AP Association for Proportional Fairness in Multirate WLANs

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***Abstract*—In this paper, we investigate the problem of achieving proportional fairness via access point (AP) association in multirate WLANs. This problem is formulated as a nonlinear programming with an objective function of maximizing the total user bandwidth utilities in the whole network. Such a formulation jointly considers fairness and AP selection. We first propose a centralized algorithm Non-Linear Approximation Optimization for Proportional Fairness (NLAO-PF) to derive the user–AP association via relaxation. Since the relaxation may cause a large integrality gap, a compensation function is introduced to ensure that our algorithm can achieve at least half of the optimal in the worst case. This algorithm is assumed to be adopted periodically for resource management. To handle the case of dynamic user membership, we propose a distributed heuristic Best Performance First (BPF) based on a novel performance revenue function, which provides an AP selection criterion for newcomers. When an existing user leaves the network, the transmission times of other users associated with the same AP can be redistributed easily based on NLAO-PF. Extensive simulation study has been performed to validate our design and to compare the performance of our algorithms to those of the state of the art.**

***Index Terms*—Access point (AP) association, bandwidth allocation, multirate WLANs, proportional fairness.**

I. Introduction

BY DEFAULT, each user in an IEEE 802.11 WLAN associates with the access point (AP) that has the largest received signal strength indicator (RSSI). As typically users are not uniformly distributed among all APs, RSSI-based approach may overload some APs while leaving others to carry very light load or even to be idle. This load unbalancing could result in unfair bandwidth allocation. Although the network is supposed to serve fairly at high performance, fairness and efficiency are often in conflict with each other. With the development of multirate WLANs, this problem has become even more challenging, as users with different bit rates intend to share the same WLAN.

It is well known that the popular 802.11 MAC protocol provides equal long-term transmission opportunities to all users associated with the same AP. Therefore, users with the same frame size and same transmission rate can achieve equal throughput (i.e., throughput-based fairness). However, in multirate WLANs, throughput-based fairness requires that users with lower bit rates occupy the channel for longer times than those with higher bit rates, drastically reducing the network throughput [1], [2]. To overcome this problem, time-based fairness is proposed such that each user can obtain an equal share of channel occupancy time. Recent research [1], [3] has shown that time-based fairness outperforms throughput-based fairness in multirate WLANs.

There exist other fairness criteria that are widely adopted in network resource assignment. *Max-min fairness* distributes resources as equally as possible among users [4]–[7]; *proportional fairness*, on the other hand, allocates bandwidth to users in proportion to their bit rates to maximize the sum of the bandwidth utilities of all users [8]–[10]. Proportional fairness has been utilized to effectively exploit the tradeoff between fairness and network performance [8], [11]. It is argued [12] that within a single saturated AP, throughput-based fairness and max-min fairness are equivalent; moreover, if all users have the same priority level, time-based fairness and proportional fairness are also equivalent.

Fairness and AP association should be jointly considered for resource management in multirate WLANs, but existing research mainly focuses on the joint study of max-min fairness and AP association. In this paper, we investigate the problem of achieving proportional fairness via AP association for performance enhancement. To achieve this goal, we formulate the problem to a nonlinear programming (**NLP**) with an objective function of maximizing the total user bandwidth utilities in the whole network and propose a centralized algorithm termed Non-Linear Approximation Optimization for Proportional Fairness (**NLAO-PF**) to solve it. Since the objective function of our **NLP** is nonlinear and the AP association is a 0–1 integer programming problem, **NLP** is NP-hard [13]. Therefore, **NLAO-PF** is decomposed into four steps to simplify the issue and improve the degree of approximation. By introducing a compensation function to the objective function of **NLP**, the total utility of the bandwidth allocation via **NLAO-PF** is

Manuscript received May 22, 2012; revised November 25, 2012; accepted January 24, 2013; approved by IEEE/ACM Transactions on Networking Editor Y. Bejerano. Date of publication February 25, 2013; date of current version February 12, 2014. This work was supported by the US National Science Foundation under Grant CNS-0831852, the NPST Program by King Saud University under Project No. 10-INF1184-02, and the National Natural Science Foundation of China under Grants No. 61272475 and No. 61003225.

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Digital Object Identifier 10.1109/TNET.2013.2245145

proved to be at least half of the optimal in the worst case. In real-world applications, **NLAO-PF** can be adopted periodically to achieve proportional fairness. To handle the case of dynamic user membership, we propose a distributed heuristic termed Best Performance First (**BPF**) based on a novel performance revenue function, which provides an AP selection criterion for the newly arriving users. When an existing user leaves the network, the transmission times of other users associated with the same AP can be redistributed easily based on **NLAO-PF**. Our comparison-based simulation study indicates that both **NLAO-PF** and **BPF** perform well when the users are distributed randomly and uniformly in the whole network. Moreover, the superiority of our algorithms is even higher compared to the most relevant ones when users are distributed in a hotspot area.

The rest of the paper is organized as follows. The most related work is discussed in Section II. The fairness criterion and our network model are introduced in Section III. The two algorithms are detailed in Sections IV and V. After presenting the evaluation results in Section VI, we conclude this paper in Section VII.

II. Related Work

Achieving fairness within a single AP via optimizing the media access procedure has been extensively studied. The fairness of CSMA/CA is analyzed by Jian and Chen in [14]. This work also proposes a rate control protocol called "Proportional Increase Synchronized Multiplicative Decrease" to achieve fair bandwidth allocation. The short-term and long-term fairness of the 802.11 DCF procedure are investigated in [15] by employing the conditional probabilities of the number of intertransmissions. A technique to estimate the fair rate from passive traffic measurements of a video application is also proposed in [15]. In [1] and [16]–[18], different media access methodologies are investigated to optimize the MAC parameters for fairness. In [19], a CSMA/CA MAC protocol without adopting the exponential backoff procedure is proposed to optimize the throughput when achieving time-based fairness by adaptively determining the contention window size based on the transmission opportunities.

AP association is another fundamental problem in wireless networks to enhance the performance. Ekici and Yongacoglu [20] propose a distributed AP selection scheme in which a user associates with an AP that provides the best performance in terms of congestion relief by considering the bit rate as well as the number of users accommodated by the AP. Abusubaih and Wolisz [21] present a centralized optimal association policy for multirate IEEE 802.11 WLANs. Their policy is based on the cell status information and can facilitate information exchange between APs. Yen *et al.* [22] model AP selection under the framework of game theory, where the sole goal of each wireless station is to maximize its achievable throughput by considering both the number of wireless stations that associate with the same AP and the set of link rates these wireless stations possess. Keranidis *et al.* [23] make the AP selection for each user in a distributed way to maximize per-user total throughput, which is defined to be the sum of the throughputs on the uplink and the downlink.

None of the works mentioned above jointly considers fairness and AP association. Achieving throughput-based max-min fairness via AP association has been studied in [6] and [24]–[26]. Gong *et al.* [24] formulate the AP selection problem in wireless mesh networks as a nonlinear optimization programming and apply a weighting parameter to obtain a tradeoff between the total throughput and the max-min fairness. In [25], a two-stage smart association control protocol is proposed. In the bandwidth allocation stage, APs collaboratively determine the number of devices they are going to associate for max-min fairness, while in the AP association stage, devices are assigned to APs for throughput maximization. Bejerano *et al.* [6] and Xu *et al.* [26] demonstrate the strong correlation between throughput-based max-min fairness and min-max load balancing and achieve the max-min fair bandwidth allocation via AP association.

