Joint Routing and Medium Access Control in Fixed Random Access Wireless Multihop Networks

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***Abstract*—We study cross-layer design in random-access-based fixed wireless multihop networks under a *physical interference model*. Due to the complexity of the problem, we consider a simple slotted ALOHA medium access control (MAC) protocol for link-layer operation. We formulate a joint routing, access probability, and rate allocation optimization problem to determine the optimal max-min throughput of the flows and the optimal configuration of the routing, access probability, and transmission rate parameters in a slotted ALOHA system. We then also adapt this problem to include an XOR-like network coding without opportunistic listening. Both problems are complex nonlinear and nonconvex. We provide extensive numerical results for both problems for medium-size mesh networks using an iterated optimal search technique. Via numerical and simulation results, we show that: 1) joint design provides a significant throughput gain over a *default* configuration in slotted-ALOHA-based wireless networks; and 2) the throughput gain obtained by the simple network coding is significant, especially at low transmission power. We also propose simple heuristics to configure slotted-ALOHA-based wireless mesh networks. These heuristics are extensively evaluated via simulation and found to be very efficient.**

***Index Terms*—Cross-layer, medium access control (MAC), network coding, routing, throughput, transmission rate.**

I. Introduction

Although the worldwide success of the Internet is partly due to the simplicity and robustness of its layered network architecture, this architecture, developed for wired networks, is not efficient for multihop wireless networks. Cross-layer approaches have been proposed [2], [3] to enhance the adaptability and performance of these networks by jointly tuning the parameters of different layers.

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One of the critical performance metrics in multihop wireless networks is throughput. It is highly dependent on the configuration of routing, medium access, and physical-layer parameters and on their interactions; see, for example, [4] and [5] in the case of a (conflict-free) scheduled network. Configuring a wireless network based on random access is much more difficult, and one might be tempted to simply use a so-called *default* configuration comprised, for example in the case of slotted ALOHA, of a minimum-hop routing and equal attempt probability. While one would expect that joint configuration of routing, medium access, and physical-layer parameters of a random access network can provide better performance than the default configuration, there is no clear indication so far on how much improvement can be achieved by joint design and how to jointly configure the parameters.

In a single-channel wireless network, during a transmission, the interference seen by a receiver is the additive interference from all the other simultaneous transmissions. As a consequence, it is essential to use a proper interference model when configuring the wireless network. The *physical interference model* based on signal-to-interference-plus-noise ratio (SINR) is the more realistic interference model for wireless networks [6]. Simpler interference models such as *primary interference model*, *protocol model*, and *capture threshold model* can provide misleading insights about the optimal configuration of routing, medium access control (MAC), and physical-layer parameters as well as throughput improvements by joint design [6].

The throughput optimization problem of any network is a link-rate constrained optimization problem [7]. For popular but complex MAC protocols such as the IEEE 802.11-based carrier-sense multiple access with collision avoidance (CSMA/CA) MAC [8], modeling the effective link rate in terms of network parameters under a realistic interference model is an open research issue in the context of multihop wireless networks. The fundamental random access protocol, slotted ALOHA, was first proposed in 1970 by Abramson [9]. It has contention characteristics similar to CSMA/CA in a WLAN [10]. Due to its simplicity of operation and analytical formulation, the protocol is often considered for understanding the contention in heavy loaded random access networks. In this paper, we first study the optimal joint configuration of routing, access probability, and transmission rate parameters in slotted ALOHA fixed wireless networks to maximize the minimum throughput of the flows under an interference model based on SINR. The critical assumption to perform this study is that the channel gains are quasi time-invariant. The objective is to provide insights on throughput gains obtained by optimized configurations over a default configuration. Note that, from this point onward, we use the term MAC in a narrow sense since we focus on a very specific MAC protocol.

Network coding has emerged as a promising technique both in wireline and wireless networks [11], [12] to improve throughput performance. Wireless networks suffer from interference due to the inherent broadcast nature of the wireless medium. Network coding is an important method that turns this apparent broadcast limitation into an advantage for better throughput performance. Network coding has been used in many contexts in wireless networks, including: 1) end-to-end multicasting [13]; 2) end-to-end unicasting [14]; 3) at the link layer [15]–[17]; and 4) physical-layer transmission [18]. The existing works in 1), 2), and 4) are mainly theoretical. Link-layer network coding is studied theoretically in [15] for unicast applications, and COPE bridges the gap between theory and practice and provides an operational protocol for general unicast traffic [16]. Due to the simplicity and practicality of link-layer network coding, this technique has attracted a lot of attention from the wireless research community.

In a wireless network, (link-layer) network coding opportunities significantly depend on the routing, MAC, and physical-layer parameters and the interactions among the three layers. It is expected that network coding opportunities as well as throughput performance can be improved significantly by joint configuration of routing, MAC, and physical-layer parameters. However, how to jointly configure the network parameters when network coding is enabled is unknown.

In a second part, we study the optimal joint configuration of routing, access probability, and transmission rate parameters in slotted ALOHA wireless networks with network coding to maximize the minimum throughput of the flows. We restrict ourselves to simple link-layer network coding without any opportunistic listening as it is too complex to analyze link-layer network coding with opportunistic listening for a wireless network and optimize the network parameters under a realistic interference model. The contributions are as follows.

