
doi:10.1017/jfm.2012.261

Numerical solution of unsteady boundary-layer separation in supersonic flow: upstream moving wall

R. Yapalparvi[1] † and L. L. Van Dommelen[2]

[1] Department of Mathematics, McMaster University, 1280 Main St W, Hamilton, L8S 4K1, Canada

[2] Department of Mechanical Engineering, FAMU-FSU College of Engineering, 2525 Pottsdamer Street, Tallahassee, FL 32310-6046, USA

(Received 7 November 2011; revised 23 April 2012; accepted 30 May 2012)

This paper is an extension of work on separation from a downstream moving wall by Ruban *et al.* (*J. Fluid. Mech.*, vol. 678, 2011, pp. 124–155) and is in particular concerned with the boundary-layer separation in unsteady two-dimensional laminar supersonic flow. In a frame attached to the wall, the separation is assumed to be provoked by a shock wave impinging upon the boundary layer at a point that moves downstream with a non-dimensional speed which is assumed to be of order $Re^{-1/8}$ where Re is the Reynolds number. In the coordinate system of the shock however, the wall moves upstream. The strength of the shock and its speed are allowed to vary with time on a characteristic time scale that is large compared to $Re^{-1/4}$. The 'triple-deck' model is used to describe the interaction process. The governing equations of the interaction problem can be derived from the Navier–Stokes equations in the limit $Re \to \infty$. The numerical solutions are obtained using a combination of finite differences along the streamwise direction and Chebyshev collocation along the normal direction in conjunction with Newton linearization. In the present study with the wall moving upstream, the evidence is inconclusive regarding the so-called 'Moore–Rott–Sears' criterion being satisfied. Instead it is observed that the pressure rise from its initial value is very slow and that a recirculation region forms, the upstream part of which is wedge-shaped, as also observed in turbulent marginal separation for large values of angle of attack.

Key words: boundary layer separation, high speed flow

1. Introduction

This study was undertaken as a further step in developing a theory of boundary-layer separation over moving walls in supersonic flow. It was motivated by numerous numerical and experimental investigations of boundary-layer separation based on Prandtl's boundary-layer theory. Prandtl (1904) was the first person to provide crucial insight into the separation phenomenon. Prandtl observed that despite the fact that 'common' gases and liquids have very low viscosity, viscous effects did play a major role in the separation phenomenon. He argued that high-Reynolds-number flow around a rigid body may be treated as inviscid except in a very thin region adjacent to the

† Email address for correspondence: ramesh.yapalparvi@gmail.com

body surface, the so-called boundary layer. This boundary layer separates when the pressure starts to rise going downstream along the body surface, and hence induces drastic changes to the flow field. It leads to a strong vortical motion and reversed flow in the region downstream of the separation point, and as a result the body experiences an increase in drag.

The analysis of Goldstein (1948) showed that the classical steady boundary-layer equations break down at the point where the flow reverses, or separates. Landau & Lifshitz (1944) had drawn a similar conclusion earlier, but Stewartson (1958) showed that their assumption that the position is a regular function of the shear was not really justified, though correct to leading order. The breakdown in the boundary layer is associated with the failure of the entire strategy of assuming an attached outer flow when in reality the global flow is separated, cf. Stewartson (1970).

A key element of the separation process, which was not fully appreciated in Prandtl's description, was an interaction between the boundary layer and the external inviscid flow, now referred to as viscous–inviscid interaction. Asymptotic theory of viscous–inviscid interaction, also known as the triple-deck theory, was formulated simultaneously by Neiland (1969) and Stewartson & Williams (1969) for the self-induced separation in supersonic flow and by Stewartson (1969) and Messiter (1970) for incompressible fluid flow near the trailing edge of a flat plate. Sychev (1972) and Smith (1977) showed that boundary-layer theory in its classical form, as formulated by Prandtl (1904), cannot be used in a small vicinity of a separation point, where viscous–inviscid interaction must be allowed for. A detailed description of applications of the theory to different forms of the boundary-layer separation may be found in the monograph by Sychev *et al.* (1998). The concept of viscous–inviscid interaction in the classical scenario has made it possible to remove the singularity and obtain a full description of small-scale separation, see Ruban (1981), Smith & Daniels (1981) and Stewartson, Smith & Kaups (1982).

Unsteady boundary-layer separation is a much more complicated phenomenon. Theoretical analysis of it requires consideration of the unsteady boundary-layer equations, such as in the case of impulsive motion of a blunt cylinder. Blasius (1908) was the first to consider the boundary-layer flow past a circular cylinder and showed that after some time the skin friction becomes zero at the rear stagnation point. Past this time, the point of zero skin friction moves upstream from the rear stagnation point and a reversed flow region exists behind it. After the work of Blasius, the case of the circular cylinder became a benchmark problem for many researchers. Proudman & Johnson (1962) and later Robins & Howarth (1972) and Van Dommelen & Shen (1985), showed that while the steady classical boundary-layer flow is singular at the point of flow reversal, Goldstein (1948), the unsteady boundary-layer solution at the rear stagnation point remains regular for all time. It was recognized, in particular by Rott (1956), Sears (1956) and Moore (1958), that unlike in steady flows, the point of vanishing shear is not associated with detachment of the boundary layer from the wall.

The analysis of Goldstein was limited to steady separation from a fixed wall. However, Moore (1958) argued that singular behaviour should also be expected in unsteady separation. To gain more insight, Moore modelled unsteady separation as steady separation from a moving wall. Such flows are unsteady when seen in a frame attached to the wall. Based on his work and similar ideas of Rott (1956) and Sears (1956), Sears & Telionis (1975) proposed that the presence of a singularity in the classical boundary-layer equations, and not flow reversal, should be used to define separation. They proposed that this separation would be characterized by a point inside

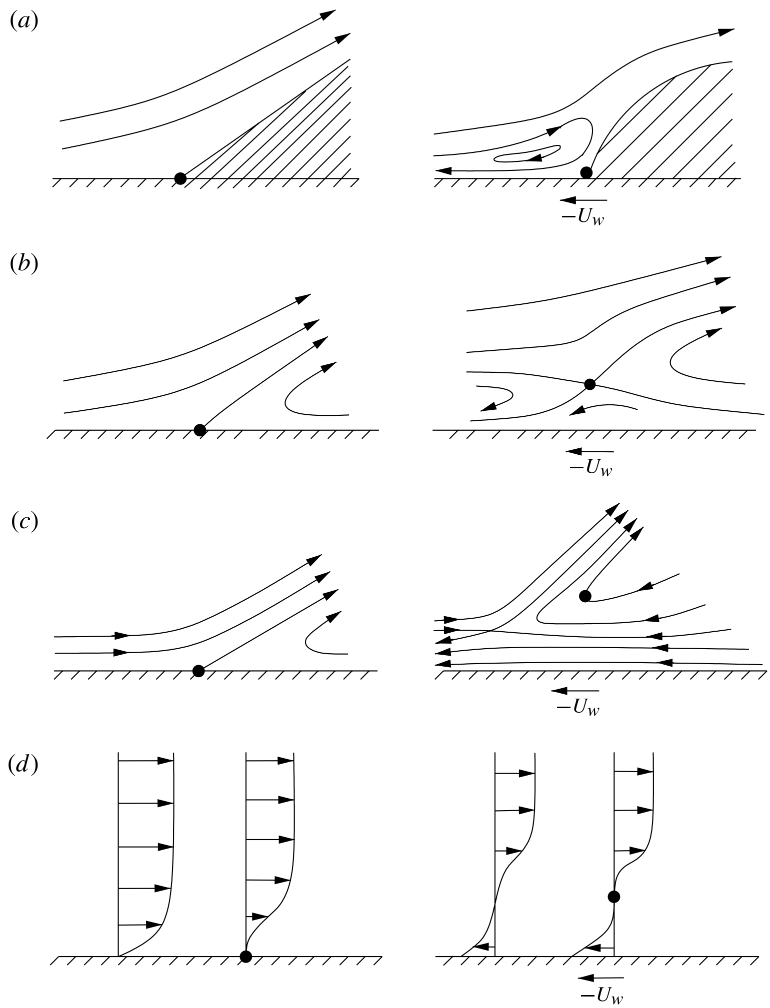

FIGURE 1. The sketches on the left show streamlines and velocity profiles for a fixed wall, and those on the right for an upstream moving wall. Streamline profiles: (*a*) Moore (1958), (*b*) Sears & Telionis (1975), (*c*) Van Dommelen & Shen (1983*a*). (*d*) Velocity profiles as proposed by Sears & Telionis (1975). Note that ● denotes the separation point.