Throughput-based max-min fairness suffers from a low network throughput in multirate WLANs [8], [11]. Proportional fairness, on the other hand, can effectively investigate the tradeoff between fairness and network throughput [8], [11]. Li *et al.* [11] formulate a nonlinear programming to achieve optimal proportional fairness in a network of APs and propose two approximate AP selection schemes, cvapPF and nlapPF, for periodic offline optimization. Both cvapPF and nlapPF rely on relaxation and rounding to obtain an integral user–AP association. Bu *et al.* [13] study the proportional fairness problem in 3G wireless data networks. This study is particularly suitable for 3G data networks and therefore is not applicable to multirate WLANs. Koukoutsidis and Siris [27] propose a branch-and-bound algorithm to investigate the network throughput under the max-min fairness and proportional fairness. However, the time complexity of this algorithm is inversely proportional to the allowable relative error from the optimal solution, resulting in an unbounded performance in the worst case. Xie *et al.* [28] formulate the problem of AP association control over vehicular networks as a convex programming in the offline setting and design a dynamic weight-based online algorithm to achieve proportional fairness. Li *et al.* [29] jointly consider AP association and power control, establish the relationship between the network utility and the AP utility according to proportional fairness, and then devise a centralized heuristic approach to optimize the network utility by increasing the average and decreasing the variance of the AP utility.

In this paper, we propose two algorithms that jointly consider AP association and fair bandwidth allocation. Our centralized problem formulation is motivated by the nonlinear programming in [11], but we adopt a completely different approach to relax the variables in our approximation algorithm design. By introducing a compensation function to the objective function to narrow down the gap caused by relaxation, we achieve an approximate solution that is at least half of the optimal in the worst case. Our second algorithm is distributed, which selects an AP for a newly arriving user based on a novel performance revenue function to achieve proportional fairness. Note that our centralized and distributed algorithms can be combined to significantly improve the performance of dynamic networks where users come and leave by their own free will.

III. Proportional Fairness and Network Model

In this section, we first briefly introduce the formal definition of proportional fairness, then we detail our network model and problem formulation.

A. Proportional Fairness

To fairly assign bandwidths to users while guaranteeing the network performance in multirate WLANs, we adopt *proportional fairness*, a fairness criterion that was proposed by Kelly [8]. According to [8], proportional fairness can be formally defined as follows. *Let b_i be the effective bandwidth of user i and $F(b_i)$ be the corresponding utility function. A vector of bandwidth assignment $\{b_1, b_2, \ldots, b_M\}$, with M being the number of users in the network, is* proportionally fair *if it is feasible and if for any other feasible vector $\{b_1^*, b_2^*, \cdots, b_M^*\}$*, the aggregate of proportional changes is either zero or negative, i.e.,

$$\sum_{i=1}^{M} \frac{b_i^* - b_i}{b_i} \leq 0.$$

Consider a small feasible perturbation $b_i \rightarrow b_i + \delta b_i$, which increases the utility function $F(b_i)$ providing that

$$\sum_{i=1}^{M} F'(b_i)\delta b_i > 0.$$

From the definition of proportional fairness, we have

$$\sum_{i=1}^{M} \frac{\delta b_i}{b_i} > 0$$

which can be rewritten as

$$\sum_{i=1}^{M} (\log(b_i))' \, \delta b_i > 0.$$

Thus, it follows that the above proportionally fair bandwidth allocation can be represented by a local maximum of the logarithmic utility function. Since the logarithmic function is differentiable and strictly concave, it has only one maximum, and therefore the local maximum is also the global maximum [8]. Accordingly, the objective of a proportionally fair bandwidth allocation can be expressed by

$$\max \sum_{i=1}^{M} \log(b_i).$$

To quantitatively evaluate the fairness degree of our bandwidth allocation, we adopt Jain's Fairness Index [30], which states that *if a system allocates resources (bandwidths in our case) to M users, with the ith user receiving an allocation b_i, the fairness index of the system is defined to be*

$$J = \frac{\left(\sum_{i=1}^{M} b_i\right)^2}{M \sum_{i=1}^{M} b_i^2}, \qquad \text{where } b_i \geq 0.$$

This index measures the "equality" of users' resource allocation $\{b_1, b_2, \ldots, b_M\}$. If all users obtain the same amount of the bandwidth, i.e., all b_i's are equal, the fairness index is 1 and the system is 100% fair. As the disparity increases, fairness decreases. An allocation scheme that favors only a few users has a fairness index close to 0.

TABLE I
NOTATIONS

Symbol	Semantics
A	The set of all access points (APs)
A_i	The set of APs associated with user i
N	$N = \|A\|$, the number of APs
U	The set of all users
M	$M = \|U\|$, the number of users
γ_{ij}	The SINR of the link from AP j to user i
g_{ij}	The channel gain from AP j to user i
p_j	The transmit power of AP j
N_0	The receiver noise power
w_i	The weight (priority) of user i, $w_i > 0$
b_i	The effective bandwidth allocated to user i
r_{ij}	The bit rate between user i and AP j
x_{ij}	The association coefficient between user i and AP j
X	The 0-1 user-AP association matrix $\{x_{ij}\}$
t_{ij}	The effective transmission time between user i and AP j
T	The transmission time allocation matrix $\{t_{ij}\}$

TABLE II
RELATIONSHIP BETWEEN SINRS AND EFFECTIVE BIT RATES IN IEEE 802.11 STANDARD

γ_{ij}(dB)	6-7.8	7.8-9	9-10.8	10.8-17	17-18.8	18.8-24	24-24.6	24.6-
r_{ij}(Mbps)	6	9	12	18	24	36	48	54

B. Network Model

Our network topology models an IEEE 802.11-based multirate WLAN that consists of multiple APs operating at the same channel. Each AP has the same limited coverage area and serves users in its area. Overlapping coverage areas of adjacent APs may exist. The union of the coverage areas of all APs forms the network coverage area. We assume that each AP transmits messages with the same power as defined by IEEE 802.11. We further assume that each user is covered by at least one AP, and each AP has at least one associated user. The notations and definitions to be utilized are summarized in Table I.

As we have known, a user in an overlapping coverage area is typically serviced by one AP and interfered with all other APs. The effective bit rate of a user in an 802.11 network is determined by the experienced signal-to-interference-plus-noise ratio (SINR) of the user. More precisely, let γ_{ij} denote the SINR of user i when associated with AP j. We have

$$\gamma_{ij} = \frac{g_{ij} p_j}{\sum_{k \in \mathbf{A_i} \cap k \neq j} g_{ik} p_k + N_0} \tag{1}$$

where g_{ij} is the channel gain from AP j to user i, p_j is the transmit power of AP j, N_0 is the additive Gaussian white noise, and $\mathbf{A_i}$ is the set of APs whose transmissions interfere with user i. Note that here we choose to focus on downlink because the data transmissions from the APs represent the dominate traffic for many real-world applications such as social networks [31], [32]. The relationship between the effective bit rates and the SINR ranges in an 802.11 network is shown in Table II [33], [34].