- We model the effective link rate for slotted ALOHA systems under an SINR-based *physical interference model* using the concept of a *conflict-free set* of nodes. These link rate models are found to be very complex and are not a convex function of their parameters.
- We formulate the joint routing, access probability, and rate allocation problem to determine the weighted max-min throughputs of the flows and the optimal configuration of routing, access probability, and transmission rate parameters in slotted-ALOHA-based wireless networks. This problem is also extended to joint routing, access probability, network coding, and rate allocation problem. These problems turn out to be very large nonlinear and nonconvex optimization problems. They are valid for any fixed wireless multihop network with quasi time-invariant channel gains.
- We solve the optimization problems numerically for several mesh[1] network scenarios with a single transmission rate at all nodes by using an iterated optimal search (IOS) technique, i.e., we study the optimal joint configuration of routing and access probability parameters in single-rate systems. Via numerical and simulation results, we show that the performance gains obtained by jointly optimizing the configuration of access probability and routing parameters over a default configuration comprising equal access probability at each node and a minimum-hop routing are very significant in slotted ALOHA systems. Specifically, we find gains on the order of 80%–300%. We also show that: 1) a significant amount of throughput improvement can be achieved by optimizing only the access probability parameters; whereas 2) a small amount of throughput improvement is achieved by optimizing only the routing parameters. Furthermore, the performance gain obtained by jointly optimizing routing, access probability, and network coding over a joint design without network coding (i.e., the gain obtained for enabling the simple network coding) is significant, especially at low transmission power. At higher transmission power, network coding becomes less attractive because there are more and more single-hop paths to the gateway. We also find that the typical rate imbalance between downlink and uplink flows in wireless mesh networks surprisingly plays a role *in favor* of network coding due to retransmissions.
- Due to their computational complexity, the optimization problems are intractable for large networks. For large single-rate wireless mesh networks, we propose simple heuristics to configure the routing and access probability parameters. We show via simulation that the max-min throughputs obtained by the heuristics are significantly higher than the max-min throughputs obtained by *default* designs and compare well to the optimal max-min throughputs.
- We solve the joint problems for multirate systems by our IOS technique and compare the throughput performance of multirate and single-rate systems. We find that by using two rates, there are some (limited) throughput improvements only for powers at which the network would not be connected if using the highest rate only.

In this paper, we study a simple MAC protocol and a simple network coding scheme to keep the formulation tractable. Our objectives are to provide insights on: 1) the interaction of routing, access probability, network coding, and transmission rate; 2) the throughput gains obtained by a joint design over a default design; and 3) throughput gains obtained by simple network coding.

The rest of the paper is organized as follows. Related work is reviewed in Section II. We formulate the optimization problems in Section III. The IOS technique is described in Section IV. In Sections V and VI, we study the cross-layer design problems for systems without and with network coding. Section VII concludes the paper.

II. Related Work

Most research on tightly coupled cross-layer design for random-access-based multihop wireless networks focuses on the transport layer and MAC layer without taking the network layer and the physical layer into account (e.g., see [19] and [20]). However, the throughput performance of a multihop wireless network depends on the interaction of the network layer, link layer, and physical layer. Tightly coupled joint

[1]We focus our numerical results on mesh networks (defined precisely in Section V) because they are typically small to medium-size, and hence our problems are computationally tractable.

design of routing, scheduling, and physical layer (as opposed to random access MAC) is addressed in many papers (e.g., see [4] and [5]).

A large number of cross-layer design studies in random access networks are based on the loosely coupled approach [21]–[25]. Since early 1990's, researchers have tried to address the problem of joint routing and MAC (JRM) for multihop ALOHA wireless networks [21], [22]. In [21], a nonlinear joint optimization problem is formulated using a simple interference model and solved by decoupling the routing and the MAC problems. For the routing problem, a heuristic is used to find the minimum-hop path with low interference, and then the MAC problem is solved by an iterative numerical method. In [22], the problem is solved by forcing the attempt probabilities to be fixed and equal for all nodes. This transforms the original problem into a linear program that can be easily solved. In both papers, the authors have decoupled the MAC and routing problems to get a workable solution.

We consider a tightly coupled joint routing, access probability, and transmission rate allocation problem based on a more sophisticated interference model and solve it by the IOS technique in single-rate and multirate systems. To the best of our knowledge, this problem is not addressed so far in any paper.

In [23]–[25], the authors address the routing problem in CSMA/CA-based wireless networks by designing different routing metrics. Routing based on these metrics improves performance in wireless networks by exploiting the MAC-layer information. However, the performance of these different routings has not been compared to an optimal solution. Our heuristic mainly focuses on the access probability parameter configuration, and its throughput performance is compared to the optimal solution of the joint routing and MAC problem.

Since the pioneering work on network coding for multicast applications on wireline networks [11], a large body of work has explored network coding for multicast as well as unicast applications on wireline and wireless networks [13], [14], [27]. These works investigate end-to-end network coding, which is complex and very difficult to implement. In [15], Wu *et al.* introduce a simple link-layer network coding, i.e., XOR-type network coding, for unicast applications. Ho *et al.* study the construction of XOR coding between a pair of flows in wireless networks with multiple unicast flows [26]. COPE [16] provides an operational protocol for XOR-type network coding with opportunistic listening in CSMA/CA networks for general unicast traffic.

Recently, the throughput performance of a two-hop relay network (i.e., three-node network) with network coding is studied in [28] and [29]. The authors demonstrate that network coding opportunities significantly depend on the configuration of MAC parameters, although these works are limited to a two-hop relay network.

In [16], the authors study network coding by using a dynamic source routing (DSR) protocol under the expected transmission time (ETT) routing metric and the default MAC parameters of 802.11. BEND, a more opportunistic link-layer network coding scheme than COPE, is proposed in [17] and is studied using a destination-sequenced distance vector (DSDV) routing protocol under the same MAC protocol. In BEND, XORed packets that are constructed from a greater number of non-network-coded packets use a smaller contention window in order to increase the efficiency of the medium access.

While it is clear that network coding opportunities in a wireless network significantly depend on the configuration of network parameters [17], [28], [29], the existing studies on random-access-based wireless networks [16], [17] do not explicitly exploit the interaction between network parameters and network coding (i.e., do not formulate and solve a joint problem).

With respect to conflict-free scheduled networks (as opposed to random access MAC), network coding has been studied in [30]–[32]. In [30], the authors study joint routing, scheduling, and network coding under a simplistic interference model and provide bounds on throughput. In [31] and [32], the authors study joint congestion control, scheduling, and bidirectional network coding.

Different from the existing works, in this paper, we study cross-layer design in slotted-ALOHA-based wireless networks with an XOR-based network coding without opportunistic listening.

III. Problem Formulations

In this section, we formulate the joint routing, access probability, and rate allocation problem (JRM-RA) and the one with network coding (JRM-NC-RA) for slotted-ALOHA-based wireless networks.

A. Joint Routing, Access Probability, and Rate Allocation

1) System Model: In this section, we describe a multirate slotted ALOHA system without network coding.

Network Topology and Flows: Consider a wireless network consisting of N stationary nodes with known locations using the same transmission power $P_{\rm t}$. The set of nodes is denoted by $\mathcal{N}$. There are F data flows in the network, belonging to the set $\mathcal{F}$. A data flow f is characterized by its source $f^{\rm s}$ and its destination $f^{\rm d}$. The rate of flow f is denoted by λ_f and is constrained to satisfy

$$\lambda_f = w_f \lambda \qquad \forall f \in \mathcal{F} \tag{1}$$

where λ is a common base throughput and w_f is a known weight. We want to maximize the achievable performance by maximizing λ, i.e., we want to maintain a preset traffic rate ratio (given by the weights $\{w_f\}$) for the flows.

