Boundary layer thicknesses

Experimentalists $\delta_{0.99}$ = y-value where u has reached 99% of the u_e value

Stupid $\delta_{1.00} = \infty$! Not typical

Theoreticians $\delta_{\text{Stokes 1}} = \sqrt{4\nu t}$ because it produces standard mathematical functions

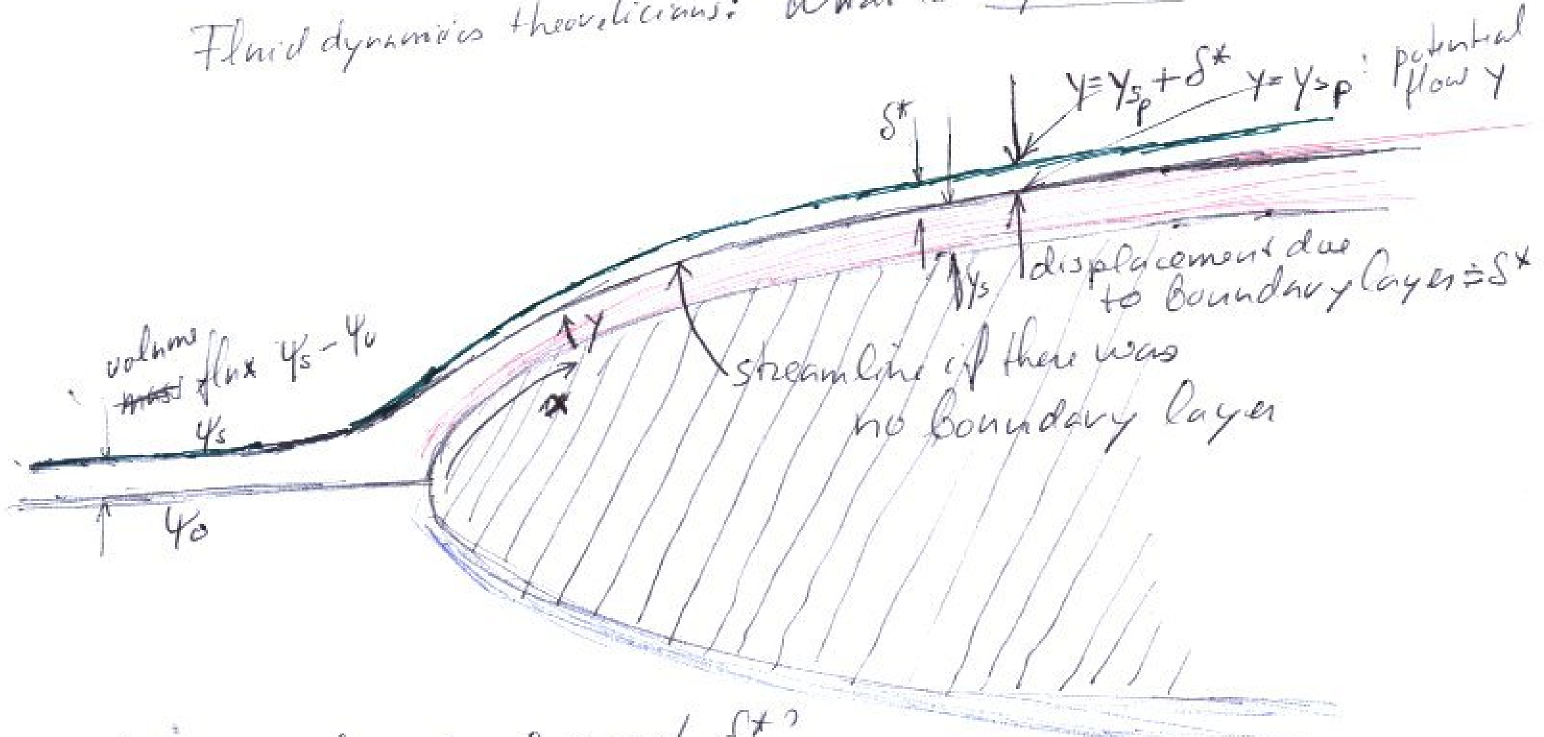
$\left(\mathrm{erf}\left(\frac{y}{\sqrt{4\nu t}}\right) = \mathrm{erf}\left(\frac{y}{\delta}\right)\right.$ not

say $\left.\mathrm{erfc}\left(\frac{1}{2}\frac{y}{\delta}\right)\right)$

smart

$\delta_{\text{Stokes}} = \delta_{0.8427}$

Fluid dynamics theoreticians: What is important??



