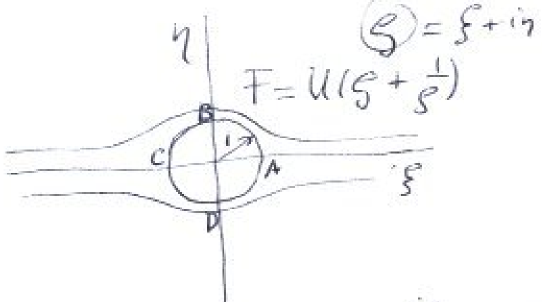
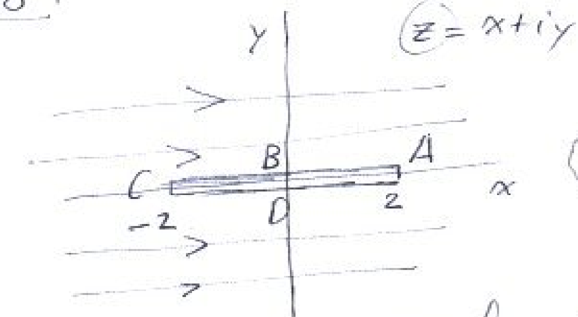
Calculus: Chain rule for change of independent variable, same F

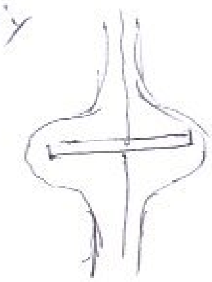
$$\frac{dF}{dz} = \frac{dF}{d\zeta}\frac{d\zeta}{dz}$$

Changing independent variable to get a different flow field is called conformal mapping

Joukowski mapping: $\boxed{z = \zeta + \frac{1}{\zeta}}$



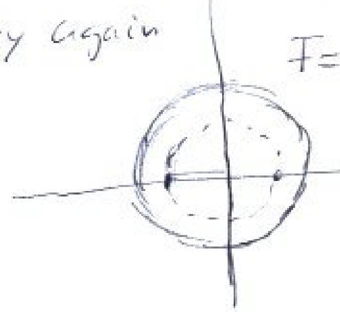


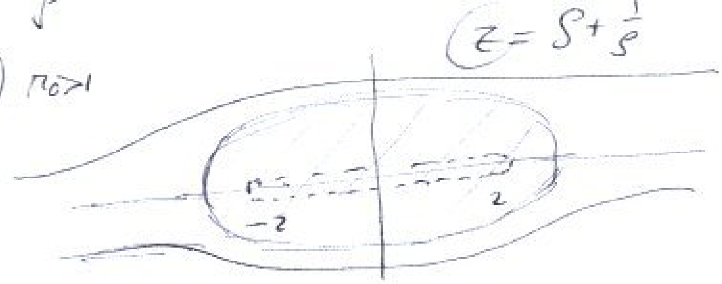


when $\zeta = e^{i\varphi}$ $\quad z = e^{i\varphi} + e^{-i\varphi} = 2\cos\varphi$ is real $\qquad F = -iV\left(\zeta - \frac{1}{\zeta}\right)$

$F = U\left(\zeta + \frac{1}{\zeta}\right)$ $\quad z = \zeta + \frac{1}{\zeta} \rightarrow F = Uz.$

try again $\quad F = U\left(\zeta + \frac{r_0^2}{\zeta}\right)$ $\quad r_0 > 1$





$\zeta = r_0 e^{i\varphi}$ $\quad z = r_0 e^{i\varphi} + \frac{1}{r_0 e^{i\varphi}} =$

$= \left(r_0 + \frac{1}{r_0}\right)\cos\varphi + i\left(r_0 - \frac{1}{r_0}\right)\sin\varphi$

ellipse!

