Complex variables

Define $\boxed{z = x + iy}$

Polar form $z = r\cos\theta + i r\sin\theta$

$= r(\cos\theta + i\sin\theta)$

$\boxed{z = re^{i\theta}}$ $\quad$ $\boxed{\text{Euler } e^{i\theta} = (\cos\theta + i\sin\theta)}$

Define $\boxed{F = \varphi + i\psi}$

For the cylinder:

$\cancel{F = U(r\cos\theta + U\frac{r_0^2}{r}}$

$$F_{cyl} = U\left(r + \frac{r_0^2}{r}\right)\cos\theta + iU\left(r - \frac{r_0^2}{r}\right)\sin\theta$$

$$= Ur(\cos\theta + i\sin\theta) + \frac{Ur_0^2}{r}(\cos\theta - i\sin\theta)$$

$$= Ure^{i\theta} + \frac{Ur_0^2}{r}e^{-i\theta}$$

$$\boxed{F_{cyl} = U\left(z + \frac{r_0^2}{z}\right)}$$

~~Any differentiable (analytical) complex function of z is a complex potential flow~~

$\boxed{\text{Any differentiable complex function } F(z) \text{ is a potential flow}}$

$$\frac{dF}{dz} = \frac{\partial \varphi}{\partial x} + i\frac{\partial \psi}{\partial x}$$
$$= -i\frac{\partial \varphi}{\partial y} + \frac{\partial \psi}{\partial y}$$

$$\Longrightarrow \boxed{\begin{array}{l} \dfrac{\partial \varphi}{\partial x} = \dfrac{\partial \psi}{\partial y} \\ \dfrac{\partial \varphi}{\partial y} = -\dfrac{\partial \psi}{\partial x} \end{array}}$$ Cauchy Riemann

$$\Rightarrow \boxed{\begin{array}{l} \dfrac{\partial^2 \varphi}{\partial x^2} + \dfrac{\partial^2 \varphi}{\partial y^2} = 0 \\ \dfrac{\partial^2 \psi}{\partial x^2} + \dfrac{\partial^2 \psi}{\partial y^2} = 0 \\ \text{harmonic, Re, Im} \end{array}}$$

Differentiable : $F = 0$ $F =$ const

$F = z$

calculus F diff $\rightarrow$ cF diff

F, G diff $\rightarrow$ $F+G$ diff

$\rightarrow$ FG diff

F diff $\rightarrow$ $\frac{1}{F}$ diff

$\rightarrow$ polynomials, Taylor series differentiable

Not differentiable : F real (or purely imaginary) and not constant

like $\mathrm{Re}(f)$ $\mathrm{Im}(f)$, $|f|$, See Cauchy Riemann