How much is displacement δ^*?

Same volume flux $\psi_s - \psi_0$ must go between streamline and wall regardless whether there is a boundary layer or not:

$$\underset{\substack{\text{real flow}\\ \text{with b.l.}}}{\int_{\psi_0}^{\psi_s} \vec{v}\cdot\vec{n}\,ds} = \underset{\substack{\text{potential flow}\\ \text{without b.l.}}}{\int_{\psi_0}^{\psi_s} \vec{v}\cdot\vec{n}\,ds}$$

$$\int_0^{y_{s_p}+\delta^*} u\,dy = \int_0^{y_{sp}} u_e\,dy = u_e\,y_{sp}$$

$$= \int_0^{y_{sp}+\delta^*} u_e\,dy + \int_0^{y_{sp}+\delta^*} (u-u_e)\,dy$$

$$\int_0^{y_{sp}+\delta^*} u\,dy = \int_0^{y_{sp}} u_e\,dy$$

$$\int_0^{y_{sp}+\delta^*} u_e\,dy + \int_0^{y_{sp}+\delta^*} (u-u_e)\,dy = \int_0^{y_{sp}} u_e\,dy$$

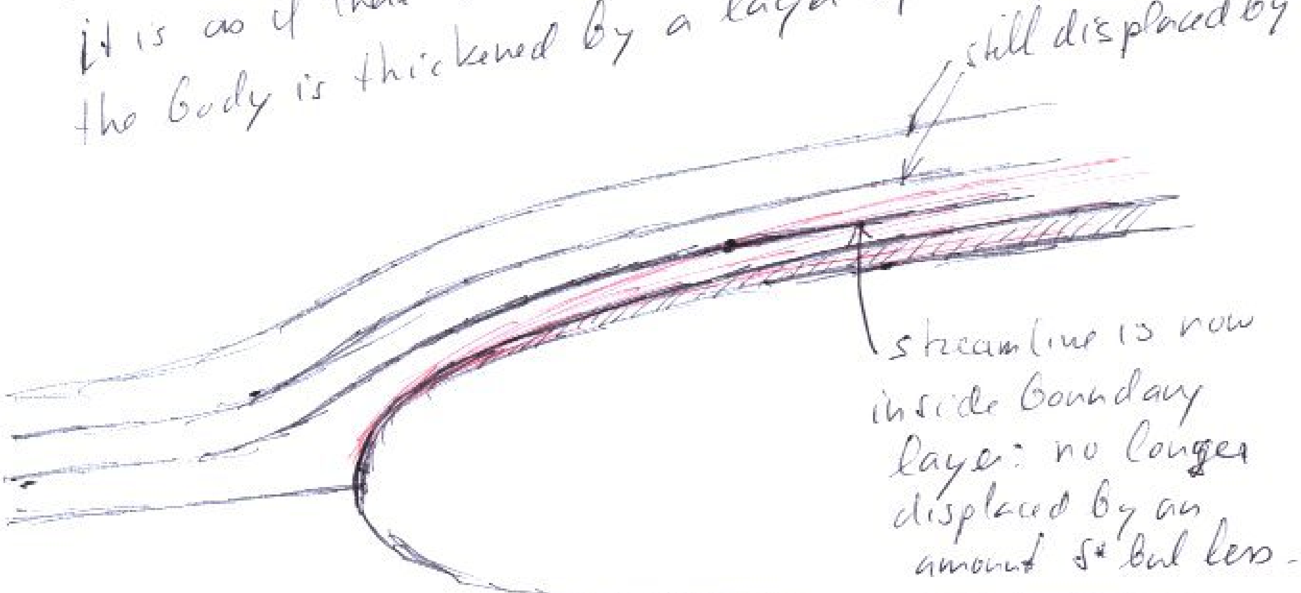
$$\cancel{u_e y_{sp}} + u_e\delta^* + \int_0^{\infty} (u-u_e)\,dy = \cancel{u_e y_{sp}}$$

$$\delta^* = \frac{1}{u_e}\int_0^{\infty} (u_e-u)\,dy$$

$$\boxed{\delta^* = \int_0^{\infty} \left(1-\frac{u}{u_e}\right)dy}$$

Gives the displacement of the potential flow streamlines just above the boundary layer!