Do implicitly $\quad W = \frac{dF}{dz}$

evaluate as $W = \frac{dF}{dz}\Big/\frac{dz}{d\zeta}$

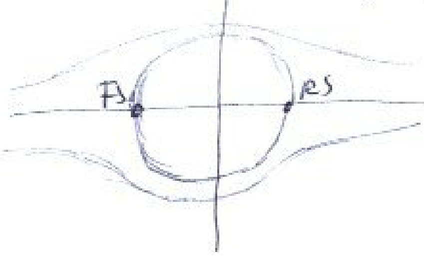
Another so ς_1 plane $\varsigma_1 = \rho_1 e^{i\varphi_1}$

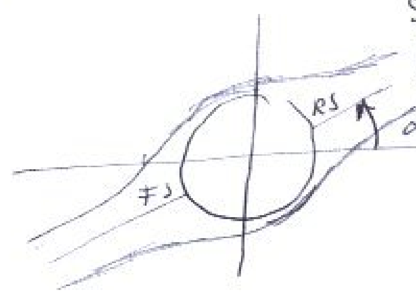
$F = U(\varsigma_1 + \frac{1}{\varsigma_1})$

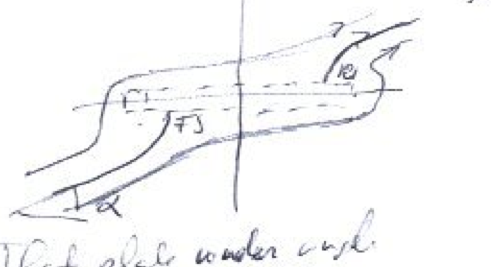
$\varsigma_2 = \varsigma_1 e^{i\alpha} \rightarrow \rho_2 = \rho_1$ but $\varphi_2 = \varphi_1 + \alpha$ → rotates plane by α

$F = U(\varsigma_2 e^{-i\alpha} + \frac{1}{\varsigma_2 e^{-i\alpha}})$

$z = \varsigma_2 + \frac{1}{\varsigma_2}$







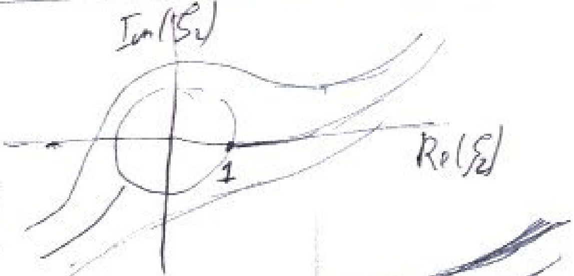
Flat plate under angle of attack α

Kutta condition

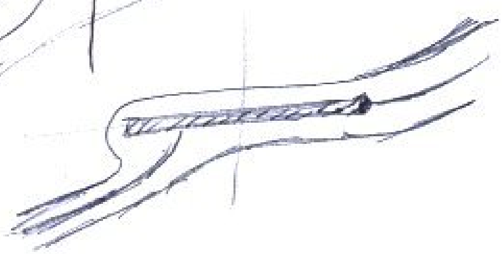
Try again add circulation to move RS to the trailing edge

$\boxed{\text{Kutta condition : Rear stagnation point must be at trailing edge}}$

$$F = U(\varsigma_2 e^{-i\alpha} + \frac{e^{i\alpha}}{\varsigma_2}) + \frac{i\Gamma}{2\pi} \ln \varsigma_2$$



$$W\Big|_{\varsigma_2=1} = 0 = \underbrace{U(e^{-i\alpha} - \frac{e^{i\alpha}}{1^2})}_{-2Ui\sin\alpha} + \frac{i\Gamma}{2\pi 1}$$



$$\rightarrow \Gamma = 4\pi U \sin\alpha$$

Lift $L = \rho U \Gamma = 4\pi \rho U^2 \sin\alpha$

Lift coefficient $c_l = \frac{L}{\frac{1}{2}\rho U^2 c \, 1}$

chord : z from -2 to 2 so $c = 4$

$\boxed{c_l = 2\pi \sin\alpha \approx 2\pi\alpha}$ α in radians

Good for thin airfoils in general

Must round off leading edge:
shift and magnify the cylinder a bit.

$$F = U\left((\zeta_2+\varepsilon)e^{-i\alpha} + \frac{e^{i\alpha}(1+\varepsilon)^2}{(\zeta_2+\varepsilon)}\right) + i\frac{\Gamma}{2\pi}\ln\zeta_2$$