It is assumed that the network is saturated such that all APs are busy all the time to send data to users. A *unit of time*, in which the network is stable, with no new user joins and no current user leaves, is to be considered. This means that under our consideration the total transmission time of an AP is equal to 1.

Each AP assigns fractional transmission times to users in accordance with proportional fairness. A user is allowed to choose one and only one AP within a unit time.

We formulate the problem of AP association based on proportional fairness as a nonlinear programming. Our goal is to construct an assignment of users to APs in a proportional manner; i.e., the assignment allocates each user a sufficient amount of bandwidth without unduly restricting the amount of bandwidth available to others. In our system model, the resources at all APs are considered as a whole when allocating bandwidth fairly to users. With this network-wide fairness objective, load balancing is automatically taken into account. Since the effective bandwidth of user i is $b_i = \sum_{j=1}^{N} x_{ij}t_{ij}r_{ij}$, we obtain the following optimization formulation:

$$\max \quad \sum_{i=1}^{M} w_i \log\left(\sum_{j=1}^{N} x_{ij}t_{ij}r_{ij}\right) \tag{2a}$$

$$\text{s.t.} \quad \sum_{j=1}^{N} x_{ij} = 1, 1 \le i \le M \tag{2b}$$

$$\sum_{i=1}^{M} x_{ij}t_{ij} = 1, 1 \le j \le N \tag{2c}$$

$$x_{ij} \in \{0,1\}, 1 \le i \le M, 1 \le j \le N \tag{2d}$$

$$t_{ij} \in [0,1], 1 \le i \le M, 1 \le j \le N. \tag{2e}$$

Equation (2) is referred to as an **NLP**. Note that our objective function (2a) considers the weights of the users, which reflects their priorities in a real network. The constraint (2b) indicates that each user can associate with one and only one AP; the constraint (2c) requires that the total transmission time of each AP j is equal to 1; the constraint (2d) assures that x_{ij} is a binary variable that is equal to 1 if and only if user i associates to AP j; and the constraint (2e) specifies the range of the variable t_{ij}. We can prove that **NLP** is NP-hard by slightly adapting the reduction procedure proposed in [13]. Note that this problem formulation is motivated by [11], but our approach to solving the problem via relaxation, as elaborated in Section IV, is fundamentally different and completely novel. Also note that the joint problem of AP association and bandwidth allocation expressed by (2) is formulated based on *time-based proportional fairness*, given the bit rates between users and APs.

IV. NLAO-PF Algorithm

Since **NLP** is NP-hard, we propose an approximation algorithm termed **NLAO-PF** to simplify the issue and improve the degree of approximation. The steps of **NLAO-PF** are outlined in Algorithm 1.

Algorithm 1: NLAO-PF

1: Obtain $\{t'_{ij}\}$ by solving **r-NLP**.
2: Get the fractional solution $\{x'_{ij}\}$ by solving **c-NLP**.
3: Get the integral solution $\{x_{ij}\}$ by a rounding process.
4: Redistribute transmission time to obtain $\{t_{ij}\}$ and calculate $\{b_i\}$.

The basic idea of **NLAO-PF** is to relax the binary variable x_{ij} such that each user is allowed to associate with multiple APs within a unit time, i.e., x_{ij} can be fractional. Actually, we further relax x_{ij} to the extent that any user is allowed to freely associate with all APs by setting $x_{ij} = 1$ at the first step. Under such a relaxed condition, we compute the optimal transmission time t'_{ij} by solving a relaxed optimization problem (Section IV-A). Taking t'_{ij} as a known parameter, we obtain x'_{ij}, the fractional user–AP association coefficient, by solving a complemented optimization problem (Section IV-B). A rounding technique is then applied to x'_{ij} to obtain an approximate integral solution of x_{ij} (Section IV-C). Based on the newly obtained x_{ij}, the transmission time t'_{ij} is redistributed for the original **NLP** problem (Section IV-D).

The relaxation involved in **NLAO-PF** may result in a large integrality gap [35]. To overcome this problem, we modify the objective function of **NLP** by adding a compensation function $g(X,T)$ in **NLAO-PF**, which is defined as follows.

Definition 1: The compensation of user i on AP j is defined by $w_i x_{ij} t_{ij} \log(r_{ij})$ if $r_{ij} \ge 1$, and 0 otherwise. Thus, the compensation of user i to all APs can be expressed by $w_i \sum_{j=1}^{N} x_{ij}t_{ij}\log(r_{ij})$. Therefore, the compensation function $g(X,T)$ can be defined correspondingly as follows:

$$g(X,T) = \sum_{i=1}^{M} w_i \sum_{j=1}^{N} x_{ij}t_{ij}\log(r_{ij}). \tag{3}$$

This compensation function is introduced to improve the lower bound of our algorithm to effectively narrow down the integrality gap caused by relaxation. The steps of **NLAO-PF** are detailed in the following subsections.

A. Relaxed Optimization Program

The first step of **NLAO-PF** is to solve the following relaxed optimization problem, denoted by **r-NLP**, to obtain an optimal $\{t'_{ij}\}$:

$$\max \quad \sum_{i=1}^{M} w_i \log\left(\sum_{j=1}^{N} t'_{ij}r_{ij}\right) + \sum_{i=1}^{M} w_i \sum_{j=1}^{N} t'_{ij}\log(r_{ij}) \tag{4a}$$

$$\text{s.t.} \quad \sum_{j=1}^{N} t'_{ij} \le 1, 1 \le i \le M \tag{4b}$$

$$\sum_{i=1}^{M} t'_{ij} = 1, 1 \le j \le N \tag{4c}$$

$$t'_{ij} \in [0,1], 1 \le i \le M, 1 \le j \le N. \tag{4d}$$

Compared to (2), **r-NLP** replaces t_{ij} by t'_{ij}, sets $x_{ij} = 1$, and includes the compensation function in its objective function. The constraint (4b) indicates that the total transmission time of user i with all APs cannot surpass 1; the constraint (4c) requires that the total transmission time of each AP is equal to 1, which means that all APs are saturated in the unit time; and the constraint (4d) defines the range of the variable t'_{ij}. Obviously, the optimal solution for $\{t'_{ij}\}$ from (4) can be computed in polynomial time [11].