We assume that all the nodes are able to use R modulation and coding schemes characterized by the set of physical transmission rates $\mathcal{R} = \{r_1, r_2, \ldots, r_R\}$. The minimum SINR necessary for using the physical transmission rate $r \in \mathcal{R}$ is given by $\gamma(r)$. Let $\mathcal{L}$ be the set of directed links in the network and $L = |\mathcal{L}|$. Clearly, the set of links depends on $P_{\rm t}$ and the modulation and coding schemes. A directed link $l \in \mathcal{L}$ is represented as $(l^{\rm o}, l^{\rm d})$, where $l^{\rm o}$ and $l^{\rm d}$ are the originating and destination nodes of the link. We denote the sets of links coming into and going out of node n by $\mathcal{L}_n^{\rm I}$ and $\mathcal{L}_n^{\rm O}$.

Channel and Interference Models: The channel gain between a transmitter and a receiver is assumed to be time-invariant. A directed link between nodes n_1 and n_2 exists if they can communicate in the absence of interference at least with the minimum physical transmission rate $r_{\min} = \min_{r\in\mathcal{R}} r$, i.e., if the signal-to-noise ratio (SNR) for the link is greater than or equal to $\gamma(r_{\min})$, i.e.,

$$\frac{G_{n_1 n_2} P_{\mathrm{t}}}{N_0} \geq \gamma(r_{\min}) \tag{2}$$

where $G_{n_1 n_2}$ is the channel gain between nodes n_1 and n_2. A rate $r \in \mathcal{R}$ is feasible on link $l \in \mathcal{L}$ if the SNR for the link is not less than $\gamma(r)$. Let $\mathcal{R}(l) \subseteq \mathcal{R}$ denote the set of feasible rates on link l. We assume that time is slotted and each node can adjust the size of a packet according to the transmission rate such that the transmission time of the packet is equal to the duration of one time-slot. Generally, in a given time-slot, a packet sent by n_1 with physical transmission rate r is considered to be successfully received by receiver n_2 if the received SINR is not less than $\gamma(r)$, i.e., a packet transmission from n_1 to n_2 using the modulation and coding scheme yielding transmission rate r is successful if

$$\frac{G_{n_1 n_2} P_{\mathrm{t}}}{N_0 + \sum\limits_{n' \neq n_1} G_{n' n_2} P_{\mathrm{t}} Y_{n'}} \geq \gamma(r) \tag{3}$$

where $Y_{n'}$ is a binary variable being equal to 1 if node n' transmits in the given slot, and 0 otherwise.

Medium Access Control: We consider a slotted ALOHA MAC protocol, where the nodes in the network are synchronized and probabilistically access the channel in each time-slot. Denote by π_n the probability that node n tries to access the channel in a given slot, i.e., the access probability, and $\boldsymbol{\pi} = [\pi_1, \pi_2, \ldots, \pi_N]$ is the corresponding probability vector. For medium access, at each slot, node n first generates a binary variable taking on value 1 with probability π_n, and zero otherwise. If the result is 1, it performs the routing operation as follows to transmit a packet; otherwise, it keeps silent.

Routing: Given that node n does try to access the channel, the routing decision is to determine which flow to send, to whom to send it, and at which rate to send. We consider a probabilistic routing strategy to select a flow, the receiver (i.e., the link), and the transmission rate. The routing operation is described by the following random variables. Given that node n does try to access the channel, we denote the conditional probability that it will select a packet of flow f to transmit on link $l \in \mathcal{L}_n^{\mathrm{O}}$ with transmission rate $r \in \mathcal{R}(l)$ by $q_{f,l}^r$ with the condition

$$\sum_{f \in \mathcal{F}, l \in \mathcal{L}_n^{\mathrm{O}}, r \in \mathcal{R}(l)} q_{f,l}^r = 1. \tag{4}$$

We assume that each node maintains a separate infinite queue for each flow.

Retransmission Strategy: We assume that a transmitter knows immediately at the end of the current slot whether its transmission is successful or not. We consider a delayed first transmission (DFT) retransmission policy, where the transmitting node keeps a copy of the packet in the queue that it is transmitting. This copy is deleted if the transmission is successful; otherwise, it is retransmitted when the transmitter selects that flow again.

2) Link Rate Model: Let $\tau_{f,l}^r$ be the probability that a packet of flow f will be transmitted on link l in a given time-slot with transmission rate $r \in \mathcal{R}(l)$. Thus

$$\tau_{f,l}^r = \pi_n q_{f,l}^r \qquad \forall n \in \mathcal{N}, \forall f \in \mathcal{F},\ \forall l \in \mathcal{L}_n^{\mathrm{O}},\ \forall r \in \mathcal{R}(l). \tag{5}$$

The collection of $\tau_{f,l}^r$ is called the transmission probability matrix, denoted by $\boldsymbol{\tau}$.

Because nodes are able to know immediately whether a collision has occurred, the effective rate of flow f on link l, $c_{f,l}$, can be expressed as

$$c_{f,l} = \sum_{r \in \mathcal{R}(l)} r \tau_{f,l}^r p_l^{\mathrm{s}}(r) \tag{6}$$

where $p_l^s(r)$ is the probability that a packet can be transmitted successfully on link l with transmission rate r, i.e., that the SINR at l_{d} is not less than the threshold $\gamma(r)$, given that the link l is active. The main difficulty of the link rate model is the calculation of $p_l^{\mathrm{s}}(r)$. We denote the effective link rate matrix by $\mathbf{c}$.