the boundary layer at which the conditions

$$u = 0, \quad \frac{\partial u}{\partial y} = 0 \tag{1.1}$$

apply when seen in a system moving along with separation. Here u is the velocity component in the direction of the wall and y the distance from the wall. Sears & Telionis named these conditions the MRS conditions, after Moore, Rott and Sears who had formulated them earlier in a more limited context (Moore 1958). See also Ludwig (1964) for a similar formulation.

The velocity profiles as sketched by Sears & Telionis (1975) are on the right in figure 1(*d*). They correspond to the streamline sketches above them and can be transformed to unsteady separation in a wall-fixed frame by simply subtracting the wall velocity. The intent of Sears & Telionis was to generalize the better understood case of plane, steady flow along a fixed wall. They assumed that the boundary-layer flow in both moving wall cases (downstream and upstream) would be bifurcated by the wake at 'some kind of stagnation point' and that, when seen in a coordinate system moving with separation, vanishing shear at a stationary point away from the wall would characterize the phenomenon.

It is important to note that while most authors agree about the MRS conditions cited above, the shape of the streamlines and the presence of a stagnation point at separation is less clear. Moore (1958) apparently did not envision a stagnation point, at least for the downstream moving wall, but proposed zero 'profile' velocity and a vertical streamline 'shoulder'. Moore was even more vague about the upstream moving wall, calling it a case 'for which speculation is difficult'. His flow sketch shown in figure 1(*a*) is quite different from figure 1(*b*) by Sears & Telionis.

On the other hand, Rott (1964, p. 432), states: 'Moore (1958) and, independently, Sears and Rott (unpublished) came to the conclusion that a sort of stagnation point, $u = v = 0$, within the boundary layer (viewed in the steady system, the walls moving) takes the significance of the laminar separation point. With boundary-layer approximations, the conditions for such a point are also very nearly $u = \partial u/\partial y = 0$'. Clearly, in this view, the stagnation point is primary and the MRS conditions secondary.

It should also be noted that Van Dommelen & Shen (1983*a*) proposed a different streamline picture, figure 1(*c*). They proposed that the separation would occur downstream of the stagnation point, rather than at it. Thus, in their proposal, the flow velocity at reversal of the component u in the direction of the wall is upwards rather than downwards, immediately before separation. In that case, near this reversal line, the initially upstream moving fluid passes into the downstream moving region. So upstream moving fluid is turned back downstream at the reversal line. Part of the thickening of the boundary layer going towards separation from the upstream side is then due to additional streamlines coming from downstream that become part of the velocity profile. Van Dommelen & Shen (1983*a*) show that a self-consistent and reasonably complete mathematical description of their proposed flow exists in the non-interactive case. This description can explain the flattening of the velocity profile at flow reversal postulated in the MRS model.

An interesting model proposed by Ludwig (1964) can make the fundamental difference between the proposals more clear. Consider the curve through the points where the streamlines are vertical. On that curve, the velocity component u in the direction of the wall crosses zero. As far as the flow above this curve is concerned, the curve can be replaced by a solid surface, an imaginary wall, through which there is a non-zero transpiration velocity. In that picture, the proposals of Moore (1958) and Sears & Telionis (1975) apply suction through the imaginary wall, while Van Dommelen & Shen (1983*a*) apply blowing. (It should be noted that Ludwig himself speaks of blowing, rather than suction. However, he only measured the total velocity, and there is no mention in the paper that the direction of the transverse velocity was actually determined. Blowing is however a more natural way to cause separation than suction.)

Both Moore and Sears & Telionis suggested that a singularity similar to the Goldstein one should occur at the point where the MRS condition holds. This singularity would be an exaggeration of the actual very rapid growth in boundary-layer thickness that is due to taking the limit of infinite Reynolds number.

For a downstream, rather than upstream, moving wall, these ideas have received considerable support. Several subsequent boundary-layer computations, as summarized in Sears & Telionis (1975), soon confirmed the MRS conditions and associated singularity.

Various more complete computations for the downstream moving wall show an MRS point near a stagnation point. Recently Ruban *et al.* (2011) demonstrated the existence of an MRS point and structure of the separated flow region for the downstream moving wall for external supersonic flow. The incompressible computations of Inoue (1981*a*,*b*) showed that the MRS conditions were satisfied at a point close to the start of the recirculating region. However, the vertical velocity was not zero at or near that point, so there was no stagnation point.

Strong theoretical support for the MRS conditions for the downstream moving wall was given by Sychev (1979), with some modifications by Van Dommelen & Shen (1982*a*, 1983*b*) and additional flow details by Elliott, Smith & Cowley (1983). This solution includes viscous–inviscid interaction. See also Ruban *et al.* (2011) and the references therein. The singularity structure is quite different from the Goldstein one envisaged by Moore and Sears & Telionis, however.

To understand unsteady separation, the major alternative to studying steady separation from a moving wall has been to study the formation of separation in an initially unseparated boundary layer. Separation is here again understood to be in the sense of breakaway of the boundary layer from the surface. It is *not* the flow reversal already described by Blasius, which is not directly associated with any unusual thickening of the boundary layer. The advantage of starting with an initially unseparated boundary layer, typically the Blasius boundary layer around an impulsively started circular cylinder, is that the pressure in the boundary layer during the evolution leading up to separation is unambiguously known. Therefore the solution is believed to be physically meaningful until right at the time of the first breakaway, for sufficiently high Reynolds number.

Telionis & Tsahalis (1974) first computed the boundary layer around the circular cylinder for longer times with the intent of finding a breakaway singularity. They found that a singularity occurs after the cylinder has moved over a 0.65 radius distance, at a position 140° from the front stagnation point. However, later authors have not been able to reproduce these data. In particular, Cebeci (1979), using a more sophisticated box scheme, carefully recomputed the range described by Telionis & Tsahalis (1974) and found the solution to be non-singular. He suggested that, unlike the theory of Sears & Telionis (1975) predicted, the solution would be smooth for all time and thicken like the Proudman & Johnson (1962) flow does at the rear stagnation point. However, using an unusual Lagrangian method, Van Dommelen & Shen (1980) continued the solution for still longer times and found that a separation singularity does occur. It forms after the cylinder has moved over a 1.5 radius distance, at a position 111° from the front stagnation point. This singularity has been confirmed by later authors using more conventional numerical methods, for example Ingham (1984), Henkes & Veldman (1987), Riley & Vasantha (1989), Puppo (1990) and Christov & Tzankov (1993). See also an earlier computation by Van Dommelen & Shen, as briefly summarized in Shen (1978), which obtained equivalent results using a simpler non-impulsive initial condition. The application of Lagrangian variables in studying unsteady boundary-layer flows, including in three dimensions, has been discussed in detail by Cowley, Van Dommelen & Lam (1990).