~~You can imagine adding a~~
As far as the potential flow streamlines are concerned it is as if there was no boundary layer, but the body is thickened by a layer of thickness δ^*



1) Can find a potential flow for the thickened body → more accurate wall pressure → better boundary layer solution

For a flat plate

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy \qquad \frac{u}{U} = f'(\eta) \qquad \eta = \frac{y}{\sqrt{x\nu/U}}$$

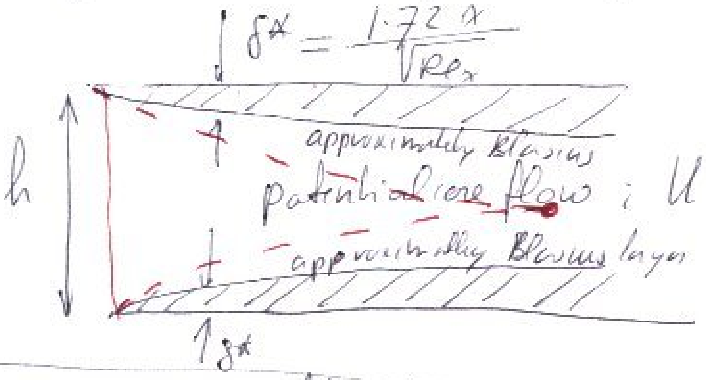
$$= \int_0^\infty (1 - f'(\eta))\, d\eta \, \sqrt{x\nu/U}$$

$$\boxed{\delta^*_{\text{Blasius}} = x \int_0^\infty (1 - f'(\eta))\, d\eta \,/ \sqrt{Re_x}}$$

Note that $\delta^* \div \sqrt{x}$: thickened plate is a thin parabola!

Numerically $\int_0^\infty (1 - f'(\eta))\, d\eta = 1.72$

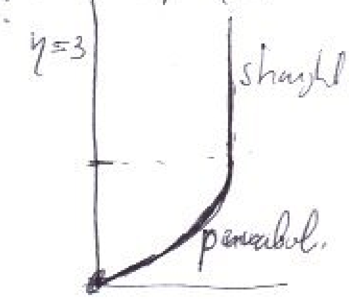
2) Displacement also affects the potential flow itself



; $U A_{\text{apparent}} = \text{constant} \rightarrow$

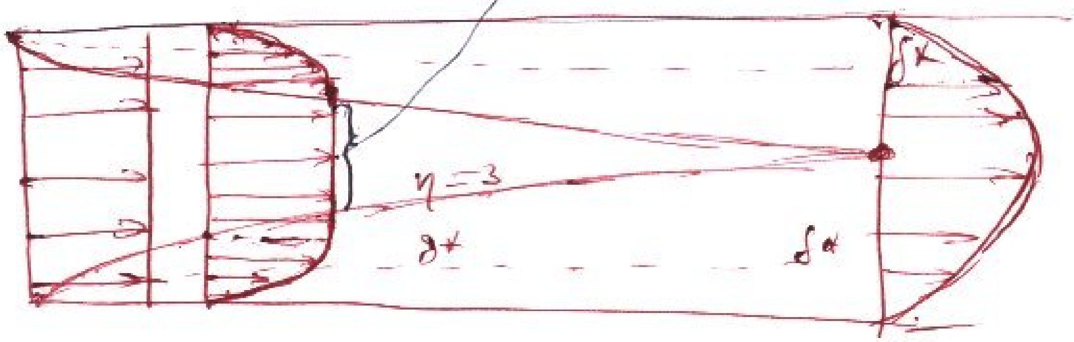
$$U_0 h = U(x)(h - 2\delta^*)$$

approximate profile

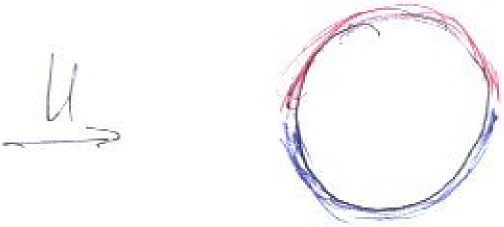


$$\eta = 3 = \frac{y}{x}\sqrt{\frac{Ux}{\nu}} \rightarrow y = 3\sqrt{\frac{\nu x}{U}}$$

