The strength of a vortex tube is constant along its length.

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Helmholtz theorems

If the Kelvin theorem applies:

1) Vortex tubes move with the fluid
2) The strength of a vortex tube is constant along its length ~~and time~~
3) A vortex tube cannot end within the fluid
   It must either: (WRONG)
   a) end ~~at~~ a ~~solid~~ boundary,
   b) form a closed loop, a "vortex ring"
   ADD → c) keep going without ever doing a) or b).
   ~~LtD~~ Poincaré
4) The strength of a vortex tube is constant in time

Derivations:

1) Because a fluid contour inside the tube surface keeps its zero circulation
2) Because $\int \vec{\omega} \cdot \vec{n}\, dA$ must be zero for any closed surface (like the vortex tube cut-off at C_1 and C_2)
3) Was wrong.
   E.g. a 2D vortex ~~in~~ fluid with expanding, moving slip walls above and below ends at the slip walls
   E.g. vortex lines could keep going around within a torus without ever hitting themselves
4) Because 1) and Kelvin.

Applications