Using the same scheme as in Cebeci (1979), Cebeci (1982) extended his computed range and achieved good agreement with Van Dommelen & Shen (1980) until shortly before the separation. However, subsequently, using a different numerical scheme, Cebeci (1986) obtained very different results from Van Dommelen & Shen (1980) and other authors, including his own earlier work, and even at quite early times. These results have not been independently confirmed.

Results very similar to those of Van Dommelen & Shen (1980) were obtained independently by Cowley (1983) using a high-order numerical series expansion. Also independently, Wang (1979), using a conventional numerical technique, obtained results that suggested a breakdown of the boundary layer at a time and location roughly similar to those of Van Dommelen & Shen (1980). However, there are significant qualitative differences between his results and those of other authors. The structure of the breakdown is unclear, but would need to be different. The interpretation of the breakdown structure as given by Wang would require a singular slope of the wall shear profile, but neither his own results nor those of others have confirmed this.

The mathematical structure of the singularity was discovered by Van Dommelen and Shen. They performed both a Lagrangian analysis of the flow, Van Dommelen (1981) and Van Dommelen & Shen (1982*b*), and an Eulerian one, Van Dommelen (1981). The two procedures gave identical results for the structure. A somewhat different but equivalent Eulerian analysis was given by Elliott *et al.* (1983). While the singularity satisfies the MRS conditions, it is not a Goldstein-type singularity in a reasonable sense. In particular, the separation process is found to be approximately inviscid where the Goldstein one is viscous. Elliott *et al.* (1983) also investigated the removal of the singularity through interactive effects and found that the boundary-layer solution breaks down earlier than suggested by Van Dommelen (1981). The first interactive stage of the unsteady boundary-layer separation occurs at a boundary-layer thickness of $O(Re^{-5/11})$, physically not that much thicker than the conventional $O(Re^{-1/2})$ thickness.

The boundary-layer profile upstream of separation is a non-interactive version of the one studied by Sychev (1979) and Van Dommelen (1981). Correspondingly, normally the wall will be moving downstream compared to the forming separation. Theoretically that is not strictly necessary, but unfortunately, even if the wall is moving downstream compared to the separation, its structure will simply not be of the form shown in figure 1. Attempts to create a downstream moving separation by studying say a rotating cylinder have not been successful so far.

Other attempts to verify the upstream moving wall case have also been difficult. Experimental support for the flattening of the tangential velocity profile as shown in figure 1(*d*) was given by Ludwig (1964), continuing work of Vidal (1959). However, his results appear to be inconsistent with the streamlines as shown in figure 1(*b*). This streamline picture should mean that the total velocity, including the transverse component, decreases to zero at the MRS point. The total velocity is in fact what Ludwig actually measured. The minimum in the measured total velocity profiles did not decrease towards zero. Instead it was approximately constant until separation and then grew in magnitude.

Some support for the streamline sketches in figure 1(*b*) at a very low Reynolds number was given by Koromilas & Telionis (1980). However, as they stated in their conclusions, ‘the case of a downstream-moving separation was attempted and some results were included here but they are rather inconclusive. (...) It was not possible to measure and observe the speed of propagation of separation and therefore it was not possible to make a quantitative comparison with the definition of Sears (1956) and the theory of Sears & Telionis (1975), Williams (1977) and Shen (1978)’.

Tsahalis (1977) was the first to study laminar-boundary-layer separation for a steady outer inviscid flow over an upstream moving wall numerically. He concluded that for a steady flow over an upstream moving wall, the separation coincides with the MRS model with a Goldstein-type singularity. He presents a velocity profile close to what is taken to be separation that shows a slope near flow reversal that is significantly lower than that at the wall, indicating an approximate MRS profile. The vertical velocity v at the flow reversal point $u = 0$ is not given. Lower in the boundary layer the vertical velocity is positive, not negative as his sketch figure 2, similar to the streamlines in figure 1(*b*), suggests. However, one problem with the computation of Tsahalis is that while the solution is affected by the flow downstream of separation, through the reversed flow near the wall, there is no physically justified downstream flow.

Inoue (1981*a*,*b*) did include a physical downstream flow in a parabolized Navier–Stokes computation, but was unable to produce results like those of Tsahalis despite an attempt to apply similar boundary conditions. Inoue (1981*a*,*b*) found in his computations that the separation bubble starts without the simultaneous vanishing of both longitudinal velocity and skin friction as predicted by the MRS conditions. The results of Tsahalis (1977) need independent confirmation.

Araki (2006) studied the problem of boundary-layer separation on a moving wall in supersonic flow based on viscous–inviscid interaction and considered both the downstream and upstream moving wall cases. For the upstream moving wall case, in a weak-shock approximation, he observed that there is a sharp pressure-drop region in the vicinity of the point where the shock interacts with the boundary layer. The pressure increases rapidly after the pressure drop near the shock impinging point. The width of the pressure drop region increases as the wall velocity $|U_w|$ increases. In the strongly nonlinear case, his results indicate that the difference between the upstream and downstream moving wall cases may induce a drastic change in the flow structure.

Most of the other studies on the upstream moving wall case that have been mentioned were concerned with incompressible fluid flow. In this paper, we consider the compressible fluid flow case on an upstream moving wall. The paper is organized as follows: § 2 gives a brief summary of the problem formulation based on the classical triple-deck theory which remains valid if the wall speed is $O(Re^{-1/8})$. The computational results for the upstream wall moving case are presented in § 3 and finally in § 4 we provide the concluding remarks.

2. Formulation of the problem

Let us consider a flat plate placed in an uniform two-dimensional supersonic flow of a perfect gas, with the plate parallel to the free-stream velocity. Let a shock wave be generated by a wedge placed above the plate and let this wedge, hence the shock, move parallel to the plate, as shown in figure 2. For consistency with the notation of Ruban *et al.* (2011), the wedge velocity will be assumed positive if the wedge moves upstream as shown.

Let the distance between the leading edge of the plate and the current position S where the shock impinges on the boundary layer, be L. Let the velocity, density, viscosity and pressure in the unperturbed free stream be U_∞, ρ_∞, μ_∞ and p_∞, respectively.

The Mach number in the free stream is given by the formula

$$M_\infty = \frac{U_\infty}{a_\infty}, \quad a_\infty = \sqrt{\gamma \frac{p_\infty}{\rho_\infty}}, \tag{2.1}$$

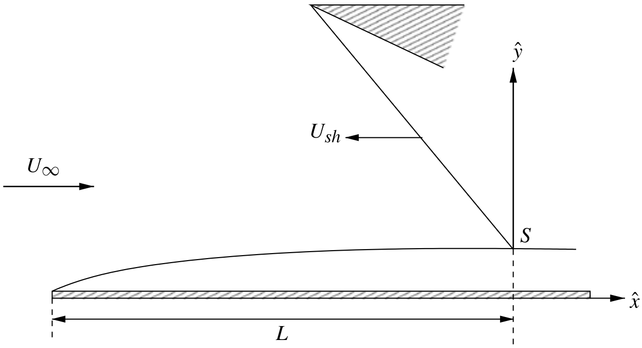

FIGURE 2. The flow layout.

where a_∞ is the speed of sound and γ is the ratio of specific heats. We shall assume that the oncoming flow is supersonic, i.e. M_∞ is an order-one quantity greater than unity. In the present analysis, the Reynolds number

$$Re = \frac{\rho_\infty U_\infty L}{\mu_\infty} \tag{2.2}$$

is assumed to be large.

The shock velocity is assumed to be of order $Re^{-1/8}$, and U_{sh} denotes the correspondingly scaled shock velocity.