B. Fractional Association

After solving **r-NLP**, we obtain the transmission time $\{t'_{ij}\}$. Now we take $\{t'_{ij}\}$ as a known input and get the fractional user–AP association $\{x'_{ij}\}$. Because of the requirements for solving convex programs, we change the linear equality constraint of **NLP** to a linear inequality constraint in the following complemented Non-Linear Programming (**c-NLP**) formulation, which does not change the solution value:

$$\max \quad \sum_{i=1}^{M} w_i \log\left(\sum_{j=1}^{N} x'_{ij} t'_{ij} r_{ij}\right) + \sum_{i=1}^{M} w_i \sum_{j=1}^{N} x'_{ij} t'_{ij} \log(r_{ij}) \tag{5a}$$

$$\text{s.t.} \quad \sum_{j=1}^{N} x'_{ij} > 0, 1 \le i \le M \tag{5b}$$

$$\sum_{i=1}^{M} x'_{ij} t'_{ij} = 1, 1 \le j \le N \tag{5c}$$

$$x'_{ij} \ge 0, 1 \le i \le M, 1 \le j \le N. \tag{5d}$$

The objective function of **c-NLP** is designed to approximate the optimal solution to **NLP**. The constraint (5b) indicates that a user should connect with at least one AP; the constraint (5c) forces the total transmission time of each AP be equal to 1; and the constraint (5d) defines the range of x'_{ij} for the case of fractional association. Note that here we take $\{t'_{ij}\}$ obtained from (4) as the input to **c-NLP** and obtain the optimal association $\{x'_{ij}\}$ for **c-NLP** given $\{t'_{ij}\}$.

We can prove that the gap introduced by our relaxation procedure is bounded. Let $f(X,T) = \sum_{i=1}^{M} w_i \log(\sum_{j=1}^{N} x_{ij} t_{ij} r_{ij})$, and $h(X,T) = f(X,T) + g(X,T)$. Then, the objective functions of **r-NLP** and **c-NLP** become $h(X=1,T)$ and $h(X,T)$, respectively. Correspondingly, $h(X=1,T')$ and $h(X',T')$ are the solutions obtained from **r-NLP** and **c-NLP**, respectively.

Theorem 1: Let (X^*,T^*) and (X',T') be the optimal solutions to **NLP** and to **c-NLP**, respectively. Then, $f(X^*,T^*) \le h(X',T') \le 2f(X^*,T^*)$.

Proof: With $\sum_{i=1}^{M} x'_{ij} t'_{ij} = 1$ [constraint (5c)] and $r_{ij} \ge 1$, $f(X',T') \ge g(X',T') \ge 0$. Thus, $h(X',T') \le 2f(X',T') \le 2f(X^*,T^*)$. Since $(X=1,T^*)$ is a feasible solution of the problem **r-NLP** and $(X=1,T')$ is the optimal solution of **r-NLP**, we have $f(X^*,T^*) = f(X=1,T^*) \le f(X=1,T^*) + g(X=1,T^*) = h(X=1,T^*) \le h(X=1,T') \le h(X',T')$, where the last inequality holds from the fact that $h(X=1,T')$ is feasible to **c-NLP**. ■

C. Rounding

In this step, we use the rounding algorithm proposed in [36] to obtain an integral association matrix X. That is, we fix the time allocation $\{t'_{ij}\}$ and replace the fractional association $\{x'_{ij}\}$ by a 0-1 variable $\{x_{ij}\}$ that encodes the desired association of users to APs. The rounding process contains two main steps: bipartite graph construction and maximum-profit matching, which are detailed as follows.

First, we construct a bipartite graph $G(x) = (U,V,E)$, where the set U represents the users in the network, and the set $V = \{v_{jk} : j = 1,2,\ldots,N, k = 1,2,\ldots,Q_j\}$, with $Q_j = \lceil \sum_{i=1}^{M} x'_{ij} \rceil$. This implies that each AP may have multiple nodes in V. The edges in $G(x)$ are constructed according to the following method. If $Q_j \le 1$, there is only one node v_{j1} corresponding to AP j. For each $x'_{ij} > 0$, add edge $e(u_i, v_{j1})$ and set $x'(u_i, v_{j1}) = x'_{ij}$, where $x'(e)$ is the fractional association weight of the corresponding user and AP. Otherwise, find the minimum index i_k such that $\sum_{i=1}^{i_k} x'_{ij} \ge k$. For $i = i_{k-1}+1, \ldots, i_k - 1$ and $x'_{ij} > 0$, add edge $e(u_i, v_{jk})$ and set $x'(u_i, v_{jk}) = x'_{ij}$. For $i = i_k$, add edge $e(u_i, v_{jk})$ and set $x'(u_i, v_{jk}) = 1 - \sum_{i=i_{k-1}+1}^{i_k - 1} x'(u_i, v_{jk})$. If $\sum_{i=1}^{i_k} x'_{ij} > k$, add edge $e(u_i, v_{j(k+1)})$ and set $x'(u_i, v_{j(k+1)}) = \sum_{i=1}^{i_k} x'_{ij} - k$. Obviously, $x'(e)$ has the following property:

$$\sum_{i=i_{k-1}+1}^{i_k} x'(u_i, v_{jk}) = \begin{cases} = 1, & k = 1,2,\ldots,Q_j - 1 \\ \le 1, & k = Q_j. \end{cases}$$

This implies that the sum of the fractional association weights on each node v_{jk} does not exceed one. The *profit* of each edge $e(u_i, v_{jk})$ in E is defined to be $w_i \log(t'_{ij} r_{ij})$.

Second, we find a maximum-profit matching $M(x)$ that matches each user node with an AP node in $G(x)$. For each edge $e(u_i, v_{jk})$ in $M(x)$, schedule user i to AP j and set $x_{ij} = 1$. Set other x_{ij}'s to be 0. Since the fractional association $\{x'_{ij}\}$ specifies a fractional matching, such a maximal matching does exist, and it determines the integral association $\{x_{ij}\}$. More details can be found in [36].

Note that $\{t'_{ij}\}$ and $\{x'_{ij}\}$ are computed from **r-NLP** and **c-NLP**, respectively. The rounding scheme constructs an integral assignment $\{x_{ij}\}$. We denote this solution by $f(X^a, T')$, which is also feasible to **NLP**. Then, we have the following.

Theorem 2: $f(X^a, T') \ge (1/2) f(X^*, T^*)$.

Proof: Note that $\{x_{ij}\}$ is obtained by employing the rounding scheme proposed by Shmoys and Tardos in [36], which proves the following property: $f(X^a,T') \ge f(X',T')$. Thus, $f(X^a,T') \ge f(X',T') \ge (1/2)[f(X',T') + g(X',T')] = (1/2)h(X',T') \ge (1/20f(X^*,T^*)$, where the last inequality holds from Theorem 1. ■

We use an example to demonstrate the rounding process. Suppose that there exist three users and two APs and that we have obtained the following fractional AP association after the second step of Algorithm 1: $x'_{11} = 1$, $x'_{22} = 1$, $x'_{31} = 1/2 = 1/2$. The corresponding bipartite graph is illustrated in Fig. 1, where $x'_{ij} = 1$ for solid edges, $x'_{ij} = 1/2$ for dashed edges, and the numbers beside the edges are their profits. More specifically, $Q_1 = Q_2 = 2$; that is, each AP has two nodes in the bipartite graph. Then, the maximum-profit matching yields $x_{11} = x_{22} = x_{31} = 1$ with a total profit of 2.7, which is larger than the fractional association profit 2.5.

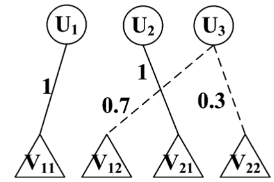


Fig. 1. Example of rounding process.

D. Transmission Time Redistribution

Since the user–AP association changes after rounding, we need to redistribute the transmission times. This is the last step of **NLAO-PF**, in which we assign transmission times to users according to proportional fairness.