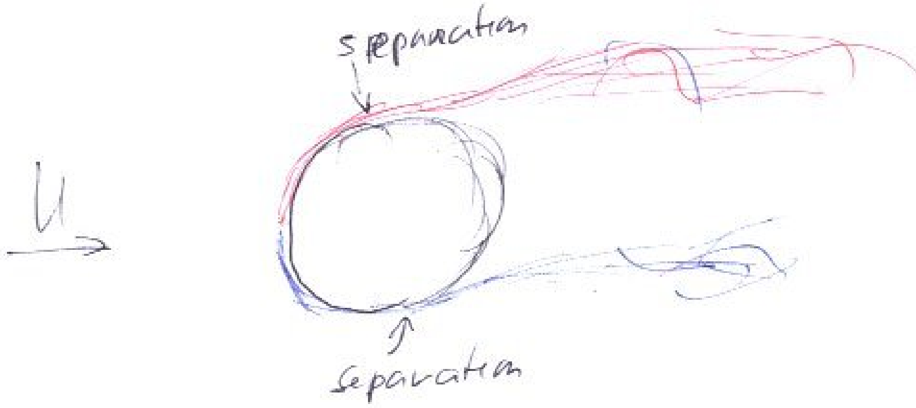
for this constant part of the velocity profile $u(h-2\delta^*) = \text{constant}$



Separation:

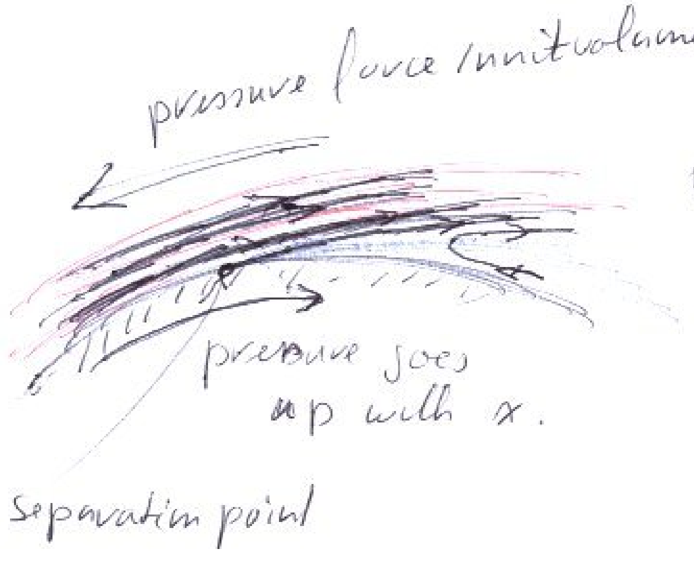


Unseparated flow: (thin)
almost no drag $c_D = O\left(\frac{1}{\sqrt{Re}}\right)$
if laminar



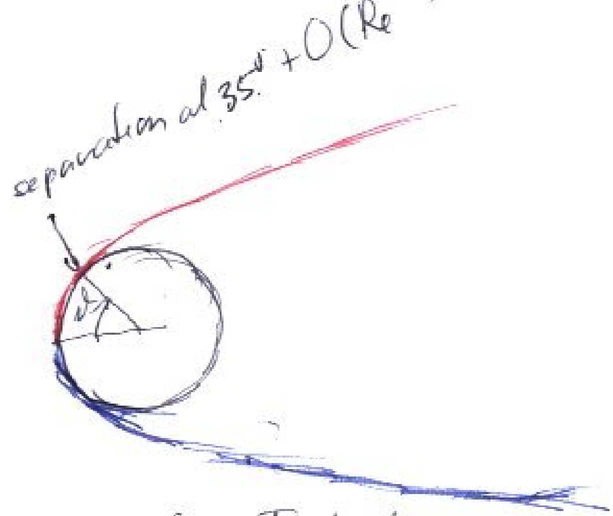
separated
finite drag
coefficient $c_D = O(1)$

Mechanics according to Prandtl



fluid particles near the wall have very little kinetic energy and after a bit of pressure rise, their motion will be reversed

Sychev - Smith description



Van Dommelen & Shen, Initial