Computation of $p_l^{\mathrm{s}}(r)$: Let $\mathcal{N}_l$ be the set of nodes excluding the transmitter of link l, i.e., $\mathcal{N}_l = \mathcal{N} \setminus l^{\mathrm{o}}$. Denote a state of $\mathcal{N}_l$ in a time-slot by σ_l, where $\sigma_l \subset \mathcal{N}_l$ is the set of active nodes in the time-slot. Because each node decides whether or not it will transmit independently of all the other nodes, the probability $P\{\sigma_l\}$ that the system is in state σ_l in a time-slot is given by

$$P\{\sigma_l\} = \prod_{i \in \sigma_l} \pi_i \prod_{j \in \mathcal{N}_l \setminus \sigma_l} (1 - \pi_j). \tag{7}$$

A transmission on link l is successful with rate r for a state σ_l depending on the received SINR at the receiver. Let $\mathcal{S}_l^r$ be the set of states for which the transmission on link l is successful with rate r. Hence, the successful transmission probability $p_l^{\mathrm{s}}(r)$ is given by

$$\begin{aligned} p_l^{\mathrm{s}}(r) &= P\left\{ \bigcup_{\sigma_l \in \mathcal{S}_l^r} \sigma_l \right\} = \sum_{\sigma_l \in \mathcal{S}_l^r} P\{\sigma_l\} \\ &= \sum_{\sigma_l \in \mathcal{S}_l^r} \prod_{i \in \sigma_l} \pi_i \prod_{j \in \mathcal{N}_l \setminus \sigma_l} (1 - \pi_j). \end{aligned} \tag{8}$$

The calculation of the successful transmission probability for a given link l and a given rate r is then made up of two parts. The first one is the enumeration of all the successful states $\mathcal{S}_l^r$. This depends on the parameters of the physical layer and on the position of the nodes, but does not depend on the $\boldsymbol{\pi}$ variables. The second step is the evaluation of the polynomial in $\boldsymbol{\pi}$ given by (8). This calculation has to be done whenever the values of $\boldsymbol{\pi}$ change, for instance during an iterative optimization procedure. The complexity in determining all the successful states is $2^{(N-1)}$. This complexity can be reduced significantly by using a suitable enumeration technique [4], [5]. The reduction in complexity depends on network topology (i.e., on node positions and on transmit power). As an example, for a 16-node network (i.e., Rand16 presented later), the number of links is found to be 92 at a transmit power $P_{\mathrm{t}} = -32$ dBm and assuming only one rate $r = 1$. The number of sets to check to determine the successful states for these 92 links are found to be 15, 15, 24, 15, 1378, 2720, 15, 15, 18 431, 718, 15, 40, 423, 499, 854 , 266, 5011, 2691, 15, 15 , 1707, 413, 1438, 1972, 24, 19, 12 573, 3094, 108, 63, 15, 18, 93, 27, 2562, 1009, 15, 15, 1186, 2293, 99, 15, 4609, 3051, 5705, 8061, 15, 22, 50, 22, 30, 22, 8271, 11 563, 15, 15, 160, 144, 15, 22, 100, 222, 15, 23, 34, 15, 41, 52, 551, 479, 78, 112, 599, 182, 158, 64, 15, 28, 24 575, 16 895, 15, 60, 612, 132, 1028, 1753, 15, 44, 40, 44, 15, and 15, respectively. On the other hand, the number of sets to check per link is 32 768 under the naive approach. The computational complexity of $p_l^{\mathrm{s}}(r)$ in

(8) depends on the number of nodes, N, and the number of successful states, $|\mathcal{S}_l^r|$, where $|\mathcal{S}_l^r|$ is given by the network topology and physical-layer parameters. The computational complexity can be further reduced significantly by applying the following proposition.

Proposition 3.1: If σ_l^1 and σ_l^2 are two successful states of the set of nodes $\mathcal{N}_l$ such that $\sigma_l^1 \cup \{n\} = \sigma_l^2$, then

$$P\{\sigma_l^1\} + P\{\sigma_l^2\} = \prod_{i \in \sigma_l^1} \pi_i \prod_{j \in \mathcal{N}_l' \setminus \sigma_l^1} (1 - \pi_j) \tag{9}$$

where $\mathcal{N}_l' = \mathcal{N}_l \setminus \{n\}$.

Proof: Using (7), we have

$$\begin{aligned} P\{\sigma_l^2\} &= \prod_{i \in \sigma_l^1 \cup \{n\}} \pi_i \prod_{j \in \mathcal{N}_l \setminus (\sigma_l^1 \cup \{n\})} (1 - \pi_j) \\ &= \frac{\pi_n}{1 - \pi_n} \prod_{i \in \sigma_l^1} \pi_i \prod_{j \in \mathcal{N}_l \setminus \sigma_l^1} (1 - \pi_j) = \frac{\pi_n}{1 - \pi_n} P\{\sigma_l^1\}. \end{aligned} \tag{10}$$

Thus, from (10) we get $P\{\sigma_l^1\} + P\{\sigma_l^2\} = P\{\sigma_l^1\}/(1 - \pi_n)$. Using this and (7), (9) can be obtained. ■

This proposition means that if two successful states satisfy the condition, they can be combined into one successful state, and hence $\mathcal{N}_l$ can be replaced by set $\mathcal{N}_l'$ for the combined state. Since a successful state is made by adding a node to another successful state, this proposition can reduce the computational complexity significantly.

3) Joint Routing, Access Probability, and Rate Allocation Optimization Problem: The JRM-RA optimization problem to maximize the common base λ is given by

$$\max_{\boldsymbol{\tau}, \boldsymbol{\pi}, \mathbf{c}} \quad \lambda \tag{11}$$

$$\text{s.t.} \sum_{l \in \mathcal{L}_n^{\mathrm{O}}} c_{f,l} - \sum_{l \in \mathcal{L}_n^{\mathrm{I}}} c_{f,l} = \begin{cases} w_f \lambda, & \text{if } n = f^{\mathrm{s}} \\ -w_f \lambda, & \text{if } n = f^{\mathrm{d}} \\ 0, & \text{otherwise} \end{cases} \quad \forall n \in \mathcal{N},\ \forall f \in \mathcal{F} \tag{12}$$

$$c_{f,l} = \sum_{r \in \mathcal{R}(l)} r \tau_{f,l}^r \left(\sum_{\sigma_l \in \mathcal{S}_l^r} \prod_{i \in \sigma_l} \pi_i \prod_{j \in \mathcal{N}_l \setminus \sigma_l} (1 - \pi_j) \right) \quad \forall f \in \mathcal{F},\ \forall l \in \mathcal{L} \tag{13}$$

$$\pi_n = \sum_{f \in \mathcal{F}, l \in \mathcal{L}_n^{\mathrm{O}}, r \in \mathcal{R}(l)} \tau_{f,l}^r \qquad \forall n \in \mathcal{N} \tag{14}$$

$$0 \le \lambda, \mathbf{c}; \qquad 0 \le \boldsymbol{\tau},\ \boldsymbol{\pi} \le 1. \tag{15}$$

The objective function in (11) is to ensure that λ is maximized. The flow conservation constraints in (12) capture that the outgoing and incoming traffic of a flow are equal at each intermediate node, that the outgoing traffic of a flow is equal to the source rate at the source node, and that the incoming traffic of a flow is equal to the source rate at the destination node. The link rate constraints in (13) ensure that the traffic rate on a link is not larger than the link rate for each flow. The equality constraints in (14) relate the attempt probabilities to the transmission probabilities. Equation (15) defines the range of the variables.