The flow will be described in a coordinate frame moving with the shock (see figure 2). In that coordinate system, the plate surface moves in the opposite direction to U_{sh} with a scaled speed $U_w = U_{sh}$. Note that U_w is negative for the cases to be studied here. As long as the variation in shock velocity and strength is asymptotically small while it moves over a $Re^{-3/8}$ triple-deck length scale, the flow can be assumed quasi-steady. The equations governing the flow are the interactive boundary-layer equations, scaled as in Ruban *et al.* (2011):

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{2.3a}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\mathrm{d}P}{\mathrm{d}X} + \frac{\partial^2 U}{\partial Y^2}. \tag{2.3b}$$

These equations have to be solved with the following boundary conditions:

$$U = U_w, \quad V = 0 \quad \text{at } Y = 0. \tag{2.3c}$$

Equations (2.3*a*) and (2.3*b*) require an initial condition

$$U = Y + U_w \quad \text{at } X \to -\infty, \tag{2.3d}$$

which follows from matching with the solution in the unperturbed boundary layer upstream of the interaction. The matching with the main part of the boundary layer leads to the following condition:

$$U \to Y + A(X) + \cdots \quad \text{as } Y \to \infty, \tag{2.3e}$$

where $A(X)$ represents the displacement produced by the viscous sublayer. The interaction law deduced from the flow analysis in the inviscid region is given by

$$P = P_s\mathscr{H}(X) - \frac{dA}{dX}, \tag{2.3f}$$

where

$$\mathscr{H}(X) = \begin{cases} 0, & X < 0 \\ 1, & X \geqslant 0. \end{cases} \tag{2.3g}$$

Taking into account that viscous–inviscid interaction allows upstream influence through the boundary layer, an additional boundary condition specifying the state of the flow downstream of the interaction region is required and is prescribed as

$$P = P_s \quad \text{at } X \to +\infty. \tag{2.3h}$$

Two additional boundary conditions can be considered as well:

$$\frac{\partial U}{\partial Y} = 1 \quad \text{at } X = -\infty, \tag{2.3i}$$

and

$$\frac{\partial U}{\partial Y} = 1 \quad \text{at } Y = \infty. \tag{2.3j}$$

To solve the above problem, we used a numerical method similar to the one used by Korolev, Gajjar & Ruban (2002) with a uniform grid. The scheme and computational parameters are similar to those used in Ruban *et al.* (2011). As before, we took a value of $Y_{\max} = 50$ to limit the computational domain in the Y-direction. Values of $X_{\min} = -100$ and $X_{\max} = 100$ were used to truncate the domain in the streamwise direction. A step size $\Delta X = 0.01$ and 64 points in the Y-direction were used. Typically, 8–10 Newton iterations were sufficient for the method to converge.

For computational purposes, a smoothing was applied to the impinging shock. In particular, the interaction law (2.3*f*) was written as

$$P = \frac{P_s}{2}\left(1 + \frac{X}{\sqrt{X^2 + r^2}}\right) - \frac{dA}{dX} \tag{2.4}$$

where the value of parameter r was chosen to be $r = 0.5$ in this study.

An assessment of the impact of using different size grids and Y_{max} on the numerical solution was performed using grid sizes of 8001×64, 1601×70 and 1201×90 and with different values of Y_{max}. The wall shear plots for these grids are shown in figures 3 and 4. These figures indicate that our results are independent of the grids used.

3. Numerical results

Before discussing the numerical results, we need to describe what the skin friction graphs indicate. Unlike for separation from a fixed wall, reversal of the sign of the skin friction does not indicate flow reversal; it merely indicates that the fluid in the boundary layer just above the wall is moving upstream slower than the wall.

Results for a wall moving upstream with $U_w = -0.2$ and $P_s = 3.5$ are shown in figures 5–9. The pressure forms a 'plateau' and the skin friction has only one significant minimum. The streamline pattern in figure 7 shows a vortex with a wedge-shape upstream region, similar to the structure observed in turbulent

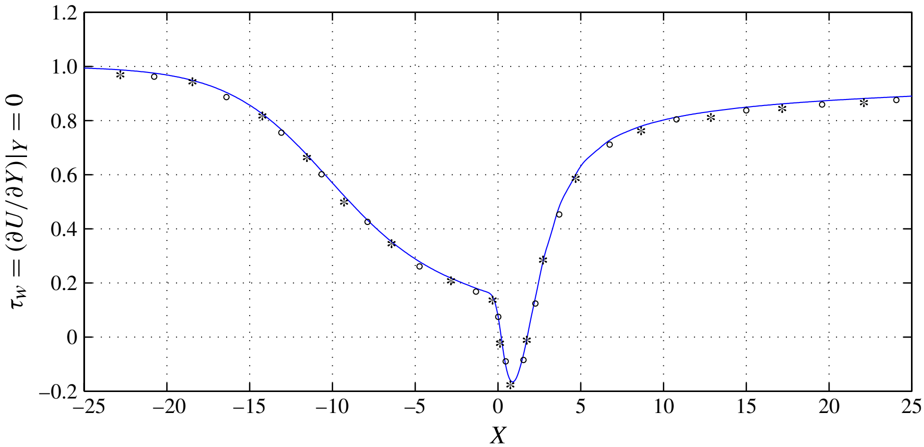

FIGURE 3. (Colour online) Skin friction distributions for various sizes of the uniform grid for $P_s = 2.0$ and $U_w = -0.5$: —, 8001 × 64; ∘, 1601 × 70; ∗, 1201 × 90.

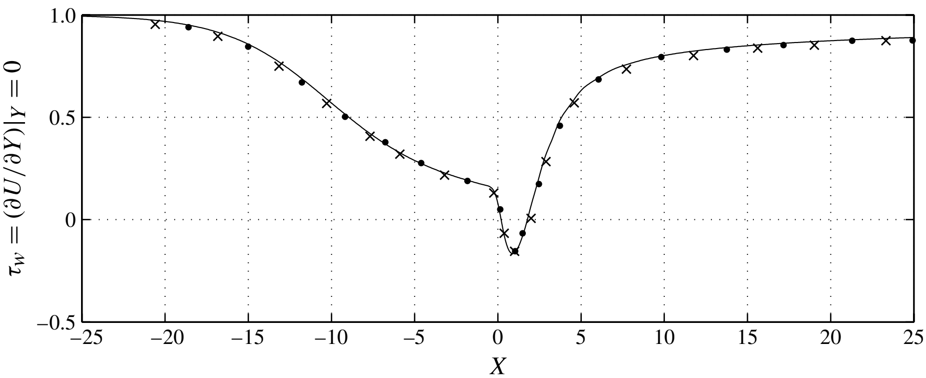

FIGURE 4. Skin friction distributions for various values of Y_{max} for $P_s = 2.0$ and $U_w = -0.5$: —, $Y_{max} = 50$; •, $Y_{max} = 15$; ×, $Y_{max} = 75$.

marginal separation, Scheichl & Kluwick (2007). The skin friction turns negative at approximately $X = -8$. The velocity profiles at various streamwise locations in figure 8 do not show any evidence of the MRS condition $\partial U/\partial Y = 0$ at $U = 0$, cf. figure 10. Although there is a zero-vorticity line $\partial U/\partial Y = 0$, figure 10 shows that it does not intersect the zero-streamwise-velocity line $U = 0$. The evidence is therefore inconclusive with respect to the MRS criterion being satisfied.

