The Relationship Between the True stress, σ_T, and Engineering Stress, s.

- True stress, σ_T is defined as the load P divided the instantaneous cross sectional area, A_i over which deformation is occurring (i.e., the neck, past the tensile point), or

$$\sigma_T = \frac{P}{A_i} \tag{1.18}$$

- Recall from equation 1-4 that the true strain is given as:

$$\epsilon_T = \ln \frac{l_i}{l_0} \tag{1.4}$$

- If no volume change occurs during deformation, then,

$$A_i l_i = A_0 l_0 \tag{1.19}$$

- It is possible to show that the true stress and engineering stress or the true strain and and engineering strain are related according to:

$$\sigma_T = \sigma(1 + \epsilon) \tag{1.20}$$

$$\epsilon_T = \ln(1 + \epsilon) \tag{1.21}$$

- The derivations above assumes both constancy of volume and a homogenous distribution of strain along the gage length of the tension specimen.

- For some metals and alloys the region of the true stress-true strain curve from the onset of plastic deformation to the point at which necking begins may be approximated by

$$\sigma_T = K\epsilon_T^n \quad (1.22a)$$

where K and n are constants.

- Equation 1-22a can also be written as:

$$\sigma_T = H\varepsilon_T^n \quad (1.22b)$$

- Equation (1.22) is known as Hollomon’s Equation

- If the stress versus strain is plotted on the log-log coordinates, this equation gives a straight line. The logarithm of Eqn (1.22b) gives:

$$\ln \sigma_T = \ln H + n \ln \varepsilon_T \qquad (1.22c)$$

  - The slope of the line is **n**, and this is called the **strain hardening exponent**.
  - lnH is the intercept at $\varepsilon = 1$.
  - The quantity *H* (or K) is called the ***strength coefficient***.

- Beyond maximum load the true strain should be based on actual area or diameter measurements.

$$\varepsilon_T = \ln \frac{A_o}{A} = \ln \frac{(pi/4)}{(pi/4)} \frac{D_o^2}{D^2} = 2 \ln \frac{D_o}{D} \qquad (1.23)$$

Instability in Tension

- Necking or localized deformation begins at maximum load.

- At necking, the increase in stress due to decrease in the cross-sectional area of the specimen becomes greater than the increase in the load-carrying ability of the metal due to strain hardening.

- This conditions of instability leading to localized deformation is defined by the condition $dP = 0$.

- Recall,

$$\boldsymbol{P = \sigma_T A_i} \qquad (1.18)$$

- The instability condition is given as:

$$dP = \sigma_T dA_i + A_i d\sigma_T = 0 \qquad (1.24a)$$

$$\therefore -\frac{dA_i}{A_i} = \frac{d\sigma_T}{\sigma_T} \qquad (1.24b)$$

From the constancy-of-volume relationship,

$$V = A_o L_o = A_i L_i = C$$

$$\therefore \quad dV = A_i dL_i + L_i dA_i = 0$$

$$\therefore \quad -\frac{dA_i}{A_i} = \frac{dL_i}{L_i} = d\varepsilon_T \qquad (1.25)$$

From equations 1-24b and 1-25, we have

$$\frac{d\sigma_T}{\sigma_T} = d\varepsilon_T \tag{1.26a}$$

Therefore, at the point of tensile instability

$$\frac{d\sigma_T}{d\varepsilon_T} = \sigma_T \tag{1.26b}$$

The above equation is referred to as the **necking or Considere** criterion.

- The necking criterion can be expressed more explicitly if engineering strain is used. Starting with Eq. (1.26b)

$$\frac{d\sigma_T}{d\varepsilon_T} = \frac{d\sigma_T}{de}\frac{de}{d\varepsilon_T} = \frac{d\sigma_T}{de}\frac{dL/L_o}{dL/L} = \frac{d\sigma_T}{de}(e+1) = \sigma_T$$

$$\frac{d\sigma_T}{de} = \frac{\sigma_T}{1+e} \qquad (1.27)$$

True Stress at Maximum Load

$$s_u = \frac{P_{\max}}{A_o}$$

and

$$\sigma_u = \frac{P_{\max}}{A_u} \qquad \varepsilon_u = \ln \frac{A_o}{A_u}$$

Eliminating P_{max} yields

$$\sigma_u = s_u \frac{A_o}{A_u}$$

and

$$\sigma_u = s_u e^{\varepsilon_u} \tag{1.28}$$

True Fracture Strain

- The true fracture strain, ε_f, is the true strain based of the original area A_o and the area after fracture A_f.

$$\varepsilon_f = \ln \frac{A_o}{A_f} \qquad (1.29)$$

- For cylindrical tensile specimens the reduction of area q is related to the true fracture strain by the relationship

$$e_f = \ln \frac{1}{1 - q} \qquad (1.30)$$