The JRM-RA optimization problem in (11)–(15) is a nonlinear optimization problem because the constraints in (13) have a strong nonlinear dependence on the $\boldsymbol{\pi}$ variables. Furthermore, constraints in (13) are not convex since both sides of the constraints turn out to be polynomials [33]. Thus, finding a global optimal solution is challenging.

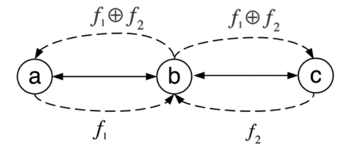

Fig. 1. Example of XOR network coding.

B. Joint Routing, Access Probability, Network Coding, and Rate Allocation

1) System Model: We consider the same kind of joint problem when a simple link-layer network coding without any opportunistic listening is enabled. In the absence of opportunistic listening, a link-layer network coding opportunity at a node involves XORing [16] exactly two packets, and these packets must enter through a pair of incoming links and leave through an opposite pair of outgoing links. In Fig. 1, for example, assume that node a (resp. c) needs to send packets of flow f_1 (resp. f_2) to node c (resp. a) through the intermediate node b. If one packet from each flow is available at node b, it can transmit both packets simultaneously by XORing them. Node a (resp. c) can then decode the packet intended for itself by XORing the packet it sent together with the received XORed packet. We assume that network coding between two packets can only be performed with the same modulation and coding scheme such that network coding operations remain simple and practical. Network topology, flows, and MAC operation are considered to be the same as defined in the system without network coding in Section III-A.1. We also consider a similar physical-layer model, i.e., a packet—single or XORed—sent by transmitter n_1 with rate r in a given time-slot is considered to be successfully received by receiver n_2 if the received SINR is not less than $\gamma(r)$. The main differences of the slotted ALOHA systems without and with network coding are in the routing and the queue maintenance operations as described in the following.

Denote by $\mathcal{R}(l_i, l_j)$ the set of common available rates in links l_i and l_j, i.e., $\mathcal{R}(l_i, l_j) = \mathcal{R}(l_i) \cap \mathcal{R}(l_j)$. Given that node n does try to access the channel, we denote the conditional probability that: 1) it will select packets of flows f_i and f_j, $f_i \ne f_j$, to transmit on links $l_i \in \mathcal{L}_n^{\mathrm{O}}$ and $l_j \in \mathcal{L}_n^{\mathrm{O}}$, respectively, $l_i \ne l_j$, using network coding with transmission rate $r \in \mathcal{R}(l_i, l_j)$ by $q_{f_i,l_i,f_j,l_j}^{\mathrm{NC}}(r)$; 2) it will select a packet of flow f_i to transmit on link $l_i \in \mathcal{L}_n^{\mathrm{O}}$ without network coding with transmission rate $r \in \mathcal{R}(l_i)$ by $q_{f_i,l_i}^{\mathrm{WNC}}(r)$. These probabilities are related by the following equation:

$$\sum_{f_i, f_j \in \mathcal{F}, l_i, l_j \in \mathcal{L}_n^{\mathrm{O}}, f_i \ne f_j, l_j < l_i, r \in \mathcal{R}(l_i, l_j)} q_{f_i,l_i,f_j,l_j}^{\mathrm{NC}}(r) + \sum_{f_i \in \mathcal{F}, l_i \in \mathcal{L}_n^{\mathrm{O}}, r \in \mathcal{R}(l_i)} q_{f_i,l_i}^{\mathrm{WNC}}(r) = 1. \tag{16}$$

We assume that each node maintains a separate infinite buffer for each flow and records the incoming link information for each packet that it received.

2) Link Rate Model: Let $\tau^{\rm NC}_{f_i,l_i,f_j,l_j}(r)$ denote the probability that packets of flows f_i and f_j will be transmitted using network coding on links l_i and l_j, respectively, in a given time-slot with transmission rate $r \in \mathcal{R}(l_i,l_j)$, and $\tau^{\rm WNC}_{f_i,l_i}(r)$ the probability that a packet of flow f_i will be transmitted on link l_i without network coding in a given time-slot with transmission rate $r \in \mathcal{R}(l_i)$. The collection of $\tau^{\rm NC}_{f_i,l_i,f_j,l_j}(r)$ and $\tau^{\rm WNC}_{f_i,l_i}(r)$ variables are denoted by $\boldsymbol{\tau}^{\rm NC}$ and $\boldsymbol{\tau}^{\rm WNC}$, respectively. For any two distinct links $l_1 \neq l_2$, we have either $l_1 < l_2$ or $l_2 < l_1$, given some (arbitrary) ordering of the links. To keep the number of variables to a minimum, with the ordered links, define

$$\boldsymbol{\tau}^{\rm NC} = \{\tau^{\rm NC}_{f_i,l_i,f_j,l_j}(r) : f_i \in \mathcal{F}, f_j \in \mathcal{F}, f_i \neq f_j, l_i \in \mathcal{L}, l_j \in \mathcal{L}, l_j < l_i, r \in \mathcal{R}(l_i,l_j)\}.$$

Thus

$$\tau^{\rm WNC}_{f_i,l_i}(r) = \pi_n q^{\rm WNC}_{f_i,l_i}(r) \quad \forall n \in \mathcal{N},\ \forall f_i \in \mathcal{F},\ \forall l_i \in \mathcal{L}^{\rm O}_n,\ \forall r \in \mathcal{R}(l_i) \tag{17}$$

and

$$\tau^{\rm NC}_{f_i,l_i,f_j,l_j}(r) = \pi_n q^{\rm NC}_{f_i,l_i,f_j,l_j}(r) \quad \forall n \in \mathcal{N},\ \forall f_i, f_j \in \mathcal{F},\ \forall l_i, l_j \in \mathcal{L}^{\rm O}_n,\ f_i \neq f_j, l_j < l_i,\ \forall r \in \mathcal{R}(l_i,l_j) \tag{18}$$

and

$$\pi_n = \sum_{f_i,f_j \in \mathcal{F}, l_i,l_j \in \mathcal{L}^{\rm O}_n, f_i \neq f_j, l_j < l_i, r \in \mathcal{R}(l_i,l_j)} \tau^{\rm NC}_{f_i,l_i,f_j,l_j}(r) + \sum_{f_i \in \mathcal{F}, l_i \in \mathcal{L}^{\rm O}_n, r \in \mathcal{R}(l_i)} \tau^{\rm WNC}_{f_i,l_i}(r). \tag{19}$$