Note also that various theories assumed a given smooth adverse pressure gradient at the location of separation, e.g. Moore (1958) and Van Dommelen & Shen (1983*a*). Figures 5 and 7 show a strong growth in the adverse pressure gradient in the region near $X = -11$ where the flow detaches from the wall. This is one potential explanation why the present results do not show formation of the MRS point where the experimental results of Ludwig (1964) for a shrouded cylinder in subsonic flow did.

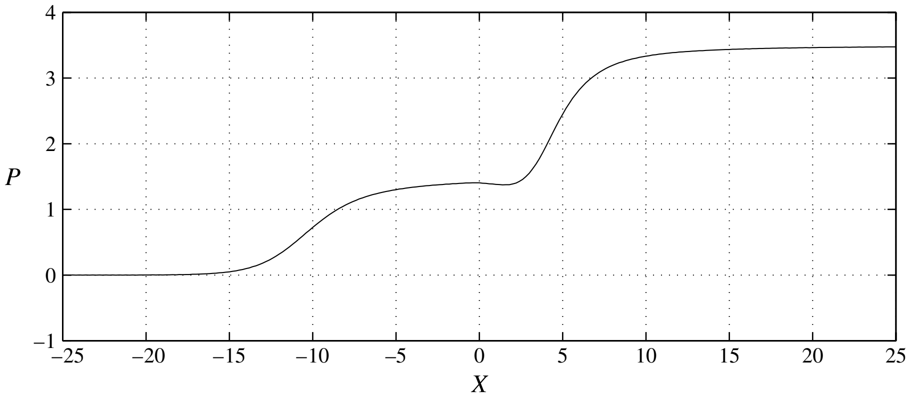

FIGURE 5. Pressure distribution for $U_w = -0.2$, $P_s = 3.5$.

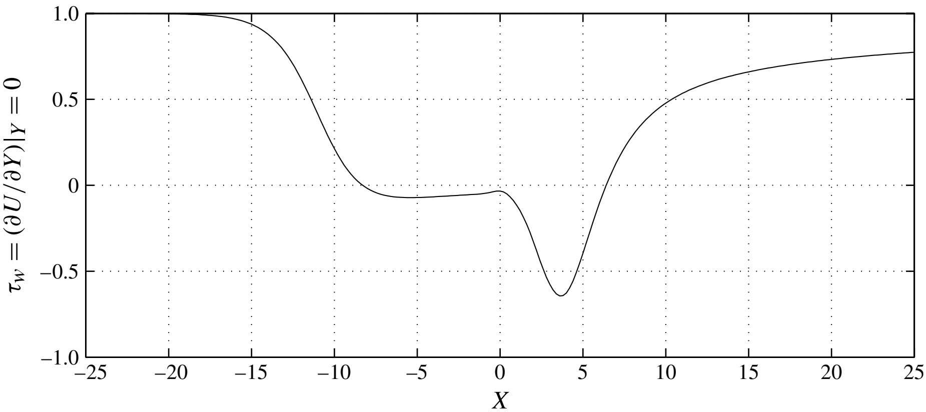

FIGURE 6. Skin friction for $U_w = -0.2$, $P_s = 3.5$.

The influence of different wall velocities on the flow in the interaction region is shown in figures 11 and 12 for pressure and skin friction. For the motionless wall, the flow undergoes separation with the separation and reattachment points clearly seen in the skin friction plot where the curve $\tau_w(X)$ intersects the abscissa (see Rizzetta, Burggraf & Jenson 1978; Korolev *et al.* 2002). At $U_w = -2$ and -5, the pressure rises very slowly from its initial value. The streamlines at $U_w = -5$ show that the recirculating region at this wall velocity extends upstream beyond $X = -100$ (using a computation with $X_{\min} = -150$.) Following (2.3*f*), this explains how the small upstream pressures can produce a significant streamline displacement. At $U_w = -5$ the pressure further shows an almost discontinuous behaviour near the shock much like the pressure distribution in the case of inviscid flow theory.

4. Concluding remarks

In this paper, we have investigated the process of a shock wave interacting with a boundary layer over an upstream moving wall for a supersonic external flow. Assuming the wall speed to be of $O(Re^{-1/8})$, the interaction process is explained

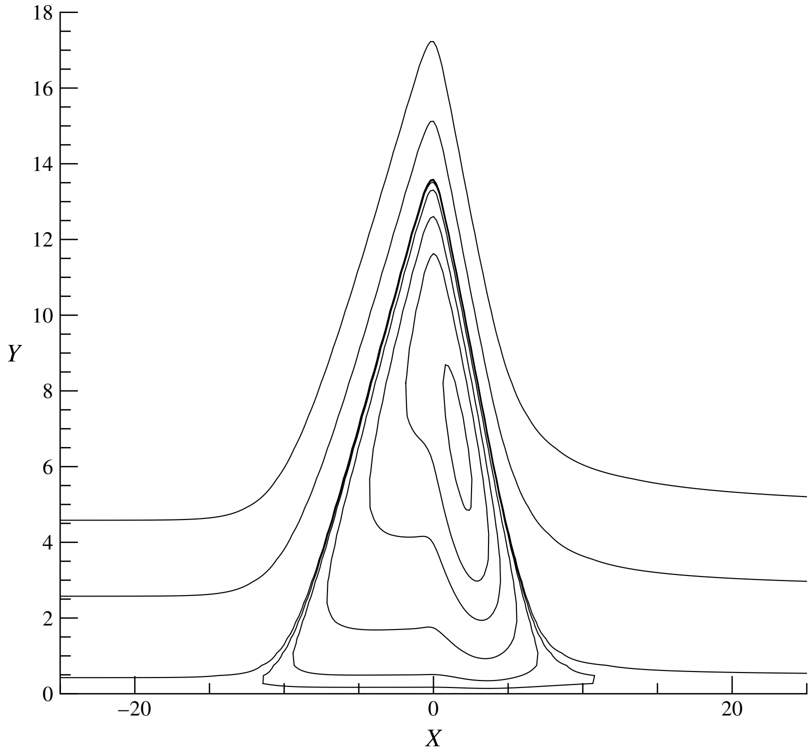

FIGURE 7. Streamline pattern for $U_w = -0.2$, $P_s = 3.5$.

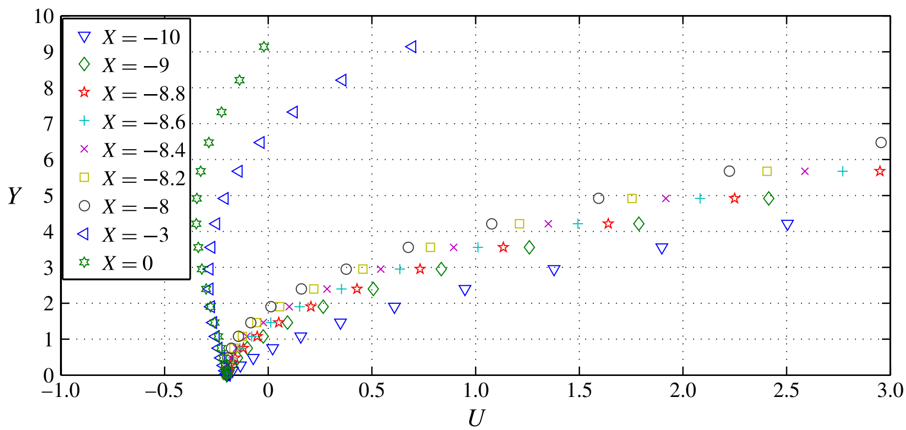

FIGURE 8. (Colour online) Velocity profiles for $U_w = -0.2$, $P_s = 3.5$.

by means of 'triple-deck theory'. At large values of the scaled wall velocity $|U_w|$, the rise in pressure from its initial value takes place very slowly over an extended range, as is seen from the pressure distributions in figure 11. A recirculation region is formed that has a wedge shape at its upstream end. As U_w is decreased, the length of the