Theorem 3: Let $\{x_{ij}\}$ be the integral user–AP association coefficients obtained from the rounding procedure outlined in Section IV-C. Given $\{x_{ij}\}$, the unique optimal transmission time assigned to user i by AP j according to proportional fairness is

$$t_{ij} = \frac{w_i x_{ij}}{\sum_{k=1}^{M} w_k x_{kj}}. \tag{6}$$

Proof:

(a) First, we consider the case of a single AP. Assume that the number of users covered by the AP is m. Since the objective function of (2) is the sum of logarithms, maximizing the total utility of the user bandwidth (2) is equivalent to maximizing

$$\prod_{i=1}^{M}(t_{ij}r_{ij})^{w_i} = \prod_{i=1}^{M}(t_{i1}r_{i1})^{w_i} = \prod_{i=1}^{M}(t_{i1})^{w_i}\prod_{i=1}^{M}(r_{i1})^{w_i}. \tag{7}$$

Note that $\{r_{i1}\}$ is the set of optimization constants. Therefore, maximizing (7) is equivalent to maximizing

$$\prod_{i=1}^{M}(t_{i1})^{w_i} = \left(\underbrace{t_{11}t_{11}\cdots t_{11}}_{w_1}\right)\left(\underbrace{t_{21}t_{21}\cdots t_{21}}_{w_2}\right)\cdots \left(\underbrace{t_{M1}t_{M1}\cdots t_{M1}}_{w_M}\right). \tag{8}$$

Since $\sum_{i=1}^{M} t_{i1} = 1$, (8) is maximized if and only if $t_{11}/w_1 = t_{21}/w_2 = t_{M1}/w_M = 1/\sum_{k=1}^{M} w_k$. Thus, we have $t_{i1} = w_i/\sum_{k=1}^{M} w_k$.

(b) Now we consider the case of multiple APs. Since x_{kj} is a 0-1 variable denoting the association coefficient between user k and AP j, $\sum_{k=1}^{M} w_k x_{kj}$ is the sum of the weights of all users associated to AP j. With a similar analysis as that of case (a), the optimal transmission time given $\{x_{ij}\}$ can be calculated by (6).

We conclude that given $\{x_{ij}\}$, our transmission time assignment based on proportional fairness is unique and optimal. ■

Note that Theorem 3 implies that proportional fairness is equivalent to time-based fairness when all users have the same weight w_i. This result is consistent with the one proposed in [12], but is obtained from a different angle.

The solution obtained from our algorithm **NLAO-PF** can be denoted as $f(X^a, T^a)$. Based on Theorems 2 and 3, we have $f(X^a, T^a) \geq f(X^a, T') \geq (1/2) f(X^*, T^*)$. That is, the approximate solution obtained from **NLAO-PF** is no less than half of the optimal solution of **NLP**.

V. BPF Algorithm

In this section, we present a distributed algorithm named **BPF** for AP association to support proportional fairness in dynamic network scenarios.

Note that without collecting the network-wide information, a user is only aware of the changes of its currently associated AP, including the join of a new user and the leave of a current user. This implies that it is impossible for a user to switch to an AP whose associated users leave the network. On the other hand, a user's utility can be increased if other users associated with the same AP leave the network, thus this user has no incentive to leave its currently associated AP. As shown in (6), within one unit of transmission time, the portion assigned to each user is proportional to its weight divided by the total weights of all users associated with the same AP. Therefore, reallocating the optimal transmission time when current users leave from the network on each AP is trivial. Thus, in this section, we focus on the case when a user intends to join the network, i.e., how to select the best AP for a new user.

A. BPF Algorighm

According to IEEE 802.11, by default, a new user selects the AP with the strongest received signal strength to associate with once entering a network. However, in some cases, this approach could unfairly overload the APs with the strongest signals and thus reduce the aggregate network throughput. Therefore, we hope to seek an AP association method such that the association of a new user results in the most positive impact on the overall network performance. Based on this observation, we construct a *performance revenue function*, which is defined to be *the difference of the bandwidth utilities* resulted from the join of the new user to an AP.

We assume that there are m_j users associated with AP j before a new user enters j's coverage area. Denote by W_j the sum of the weights of all users on AP j. Let b_{ij} denote the bandwidth of user i obtained from AP j. According to (6), we have $b_{ij} = w_i r_{ij}/W_j$, where $W_j = \sum_{k=1}^{m_j} w_k$. Let r_{0j} be the effective bit rate of the new user, and w_0 be its weight. If the new user is allowed to join AP j, its bandwidth from AP j should become $b'_{ij} = w_i r_{ij}/(W_j + w_0)$. Then, the difference of the bandwidth utility on AP j can be computed as follows:

$$\begin{aligned}
\delta_j &= \sum_{i=1}^{m_j} w_i \log\left(b'_{ij}\right) + w_0 \log\left(b'_{0j}\right) - \sum_{i=1}^{m_j} w_i \log(b_{ij}) \\
&= \log\left(\prod_{i=1}^{m_j}\left(b'_{ij}\right)^{w_i}\right) + \log\left(b'_{0j}\right)^{w_0} - \log\left(\prod_{i=1}^{m_j}(b_{ij})^{w_i}\right) \\
&= \log\left(\prod_{i=1}^{m_j}\left(\frac{b'_{ij}}{b_{ij}}\right)^{w_i}\right) + \log\left(b'_{0j}\right)^{w_0} \\
&= \log\left(\prod_{i=1}^{m_j}\left(\frac{W_j}{W_j + w_0}\right)^{w_i}\right) + \log\left(\frac{r_{0j}w_0}{W_j + w_0}\right)^{w_0} \\
&= \log\left(\frac{W_j}{W_j + w_0}\right)^{W_j} + \log\left(\frac{r_{0j}w_0}{W_j + w_0}\right)^{w_0} \\
&= \log\left(\left(\frac{r_{0j}w_0}{W_j + w_0}\right)^{w_0}\left(\frac{W_j}{W_j + w_0}\right)^{W_j}\right).
\end{aligned} \tag{9}$$

Note that δ_j reflects the improvement of the network performance in terms of bandwidth utility defined in **NLAO-PF** when the new user joins the basic service set of AP j. Therefore, intuitively the new user should choose the AP with the highest δ value instead of the one with the strongest signal strength. This idea is summarized by the BPF algorithm outlined in Algorithm 2, which adopts δ as the selection criteria for a new user to choose an AP for association.

Algorithm 2: Best Performance First (BPF)

1: After a new user enters the network, it sends its weight to each reachable AP j, collects the sum of the weights W_j from AP j, and computes its effective bit rate r_{0j}.
2: For each reachable AP j, calculate the difference of bandwidth utility δ_j.
3: Select the AP with the largest δ_j.

Also note that the computation of δ_j is based on (6), which computes the optimal transmission time given a user–AP association to achieve proportional fairness in **NLAO-PF**. Considering the fact that **BPF** assigns the AP to a new user that can achieve the best bandwidth utility improvement while **NLAO-PF** intends to find the AP association and transmission time assignment such that the total bandwidth utilities can be maximized, we claim that **BPF** and **NLAO-PF** employ the same fundamental theory to achieve proportionally fair AP association.