Let $\mathcal{L}^{\rm O}_n(r) \subseteq \mathcal{L}^{\rm O}_n$ denote the set of feasible links with transmission rate r going out from node n. For the transmissions associated with the transmission probabilities $\tau^{\rm NC}_{f_i,l_i,f_j,l_j}(r)$, with $n \in \mathcal{N}, l_i, l_j \in \mathcal{L}^{\rm O}_n, r \in \mathcal{R}(l_i,l_j), l_j < l_i, f_i \neq f_j$, the effective rate of flow f_i on link l_i, $c^{\rm NC}_{f_i,l_i,f_j,l_j,r}(f_i,l_i)$, is given by

$$c^{\rm NC}_{f_i,l_i,f_j,l_j,r}(f_i,l_i) = r\tau^{\rm NC}_{f_i,l_i,f_j,l_j}(r)p^{\rm s}_{l_i}(r) \tag{20}$$

and the effective rate of flow f_j on link l_j, $c^{\rm NC}_{f_i,l_i,f_j,l_j,r}(f_j,l_j)$, is given by

$$c^{\rm NC}_{f_i,l_i,f_j,l_j,r}(f_j,l_j) = r\tau^{\rm NC}_{f_i,l_i,f_j,l_j}(r)p^{\rm s}_{l_j}(r). \tag{21}$$

For the transmissions associated with the transmission probabilities $\tau^{\rm WNC}_{f_i,l_i}(r)$, the effective rate of flow f_i on link l_i is given by

$$c^{\rm WNC}_{f_i,l_i,r} = r\tau^{\rm WNC}_{f_i,l_i}(r)p^{\rm s}_{l_i}(r). \tag{22}$$

Thus, the effective rate of flow f_i on link l_i with rate $r \in \mathcal{R}(l_i)$ for the system with network coding can be written as

$$c^r_{f_i,l_i} = \left[\tau^{\rm WNC}_{f_i,l_i}(r) + \sum_{f_j \in \mathcal{F}, f_j \neq f_i, l_j \in \mathcal{L}^{\rm O}_{l_i^{\rm o}}(r), l_j < l_i} \tau^{\rm NC}_{f_i,l_i,f_j,l_j}(r) + \sum_{f_j \in \mathcal{F}, f_j \neq f_i, l_j \in \mathcal{L}^{\rm O}_{l_i^{\rm o}}(r), l_j > l_i} \tau^{\rm NC}_{f_j,l_j,f_i,l_i}\right] r p^{\rm s}_{l_i}(r). \tag{23}$$

The effective rate of flow f_i on link l_i is then given by

$$c_{f_i,l_i} = \sum_{r \in \mathcal{R}(l_i)} c^r_{f_i,l_i}. \tag{24}$$

3) Joint Routing, Access Probability, Network Coding, and Rate Allocation Optimization Problem: We now formulate the JRM-NC-RA optimization problem. In Section III-B.2, we derive the expression of the effective rate of a flow on a given link by combining the rates achieved by both types of transmissions (with and without network coding). Similar to the JRM-RA optimization problem in (11)–(15), we will use this expression to model the link rate constraints. In the JRM-RA optimization problem, we use the flow conservation constraints such that the arrival rate of a flow is equal to the service rate of the flow at an intermediate node. Unfortunately, these constraints are not sufficient to forbid a node to do more network coding than allowed with the available packets. To ensure that a node cannot do more network coding than allowed, we add network coding constraints to the optimization problem as described in the following. Since the packets in an XORed transmission must enter through a pair of incoming links and leave through an opposite pair of outgoing links, considering only the transmission probability $\tau^{\rm NC}_{f_i,l_i,f_j,l_j}(r)$, the effective rates of flow f_i on link l_i, $c^{\rm NC}_{f_i,l_i,f_j,l_j,r}(f_i,l_i)$, and flow f_j on link l_j, $c^{\rm NC}_{f_i,l_i,f_j,l_j,r}(f_j,l_j)$, are restricted by the rates of flow f_i on link $\bar{l}_j$, $c_{f_i,\bar{l}_j}$, and flow f_j on link $\bar{l}_i$, $c_{f_j,\bar{l}_i}$, respectively, where the opposite link of l is denoted by $\bar{l}$, i.e., $l^{\rm o} = \bar{l}^d$ and $l^{\rm d} = \bar{l}^{\rm o}$. The network coding constraints for node n can be written as

$$c^{\rm NC}_{f_i,l_i,f_j,l_j,r}(f_i,l_i) \le c_{f_i,\bar{l}_j} \tag{25}$$

$$c^{\rm NC}_{f_i,l_i,f_j,l_j,r}(f_j,l_j) \le c_{f_j,\bar{l}_i}. \tag{26}$$

Note that the effective rate of flow f_i on link l_i (resp. flow f_j on link l_j) and the arrival rate $c_{f_j,\bar{l}_i}$ ($c_{f_i,\bar{l}_j}$) depend on each other due to the common transmission probability $\tau^{\rm NC}_{f_i,l_i,f_j,l_j}(r)$ [see (20) and (21)]. Thus, if we can derive a network coding constraint for the arrival flow f_i at node n through the incoming link $\bar{l}_j$, the constraint for any arrival flow at node n through any incoming link can be written in a similar way. Packets of the arrival flow f_i at node n through the incoming link $\bar{l}_j$ are transmitted with the transmission probabilities $\{\tau^{\rm NC}_{f_i,l_i,f_j,l_j}(r) : f_j \in \mathcal{F}, l_i \in \mathcal{L}^{\rm O}_n, f_i \neq f_j, l_j < l_i, r \in \mathcal{R}(l_i,l_j)\}$ and the total effective rate of flow f_i achieved by all of these transmission probabilities is restricted by flow rate $c_{f_i,\bar{l}_j}$. Thus, the network coding constraint for $n \in \mathcal{N}$, $f_i \in \mathcal{F}$, $l_j \in \mathcal{L}^{\rm O}_n$ can be written as