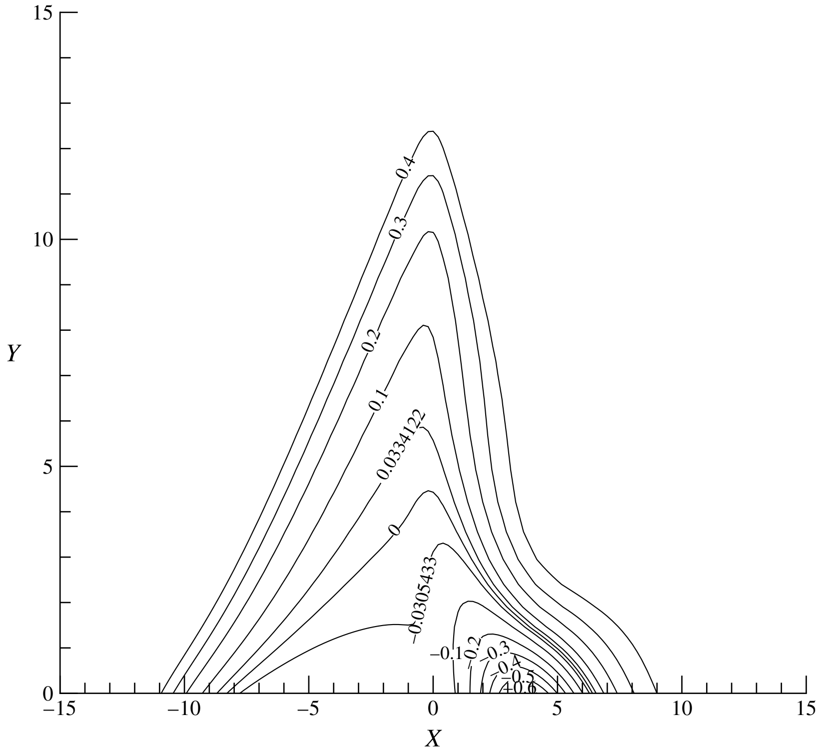

FIGURE 9. Vorticity distribution for $U_w = -0.2$, $P_s = 3.5$.

recirculation region increases. A similar behaviour is observed in turbulent marginal separation for large values of angles of attack, Scheichl & Kluwick (2007).

Our other task was to examine the Moore–Rott–Sears criterion for the flow considered. The velocity and vorticity distribution do not show evidence of the MRS criterion. In that respect, the present results seem to be quite similar to those of Inoue (1981*a*,*b*), even though the flow is very different. The theoretical work of Araki (2006) which has a similar wall pressure distribution also failed to provide sufficient evidence of the MRS point. We can therefore say that the MRS criterion does not seem applicable for the upstream moving wall case in the flow computed here.

As a referee pointed out, there is a question as to how far a flow of the type considered in this paper describes separation in a meaningful sense. The physical scalings of the considered flow, together with the attached downstream boundary condition, make the effects of the recirculating flow on the inviscid flow above the boundary unavoidably relatively minor. More fundamentally, as Sears & Telionis in particular have emphasized, a meaningful definition of separation in all but the most trivial cases requires a limit process. Loosely speaking, the definition of separation (if physically meaningful) is that the boundary layer moves significantly away from the wall at a well-defined location. But ‘significantly’ is not an objective mathematical term. There are no concepts like large or small in mathematics, but only limit processes. For the standard Sychev–Smith separation, the limit process is that the body-scale Reynolds number tends to infinity. But that Reynolds number does not appear unambiguously in the present scaled flow. However, consider figure 13

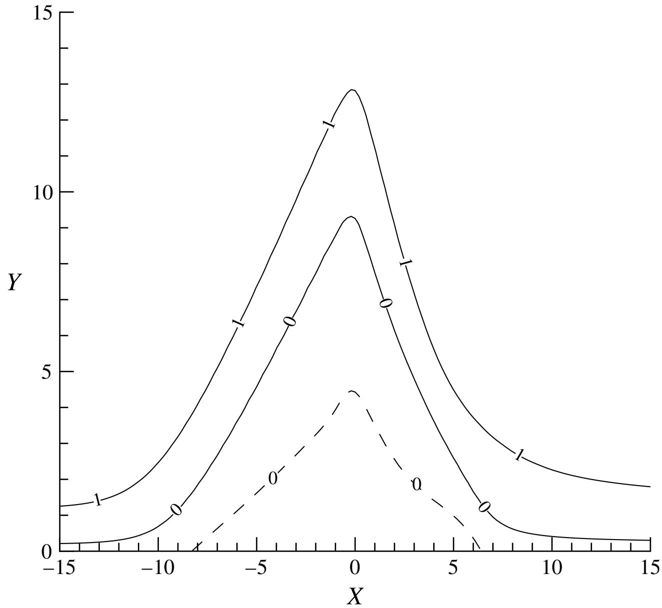

FIGURE 10. Velocity and vorticity distribution for $U_w = -0.2$, $P_s = 3.5$ showing: $U = 0$ and 1 (—, with labels 0 and 1 respectively); and $\partial U/\partial Y = 0$ (- - -).

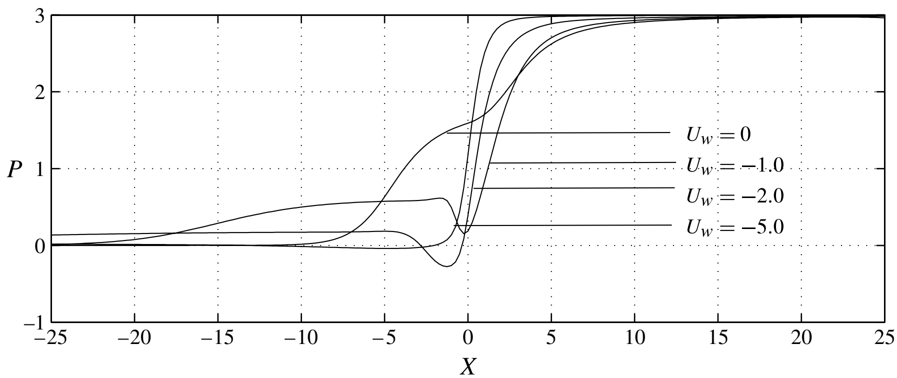

FIGURE 11. Pressure P for the shock strength $P_s = 3$ and various values of the wall velocity U_w.

where $-\mathrm{d}^2A/\mathrm{d}X^2$ is plotted against X. Recall that the normal Sychev–Smith separation is characterized by a localized positive peak in this parameter at the location of separation. It describes the rapid curving of the boundary-layer streamlines away from the wall in the region of separation. In the present flow, a similar positive peak seems to develop at negative X when the shock pressure jump P_s increases. The rapid curvature away from the wall of the main boundary-layer streamlines is also qualitatively evident in the streamlines overlying the viscous sublayer in figure 7.