B. Theoretical Analysis

Equation (6) indicates that the transmission time of a user is a function of the weights of all users associated with the same AP. Users always intend to obtain a higher bit rate and a larger transmission time to improve their own bandwidth. However, simply selecting the AP with the highest signal strength ignores the transmission time, which may make the network performance declined due to the decreased transmission time of other users associated with the same AP. The bandwidth utility difference defined by (9) considers both the effective bit rate and the weight, which reflects the received signal strength and transmission time, respectively, and therefore providing a better AP selection criterion to achieve proportional fairness.

Furthermore, according to (9), the total network utility increases ($\delta_j > 0$) after accepting a new user only if the following condition is satisfied, i.e.:

$$(r_{0j}w_0)^{w_0} > \frac{(W_j+w_0)^{W_j+w_0}}{W_j^{W_j}}. \tag{10}$$

Let α_j be the ratio of W_j to w_0, i.e., $\alpha_j = W_j/w_0$. Note that $\alpha_j = 0$ if and only if AP j does not have any user associated with it before the new user comes, which is a trivial case. We can express the relationship between the user bit rate and the network utility by Theorem 4.

Theorem 4: If a new user is associated with AP j, the total network utility increases when the new user's bit rate r_{0j} is larger than a threshold value θ_j

$$\theta_j = \begin{cases} 1, & W_j = 0 \\ (1+\alpha_j)\left(1+\frac{1}{\alpha_j}\right)^{\alpha_j}, & W_j > 0. \end{cases} \tag{11}$$

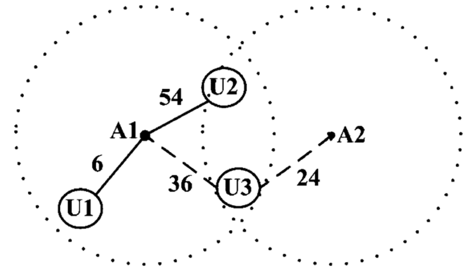


Fig. 2. AP association of the new user U3.

Proof: If $W_j = 0$, AP j has no user associated with it. According to the property of the logarithm function, the network utility increases only if $r_{0j} > 1$, i.e., $\theta_j = 1$.

If $W_j > 0$, there are certain users on AP j. Then, we have

$$\frac{(W_j+w_0)^{W_j+w_0}}{W_j^{W_j}w_0^{w_0}} = \frac{(W_j+w_0)^{W_j}}{W_j^{W_j}} \times \frac{(W_j+w_0)^{w_0}}{w_0^{w_0}} = \left(1+\frac{w_0}{W_j}\right)^{W_j}\left(1+\frac{W_j}{w_0}\right)^{w_0}. \tag{12}$$

In order to make (10) hold, $r_{0j}^{w_0} > (1+(w_0/W_j))^{W_j}(1+(W_j/w_0))^{w_0}$. Thus, we have

$$r_{0j} > \left(1+\frac{w_0}{W_j}\right)^{\frac{W_j}{w_0}}\left(1+\frac{W_j}{w_0}\right) = (1+\alpha_j)\left(1+\frac{1}{\alpha_j}\right)^{\alpha_j}. \tag{13}$$

■

Fig. 2 illustrates the impact of different AP selection algorithms on the network performance. A1 and A2 are APs with the same coverage area denoted by circular regions. We assume that users U1, U2, and U3 have the same weight. The solid line labels the AP association for current users, and the dashed lines indicate the possible AP associations for the new user U3. The number beside each line is the effective bit rate if the user is associated with the corresponding AP. Before U3 joins the network, the total throughput is 30 and the network utility is 4.4 according to proportional fairness [each user is allocated half of the transmission time based on (6)]. When U3 comes, our BPF algorithm selects A2 for U3, which achieves a total network throughput of 54 and a network utility of 7.57. If the strongest-signal-first policy (the default user–AP association method in 802.11) is adopted, A1 is selected for U3, yielding a total throughput of 32 and a network utility of 6.07, which are far less than those obtained from BPF.

On the other hand, the threshold values for U3 to associate with A1 and A2 are $\theta_1 = 6.75$ and $\theta_2 = 1$, respectively. This indicates that the total network utility could be increased if the bit rate between U3 and A1 is larger than 6.75, or if the bit rate between U3 and A2 is larger than 1; otherwise, the total utility would decrease after U3 joins the network. For example, if the bit rate between U3 and A1 (A2) is 6 (0.5), the new network throughput becomes 22 (30.5), and the utility is reduced from 4.4 to 4.2 (3.7) if U3 is associated with A1 (A2). For such a case, the network utility decreases, resulting in a lower degree of fairness in bandwidth allocation.

VI. Evaluation

In this section, we report our simulation results for the scenarios where the network contains either static or mobile users. We also compare the performance of our algorithms (the proportional fairness algorithm NLAO-PF and the AP selection algorithm BPF proposed in this paper) with those of the following ones:

- cvapPF: a time-based proportional fairness algorithm proposed in [11]. This algorithm is selected for comparison because it targets the same nonlinear problem formulation as that of NLAO-PF but adopts a different relaxation and rounding procedures.
- Strongest Signal First (SSF): the default user–AP association mechanism in the 802.11 standard.
- Norm Load-based Best AP Selection (NLB): an online distributed max-min fairness algorithm designed by Xu *et al.* [26]. By comparing to this scheme, the difference between max-min fairness and proportional fairness in multirate WLANs can be well demonstrated.

All these algorithms are examined carefully according to the following performance metrics:

- per-user throughput in Mbps and the corresponding statistical information;
- Jain's Fairness Index [30], which is defined to be

$$J = \frac{\left(\sum_{i=1}^{M} b_i\right)^2}{M\left(\sum_{i=1}^{M} b_i^2\right)} \tag{14}$$

where b_i is the effective bandwidth allocated to user i. Note that a larger value of $J \in [0,1]$ indicates a better fairness.

For ease of comparison, we employ the same simulation settings as those in [11], which are detailed as follows.

Our network contains a total of 20 APs placed on a 5×4 grid, with each on a grid point. The coverage area of an AP is set to 150 m, and the distance between two adjacent APs is 100 m. Assume that the transmission power of an AP is 20 dBm [37]. There are $50 \sim 300$ users residing in the network, resulting in different levels of network loads. For simplicity, we assume that all users have the same weight. Two types of user distributions are considered: 1) users are randomly and uniformly distributed within the coverage area of the network; 2) users are randomly positioned in a circle-shaped hotspot area with a radius of 100 m near the center of the 20-AP network. The former simulates the scenario with a balanced user distribution, while the latter simulates the scenario where users are distributed in a particular focused area.

We employ a simple wireless channel model in which the user bit rate only depends on the experienced SINR. The values commonly advertised by 802.11a/g are employed in our simulation. Therefore, we assume that the bit rate of the users is determined according to Table II. The link gains are modeled by the following equation:

$$g_{ij} = s_{ij} d_{ij}^{-4} \tag{15}$$

where s_{ij} is a log-normally distributed shadowing factor, and d_{ij} is the distance between user i and AP j. Shadowing factors are generated according to the Viterbi model [38], with $E(s_{ij}) = 0$ dB and $\sigma(s_{ij}) = 10$ dB.