$$\sum_{\substack{f_j \in \mathcal{F}, f_i \neq f_j, l_i \in \mathcal{L}^{\rm O}_n \\ l_j < l_i, r \in \mathcal{R}(l_i,l_j)}} \tau^{\rm NC}_{f_i,l_i,f_j,l_j}(r) r p^{\rm s}_{l_i}(r) + \sum_{\substack{f_j \in \mathcal{F}, f_i \neq f_j, l_i \in \mathcal{L}^{\rm O}_n \\ l_j > l_i, r \in \mathcal{R}(l_i,l_j)}} \tau^{\rm NC}_{f_j,l_j,f_i,l_i}(r) r p^{\rm s}_{l_i}(r) \le c_{f_i,\bar{l}_j} \tag{27}$$

where the left-hand side represents the total effective rate of flow f_i on all the outgoing links (except l_j) of node n for network coding with the traffic of flows other than f_i on link l_j.

To compute π_n from (19), the number of additive terms is $O(RF^2L^2)$, which is very high and limits the size of the network that we can handle numerically. To reduce the computational complexity of π_n, we rewrite (19) as follows:

$$\pi_n = \frac{1}{2}\left(2\sum_{f_i\in\mathcal{F},l_i\in\mathcal{L}_n^{\mathrm{O}},r\in\mathcal{R}(l_i)}\ \sum_{\substack{f_j\in\mathcal{F},l_j\in\mathcal{L}_n^{\mathrm{O}}(r)\\ f_i\neq f_j,l_j<l_i}}\tau^{\mathrm{NC}}_{f_i,l_i,f_j,l_j}(r)\right.$$
$$\left.+\sum_{f_i\in\mathcal{F},l_i\in\mathcal{L}_n^{\mathrm{O}},r\in\mathcal{R}(l_i)}\tau^{\mathrm{WNC}}_{f_i,l_i}(r)\right)$$
$$=\frac{1}{2}\left[\sum_{\substack{f_i\in\mathcal{F},l_i\in\mathcal{L}_n^{\mathrm{O}}\\ r\in\mathcal{R}(l_i)}}\left\{\sum_{\substack{f_j\in\mathcal{F},l_j\in\mathcal{L}_n^{\mathrm{O}}(r)\\ f_i\neq f_j,l_j<l_i}}\tau^{\mathrm{NC}}_{f_i,l_i,f_j,l_j}(r)\right.\right.$$
$$\left.\left.+\sum_{\substack{f_j\in\mathcal{F},l_j\in\mathcal{L}_n^{\mathrm{O}}(r)\\ f_i\neq f_j,l_j>l_i}}\tau^{\mathrm{NC}}_{f_j,l_j,f_i,l_i}(r)\right\}\right]+\sum_{\substack{f_i\in\mathcal{F},l_i\in\mathcal{L}_n^{\mathrm{O}}\\ r\in\mathcal{R}(l_i)}}\tau^{\mathrm{WNC}}_{f_i,l_i}(r). \tag{28}$$

Using (23) in (28), we have

$$\pi_n = \frac{1}{2}\sum_{f_i\in\mathcal{F},l_i\in\mathcal{L}_n^{\mathrm{O}},r\in\mathcal{R}(l_i)}\frac{c^r_{f_i,l_i}}{rp^{\mathrm{s}}_{l_i}(r)}$$
$$+\frac{1}{2}\sum_{f_i\in\mathcal{F},l_i\in\mathcal{L}_n^{\mathrm{O}},r\in\mathcal{R}(l_i)}\tau^{\mathrm{WNC}}_{f_i,l_i}(r). \tag{29}$$

Thus, using (29), the number of additive terms in the computation of π_n is reduced from $O(RF^2L^2)$ to $O(RFL)$.

Let $\mathbf{p}^{\mathrm{s}}$ represent the vector of successful transmission probabilities on the links, and let ϵ be a very small positive constant. We formulate the JRM-NC-RA optimization problem as

$$\max_{\boldsymbol{\tau}^{\mathrm{NC}},\boldsymbol{\tau}^{\mathrm{WNC}},\boldsymbol{\pi},\mathbf{p}^{\mathrm{s}},\mathbf{c}}\ \lambda \tag{30}$$

$$\text{s.t.}\ \sum_{l\in\mathcal{L}_n^{\mathrm{O}}}c_{f,l}-\sum_{l\in\mathcal{L}_n^{\mathrm{I}}}c_{f,l}=\begin{cases}w_f\lambda, & \text{if } n=f^{\mathrm{s}}\\ -w_f\lambda, & \text{if } n=f^{\mathrm{d}}\\ 0, & \text{otherwise}\end{cases}\quad\forall n\in\mathcal{N},\forall f\in\mathcal{F} \tag{31}$$

$$c_{f_i,l_i}=\sum_{r\in\mathcal{R}(l_i)}\left(\tau^{\mathrm{WNC}}_{f_i,l_i}(r)+\sum_{f_j\in\mathcal{F},l_j\in\mathcal{L}_n^{\mathrm{O}}(r),f_i\neq f_j,l_j<l_i}\tau^{\mathrm{NC}}_{f_i,l_i,f_j,l_j}(r)\right.$$
$$\left.+\sum_{f_j\in\mathcal{F},l_j\in\mathcal{L}_n^{\mathrm{O}}(r),f_i\neq f_j,l_j>l_i}\tau^{\mathrm{NC}}_{f_j,l_j,f_i,l_i}(r)\right)rp^{\mathrm{s}}_{l_i}(r)$$
$$\forall n\in\mathcal{N},\forall f_i\in\mathcal{F},\forall l_i\in\mathcal{L}_n^{\mathrm{O}} \tag{32}$$