The localization of the peak with increasing pressure jump is perhaps more clearly seen in figure 14, in which the horizontal and vertical axes have been rescaled to keep

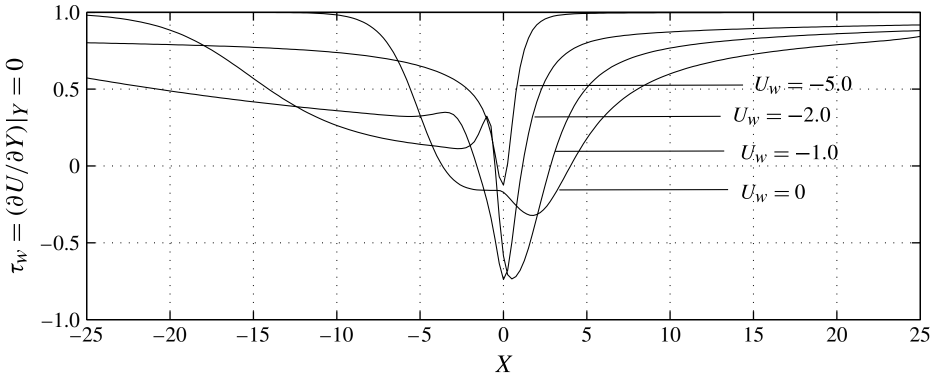

FIGURE 12. Skin friction τ_w for the shock strength $P_s = 3$ and various values of the wall velocity U_w.

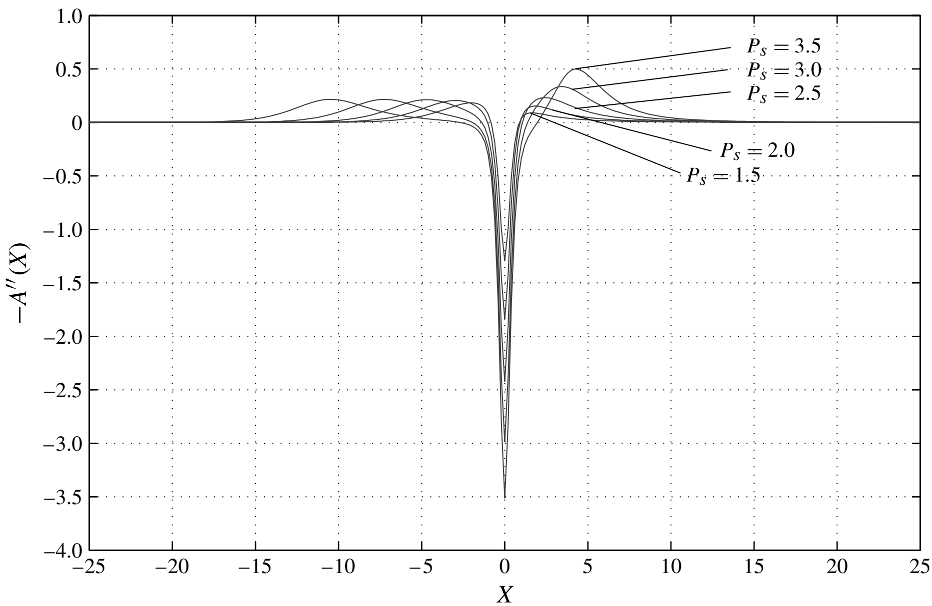

FIGURE 13. Negative second derivative of the displacement function $A(X)$ for the wall velocity $U_w = -0.2$ and various values of shock strength P_s.

the maximum at the same position. The present authors would therefore argue that for larger pressure jumps, a well-defined location can be identified for which 'separation' seems to occur in a mathematically and physically meaningful sense (i.e. within the setting of $P_s \to \infty$). Yet there is no sign of a corresponding approximate establishment of the MRS conditions in this region. The classical MRS velocity profile has an interior vorticity minimum. However, our results show a monotonic increase in vorticity away from the wall, figure 9.

Further support for this conclusion would have to come from a complete asymptotic description of the flow in the limit of infinite P_s. However, such an analysis is outside

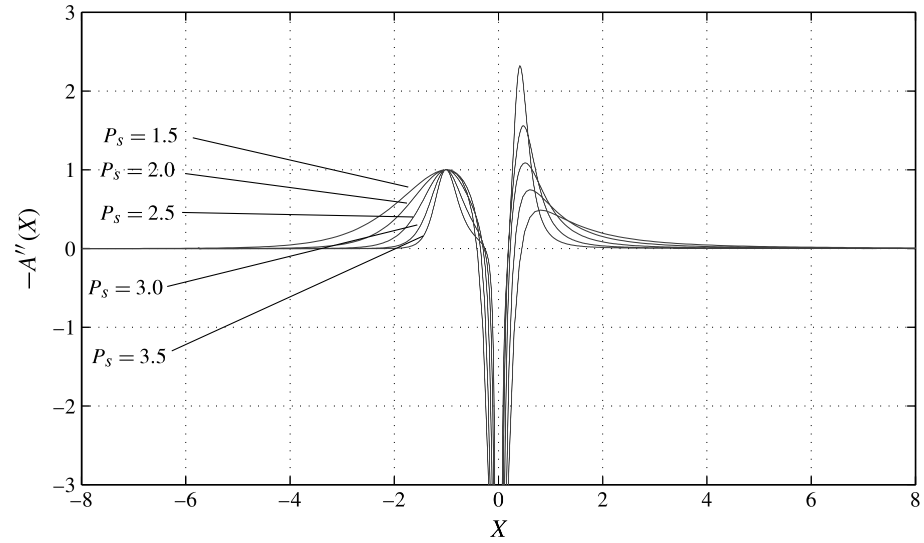

FIGURE 14. Scaled negative second derivative of the displacement function $A(X)$ for the wall velocity $U_w = -0.2$ and various values of shock strength P_s .

the scope of this study. And even if the Moore–Rott Sears conditions were to become meaningful in a limited range of very large values of P_s, clearly their practical value would still be significantly limited for the flows considered here.

Acknowledgements

R.Y. thanks Professor A. I. Ruban and Professor J. S. B. Gajjar for useful discussions. Professor F. T. Smith is thanked for useful discussion during the study of this problem.

REFERENCES

ARAKI, D. 2006 Boundary-layer separation on moving surface in supersonic flow. PhD thesis, University of Manchester.

BLASIUS, H. 1908 Grenzschichten in Flüssigkeiten mit kleiner Reibung. *Z. Math. Phys.* **56**, 1–37, Engl. transl. *NACA Tech. Mem.* 1256.

CEBECI, T. 1979 The laminar boundary layer on a circular cylinder impulsively started from rest. *J. Comput. Phys.* **31**, 153.

CEBECI, T. 1982 Unsteady separation. In *Numerical and Physical Aspects of Aerodynamic Flows* (ed. T. Cebeci), pp. 265–267. Springer.

CEBECI, T. 1986 Unsteady boundary layers with an intelligent numerical scheme. *J. Fluid Mech.* 129–140.

CHRISTOV, C. I. & TZANKOV, I. T. 1993 Numerical investigation of the laminar boundary layer flow around an impulsively moved circular cylinder. *Comput. Meth. Appl. Mech.* **109**, 1–15.

COWLEY, S. J. 1983 Computer extension and analytic continuation of Blasius expansion for impulsive flow past cylinder. *J. Fluid Mech* **135**, 389–405.

COWLEY, S. J., VAN DOMMELEN, L. L. & LAM, S. T. 1990 On the use of Lagrangian variables in description of unsteady boundary-layer separation. *Phil. Trans. R. Soc. Lond.* A **333**, 343–378.

ELLIOTT, J. W., SMITH, F. T. & COWLEY, S. J. 1983 Breakdown of boundary layers (i) on moving surfaces; (ii) in semi-similar flows; (iii) in fully unsteady flow. *Geophys. Astrophys. Fluid Dyn.* **25**, 77–138.

GOLDSTEIN, S. 1948 On laminar boundary-layer flow near a position of separation. *Q. J. Mech. Appl. Maths* **1** (1), 43–69.

HENKES, R. & VELDMAN, A. 1987 On the breakdown of the steady and unsteady interacting boundary-layer description. *J. Fluid Mech.* **179**, 513–530.

INGHAM, D. B. 1984 Unsteady separation. *J. Comput. Phys.* **53**, 90–99.