TABLE III
Simulation Parameters

Parameter	Value
Simulation Time	300s
Packet Size	512 bits
Transmit Power	100mW
SINR Threshold	6dB
Noise	-80dB

We use OMNetpp as the simulator with the corresponding parameters shown in Table III.

A. Static Network Scenario

We first report our simulation results for the static network scenario. Assume that all users stay in the network at fixed positions during the whole simulation time.

It is not surprising to observe that by varying the number of users, we obtain quantitatively similar results. Therefore, in this section we only report the results for the 200-user case. The statistics of the achieved throughput and the Jain's Fairness Index of different algorithms are presented in Table IV. In the following, we give a detailed analysis on the results obtained from different scenarios.

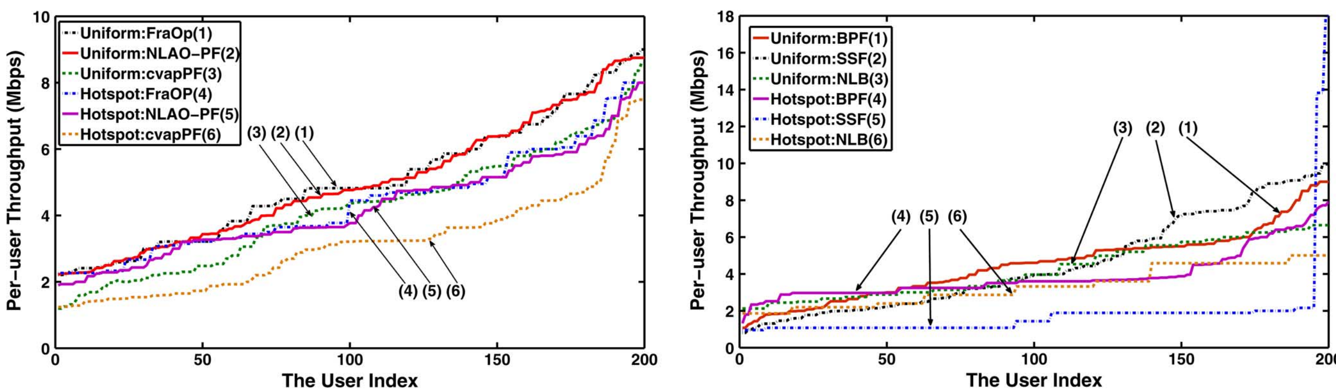
First, we compare the two centralized global optimization algorithms: NLAO-PF and cvapPF. Since the problem of AP association based on proportional fairness is NP-hard, these two algorithms can only obtain approximate solutions. For comparison purpose, we use the numerical result obtained from r-NLP without the compensation function $g(X,T)$ as a benchmark and call it FraOp.

Fig. 3 plots the achieved per-user throughput in Mbps versus user index, with the users sorted by their throughputs in a nondecreasing order. In Fig. 3(a), we observe that in the two user distribution cases, NLAO-PF and cvapPF can both achieve proportional fairness, but NLAO-PF outperforms cvapPF in terms of throughput with a value closer to FraOp. As shown in Table IV, the improvements of NLAO-PF over cvapPF in terms of average throughput are 18.8% and 35.0% for the cases of users being uniformly distributed in the whole network and in a hotspot area, respectively. On the other hand, the fairness index of these two algorithms is almost the same. Moreover, in both cases, the average throughputs of NLAO-PF are 99.4% and 96.8%, while those of cvapPF are 83.7% and 71.7%, normalized over FraOp. Therefore, we conclude that NLAO-PF outperforms cvapPF.

Second, we compare BPF with two other distributed AP association heuristics: SSF and NLB. Fig. 3(b) plots the achieved per-user throughput in Mbps versus user index, with the users sorted by their throughputs in a nondecreasing order. We observe that when users are randomly and uniformly distributed in the whole network [Fig. 3(b), curves (1)–(3)], the average throughput of SSF is a little bit higher than those of BPF and NLB. However, SSF demonstrates a much larger variance in the user throughput and its fairness is poorer (see Table IV). Nevertheless, the advantages of BPF and NLB cannot be easily man-

TABLE IV
STATISTICS OF THE RESULTS IN STATIC NETWORK SCENARIO (200 USERS)

Case	Algorithm	Max. (Mbps)	Min. (Mbps)	Mean. (Mbps)	Std. (Mbps)	Jain's Fairness Index
Uniform	FraOp	9.01	2.21	5.02	1.87	0.86
	NLAO-PF	**8.75**	**2.21**	**4.99**	**1.88**	**0.85**
	BPF	**8.99**	**1.08**	**4.51**	**1.92**	**0.84**
	cvapPF	8.58	1.19	4.20	1.87	0.86
	SSF	9.95	0.81	4.53	2.62	0.67
	NLB	6.65	2.12	4.16	1.57	0.90
Hotspot	FraOp	8.11	2.26	4.38	1.45	0.94
	NLAO-PF	**8.05**	**1.92**	**4.24**	**1.47**	**0.94**
	BPF	**7.92**	**1.32**	**3.89**	**1.54**	**0.91**
	cvapPF	7.49	1.24	3.14	1.50	0.91
	SSF	17.99	0.96	2.14	2.25	0.70
	NLB	5.01	1.86	3.03	1.26	0.95



(a) (b)

Fig. 3. Per-user throughput of different algorithms. (a) Per-user throughput of centralized algorithms. (b) Per-user throughput of distributed algorithms.

ifested under this user distribution scenario because the traffic load is more balanced.

The situation changes in the hotspot case as shown in Fig. 3(b) [curves (4)–(6)]. In this case, users reside in the vicinity of certain APs, leading to a more intensive competition for resources and a more imbalanced network load. Obviously, SSF aggravates the extent of load imbalance and enlarges the user throughput variances without considering fairness. Thus, by taking into account the traffic loads of APs and the achievable transmission rates, both BPF and NLB can significantly improve the user throughput and fairness. More specifically, the average throughput improvement of BPF is 81.8% compared to SSF, and 28.4% compared to NLB.

When comparing from the viewpoint of fairness criteria, it can be seen that although max-min fairness (NLB) can obtain a higher fairness index value, it reduces the network throughput. This result confirms the fact that proportional fairness (BPF) provides a more effective tradeoff between fairness and network throughput in a multirate WLAN.

B. Dynamic Network Scenario

In this section, we report the simulation results of the three distributed algorithms—namely BPF, SSF, and NLB—for the dynamic network scenario where users are mobile.

We consider the case when users join the network one by one. After a user enters the network, it starts to move according to the following *Random Waypoint Mobility Model* [39], [40]: The user moves at a random speed to a random waypoint that is uniformly chosen from the given area and pauses at each waypoint for a random interval ranging from 0 to 30 s. The random speed is uniformly selected from the range [0, 15 m per second].

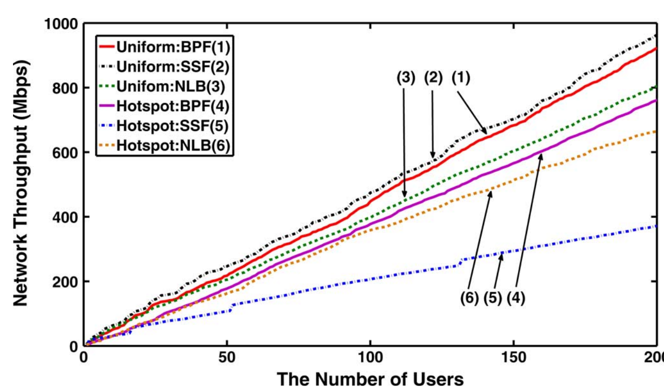


Fig. 4. Aggregated throughput versus the number of users.

Fig. 4 reports the aggregated throughput in Mbps versus the number of users that have joined the network. From the curves (1)–(3) in Fig. 4, we observe that SSF obtains a slightly higher aggregated throughput when users are uniformly distributed in the whole network. This is attributed to the more balanced network load, for which case the advantages of BPF and NLB cannot be easily manifested.

For the hotspot case shown in Fig. 4 [curves (4)–(6)], fairness becomes a key factor because of the load imbalance. Thus,

TABLE V
STATISTICS OF THE RESULTS (PER-USER THROUGHPUT AND FAIRNESS) IN DYNAMIC NETWORK SCENARIO (200 USERS)

Case	Algorithm	Max. (Mbps)	Min. (Mbps)	Mean. (Mbps)	Std. (Mbps)	Jain's Fairness Index
Uniform	**BPF**	**7.59**	**3.89**	**6.27**	**1.24**	**0.96**
	SSF	7.69	3.34	6.20	1.52	0.91
	NLB	7.32	4.47	6.21	0.96	0.98
Hotspot	**BPF**	**6.29**	**3.97**	**5.48**	**0.99**	**0.98**
	SSF	6.51	1.07	4.09	2.19	0.70
	NLB	6.02	3.34	4.63	0.81	0.99

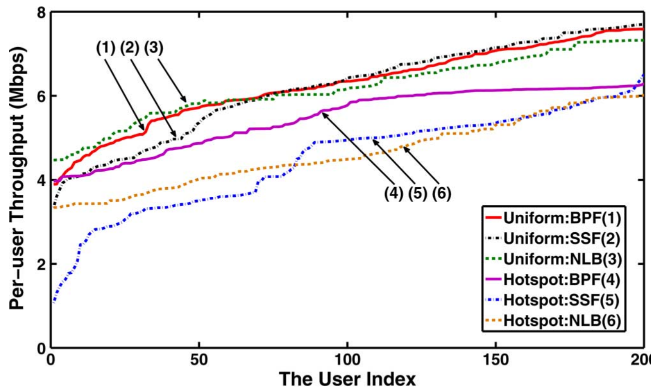


Fig. 5. Per-user throughput of the distributed algorithms in the dynamic network scenario.

BPF and NLB outperform SSF. On the other hand, it is difficult for NLB to effectively enhance the aggregated throughput since it takes into account the max-min fairness and ignores the network efficiency. Therefore, we conclude that BPF provides a more effective tradeoff between the aggregated throughput and fairness.

The impact of user mobility on the network performance is presented in Fig. 5 and Table V. Fig. 5 plots the achieved per-user throughput in Mbps versus user index, with the users sorted by their throughputs in a nondecreasing order. Although the average throughput of SSF is almost the same as that of BPF for the uniform case, its fairness (Jain's Index) is the lowest. In particular, its performance is the worst for the hotspot case in terms of per-user throughput and fairness because it does not consider load balancing in the network. Since NLB aims at maximizing the minimum user throughput and guarantees that all users on the same AP obtain equal throughput, it cannot significantly enhance the network throughput. On the other hand, BPF associates users to APs according to proportional fairness, thus achieving an effective tradeoff between throughput and fairness. The statistics of per-user throughput for the dynamic network scenario (Table V) indicate that the improvement of the average per-user throughput of BPF is 34.0% compared to that of SSF, and 18.4% compared to that of NLB in the hotspot case. Thus, we conclude that BPF performs better than SSF and NLB in terms of per-user throughput.

The numbers of AP reassociations due to user mobility for the three distributed algorithms are listed in Table VI. Note that each reassociation involves a deassociation from the previous AP. In BPF and NLB, a user performs deassociation and then reassociation to a different AP by considering both the load of the AP and the data rate, while SSF takes into account the value of SINR only. As a result, BPF and NLB conduct smaller numbers of deassociations/reassociations compared to SSF. Note that more AP reassociations implies less time for user transmissions, leading to a shorter effective transmission time and a lower throughput. Thus, with the smallest number of AP reassociations compared to SSF and NLB, BPF can effectively improve the user throughput. Moreover, the least number of AP switching operations also implies that BPF has the highest degree of stability compared to SSF and LLF, which is important as typically users prefer networks providing stable communications.

TABLE VI
NUMBER OF AP REASSOCIATIONS IN DYNAMIC NETWORK SCENARIO (200 USERS)

Case	Algorithm	Max. Times	Min. Times	Mean. Times
Uniform	**BPF**	**14**	**8**	**11**
	SSF	18	10	16
	NLB	14	9	12
Hotspot	**BPF**	**10**	**7**	**8**
	SSF	16	9	12
	NLB	12	7	10

VII. CONCLUSION

The wide spread of multirate WLAN applications makes the network management more complex and critical. Fairness and AP association are two hot issues. In multirate WLANs, some users may get starved if fairness is not carefully considered. In this paper, we investigate how to optimize user–AP association to achieve proportional fairness. We propose two AP association algorithms, namely **NLAO-PF** and **BPF**. Although the problem of optimizing user–AP association to achieve proportional fairness is NP-hard, **NLAO-PF** obtains a result that is guaranteed to be at least half of the optimal via a compensation function. On the other hand, **BPF** provides a new AP selection criterion based on a performance revenue function obtained when new users join the network. Simulations confirm that our schemes can achieve proportional fairness in bandwidth allocation and effectively enhance the aggregated throughput. Moreover, our algorithms achieve even better performance when users are distributed in a hotspot area with imbalanced network load.

In our future research, we will consider a more general network model that contains both the uplink and the downlink traffic; moreover, we will investigate the impact of interference on proportional fairness in multirate WLANs.

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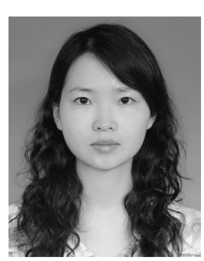

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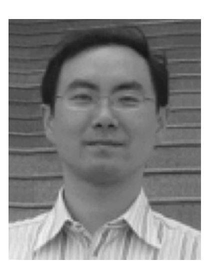

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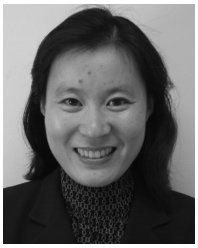

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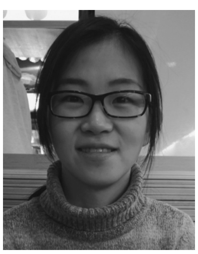

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