INOUE, O. 1981*a* MRS criterion for flow separation over moving walls. *AIAA J.* **19**, 1108–1111.

INOUE, O. 1981*b* A numerical investigation of flow separation over moving walls. *J. Phys. Soc. Japan* **50** (3), 1002–1008.

KOROLEV, G., GAJJAR, J. S. B. & RUBAN, A. I. 2002 Once again on the supersonic flow separation near a corner. *J. Fluid. Mech.* **463**, 173–199.

KOROMILAS, C. A. & TELIONIS, D. P. 1980 Unsteady laminar separation: an experimental study. *J. Fluid Mech.* **97** (2), 347–384.

LANDAU, L. & LIFSHITZ, E. 1944 *Mechanics of Continuous Media*. Gostekhizdat.

LUDWIG, G. R. 1964 An experimental investigation of laminar separtion from a moving wall. *AIAA Paper* 64-6.

MESSITER, A. F. 1970 Boundary-layer flow near the trailing edge of a flat plate. *SIAM J. Appl. Maths* **18** (1), 241–257.

MOORE, F. K. 1958 On the separation of the unsteady laminar boundary layer. In *Boundary Layer research* (ed. H. Görtler), pp. 431–432. Springer.

NEILAND, V. YA 1969 Theory of laminar boundary-layer separation in supersonic flow. *Izv. Akad. Nauk SSSR Mekh. Zhidk. Gaza* **4**, 53–57, English transl. *Fluid Dyn.***4 (4), 33–35**.

PRANDTL, L. 1904 Über Flüssigkeitsbewegung bei sehr kleiner Reibung. In *Verh. III Intl. Math. Kongr., Heidelberg*, pp. 484–491. Teubner, Leipzig, 1905, English transl. *NACA Tech. Memo.* 452.

PROUDMAN, I. & JOHNSON, K. 1962 Boundary-layer growth near a rear stagnation point. *J. Fluid Mech.* **12**, 161–168.

PUPPO, G. 1990 Singularity formation in Prandtl's equations and a new numerical method. PhD thesis, New York University..

RILEY, N. & VASANTHA, R. 1989 Unsteady high Reynolds number flows. *J. Fluid Mech.* **205**, 243–262.

RIZZETTA, D. P., BURGGRAF, O. R. & JENSON, R. 1978 Triple-deck solutions for viscous supersonic and hypersonic flow past corners. *J. Fluid Mech.* **89** (3), 535–552.

ROBINS, A. J. & HOWARTH, J. A. 1972 Boundary-layer development at a two-dimensional rear stagnation point. *J. Fluid Mech.* **56**, 161–171.

ROTT, N. 1956 Unsteady viscous flow in the vicinity of a stagnation point. *Q. J. Appl. Maths* **13**, 444–451.

ROTT, N. 1964 Theory of time-dependent laminar flows. In *Theory of Laminar Flows* (ed. F. Moore), pp. 395–438. Princeton.

RUBAN, A. I. 1981 Singular solutions of the boundary layer equations which can be extended continously therough the point of zero surface friction. *Fluid Dyn.* **16** (6), 835–843.

RUBAN, A. I., ARAKI, D., YAPALPARVI, R. & GAJJAR, J. S. B. 2011 On unsteady boundary layer separation in supersonic flow. Part 1. Upstream moving separation point. *J. Fluid. Mech.* **678**, 124–155.

SCHEICHL, B. & KLUWICK, A. 2007 Turbulent marginal separation and turbulent Goldstein problem. *AIAA* **45** (1), 20–36.

SEARS, W. R. 1956 Some recent developments in airfoil theory. *J. Aeronaut. Sci* **23**, 490–499.

SEARS, W. R. & TELIONIS, D. P. 1975 Boundary-layer separation in unsteady flow. *SIAM J. Appl. Maths* **23**, 215–235.

SHEN, S. 1978 Unsteady separation according to the boundary-layer equation. *Adv. Appl. Mech.* **13**, 177.

SMITH, F. T. 1977 The laminar separation of an incompressible fluid streaming past a smooth surface. *Proc. R. Soc.* A **356**, 443–463.

SMITH, F. T. & DANIELS, P. G. 1981 Removal of Goldstein singularity at separation, in flow past obstacles in wall layers. *J. Fluid Mech.* **110**, 1–37.

STEWARTSON, K. 1958 On Goldstein's theory of laminar separation. *Q. J. Mech. Appl. Maths* **11**, 399–410.

STEWARTSON, K. 1969 On the flow near the trailing edge of a flat plate ii. *Mathematika* **16** (1), 106–121.

STEWARTSON, K. 1970 Is the singularity at separation removable? *J. Fluid Mech.* **44**, 347–364.

STEWARTSON, K., SMITH, F. T. & KAUPS, K. 1982 Marginal separation. *Stud. Appl. Maths* **67** (1), 45–61.

STEWARTSON, K. & WILLIAMS, P. G. 1969 Self-induced separation. *Proc. R. Soc. Lond.* A **312**, 181–206.

SYCHEV, V. V. 1972 Laminar separation. *Izv. Akad. Nauk. SSSR Mekh. Zhidk. Gaza* **3**, 47–59.

SYCHEV, V. V. 1979 Asymptotic theory of non-stationary separation. *Izv. Akad. Nauk SSSR Mekh. Zhidk. Gaza* **6**, 21–32, English transl. *Fluid Dyn.* (1980) 14, 829–838.

SYCHEV, V. V., RUBAN, A. I., SYCHEV, V. V. & KOROLEV, G. L. 1998 *Asymptotic Theory of Separated Flows*. Cambridge University Press.

TELIONIS, D. & TSAHALIS, D. 1974 Unsteady laminar separation over impulsively moved cylinders. *Acta Astronaut.* **1**, 1487–1505.

TSAHALIS, D. 1977 Laminar boundary-layer separation from an upstream moving wall. *AIAA J.* **15**, 561–566.

VAN DOMMELEN, L. L. 1981 Unsteady separation. PhD thesis, Cornell university.

VAN DOMMELEN, L. L. & SHEN, S. F. 1980 The spontaneous generation of a singularity in a separating boundary layer. *J. Comput. Phys.* **38**, 125–140.

VAN DOMMELEN, L. L. & SHEN, S. F. 1982*a* A bifurcation-free interaction solution for steady separation from a downstream moving wall. *AIAA*-82-0347.

VAN DOMMELEN, L. L. & SHEN, S. F. 1982*b* The genesis of separation. In *Symposium on Numerical and Physical Aspects of Aerodynamic Flows* (ed. T. Cebeci), pp. 293–311. Springer.

VAN DOMMELEN, L. L. & SHEN, S. F. 1983*a* Boundary layer separation singularities for an upstream moving wall. *Acta Mechanica* **49**, 241–254.

VAN DOMMELEN, L. L. & SHEN, S. F. 1983*b* An unsteady interactive separation process. *AIAA J.* **21**, 358–362.

VAN DOMMELEN, L. L. & SHEN, S. F. 1985 The flow at a rear stagnation point is eventually determined by exponentially small values of the velocity. *J. Fluid Mech.* **157**, 1–16.

VIDAL, R. J. 1959 Research on rotating stalling axial flow compressor; part III-experiments on laminar separation from a moving wall. *Tech. Rep.* 59–75. Wright Air Dev. Cent.

WANG, K. C. 1979 Unsteady boundary-layer separation. *Tech. Rep.* MML TR 79-16c. Martin Marietta Laboratory.

WILLIAMS, J. C. III 1977 Incompressible boundary-layer separation. *Annu. Rev. Fluid. Mech.* **9**, 113–144.