$$\sum_{f_j\in\mathcal{F},f_i\neq f_j,l_i\in\mathcal{L}_n^{\mathrm{O}},l_j<l_i,r\in\mathcal{R}(l_i,l_j)}\tau^{\mathrm{NC}}_{f_i,l_i,f_j,l_j}(r)rp^{\mathrm{s}}_{l_i}(r)$$
$$+\sum_{f_j\in\mathcal{F},f_i\neq f_j,l_i\in\mathcal{L}_n^{\mathrm{O}},l_j>l_i,r\in\mathcal{R}(l_i,l_j)}\tau^{\mathrm{NC}}_{f_j,l_j,f_i,l_i}(r)rp^{\mathrm{s}}_{l_i}(r)$$
$$\leq c_{f_i,\bar{l}_j}\quad\forall n\in\mathcal{N},\ \forall f_i\in\mathcal{F},\ \forall l_j\in\mathcal{L}_n^{\mathrm{O}} \tag{33}$$

$$\pi_n=\frac{1}{2}\sum_{f_i\in\mathcal{F},l_i\in\mathcal{L}_n^{\mathrm{O}},r\in\mathcal{R}(l_i)}\frac{c^r_{f_i,l_i}}{rp^{\mathrm{s}}_{l_i}(r)}$$
$$+\frac{1}{2}\sum_{f_i\in\mathcal{F},l_i\in\mathcal{L}_n^{\mathrm{O}},r\in\mathcal{R}(l_i)}\tau^{\mathrm{WNC}}_{f_i,l_i}(r)\quad\forall n\in\mathcal{N} \tag{34}$$

$$p^{\mathrm{s}}_l(r)=\sum_{\sigma_l\in\mathcal{S}_l^r}\prod_{i\in\sigma_l}\pi_i\prod_{j\in\mathcal{N}_l\backslash\sigma_l}(1-\pi_j)\quad\forall l\in\mathcal{L},\forall r\in\mathcal{R}(l) \tag{35}$$

$$0\leq\lambda,\mathbf{c};\qquad 0\leq\boldsymbol{\tau}^{\mathrm{NC}},\boldsymbol{\tau}^{\mathrm{WNC}},\ \boldsymbol{\pi}\leq 1 \tag{36}$$

$$\epsilon\leq\mathbf{p}^{\mathrm{s}}\leq 1. \tag{37}$$

In (33), we include network coding constraints to ensure that a node cannot do network coding more often than the packet arrivals allow. We also include boundary constraints in (37) for the $p^{\mathrm{s}}_l(r)$ variables. We use ϵ as a lower bound of $p^{\mathrm{s}}_l(r)$ since the constraints in (34) become infeasible at $p^{\mathrm{s}}_l(r)=0$ and for a practical network usually $p^{\mathrm{s}}_l(r)>0\ \forall l\in\mathcal{L}(r), r\in\mathcal{R}$, where $\mathcal{L}(r)\subseteq\mathcal{L}$ is the set of feasible links with rate r. Thus, we do not consider the case where $p^{\mathrm{s}}_l(r)=0$ for any link $l\in\mathcal{L}(r)$. Similar to the JRM-RA optimization problem, the JRM-NC-RA optimization problem is nonlinear and nonconvex due to the nonlinear and nonconvex constraints in (32)–(35), but the computational complexity significantly increases in this problem

4) Problem Simplification: To reduce the complexity of the JRM-NC-RA optimization problem in (30)–(37), one can restrict network coding to bidirectional flows.[2]

We define bidirectional network coding as follows. Let $\overline{f}_i\in\mathcal{F}$ denote the corresponding uplink (resp. downlink) flow of the downlink (resp. uplink) flow $f_i\in\mathcal{F}$. Nodes are allowed to do network coding only between $f_i\in\mathcal{F}$ and $\overline{f}_i\in\mathcal{F}$. Given that node n does try to access the channel, denote the conditional probability that: 1) it will select packets of flows $f_i\in\mathcal{F}$ and $\overline{f}_i\in\mathcal{F}$ to transmit on links $l_i\in\mathcal{L}_n^{\mathrm{O}}$ and $l_j\in\mathcal{L}_n^{\mathrm{O}}$, resp., $l_j<l_i$, with rate $r\in\mathcal{R}(l_i,l_j)$ using network coding by $q^{\mathrm{NC}}_{f_i,l_i,\overline{f}_i,l_j}(r)$; 2) it will select a packet of flow $f_i\in\mathcal{F}$ to transmit on link $l_i\in\mathcal{L}_n^{\mathrm{O}}$ without network coding at rate $r\in\mathcal{R}(l_i)$ by $q^{\mathrm{WNC}}_{f_i,l_i}(r)$ with the condition

$$\sum_{\substack{f_i\in\mathcal{F},l_i\in\mathcal{L}_n^{\mathrm{O}},l_j\in\mathcal{L}_n^{\mathrm{O}}\\ l_j<l_i,r\in\mathcal{R}(l_i,l_j)}}q^{\mathrm{NC}}_{f_i,l_i,\overline{f}_i,l_j}(r)+\sum_{\substack{f_i\in\mathcal{F},l_i\in\mathcal{L}_n^{\mathrm{O}}\\ r\in\mathcal{R}(l_i)}}q^{\mathrm{WNC}}_{f_i,l_i}(r)=1.$$

One can formulate the joint routing, access probability, (bidirectional) network coding, and rate allocation (JRM-BiNC-RA) optimization problem, by replacing $f_j\in\mathcal{F}$ with $\overline{f}_i$ in all the constraints in the JRM-NC-RA problem formulation (30)–(37). Note that by using network coding on bidirectional flows, the number of additive terms for each of the link rate constraints is reduced to $O(LR^2)$ from $O(FLR^2)$ and the number of additive terms to compute π_n is reduced to $O(RL)$ from $O(RFL)$.

IV. Solution Technique

We choose to solve the nonlinear and nonconvex optimization problems by the IOS technique [1], which is an iterated local

[2]Two flows f and g are called bidirectional if $f^{\mathrm{s}}=g^{\mathrm{d}}$ and $f^{\mathrm{d}}=g^{\mathrm{s}}$. We choose one randomly to be called uplink flow, while the other one is called downlink flow.

search technique [34]. However, we will only be able to obtain solutions in the case of small to medium-size networks.

A. *Iterated Optimal Search Algorithm*

For a given problem, the IOS algorithm finds a sequence of local maxima by starting from different initial values at each iteration. The main feature of this method is that the initial values of a local search are chosen using the best solution of the previous iterations. Denote by M the total number of iterations of the algorithm. Furthermore, let $\mathbf{x}$ be the vector of variables of the optimization problem and $\mathbf{x}_m$ be the initial values of the variables for the mth iteration. At each iteration, we use MINOS 5.51 [35] to compute the local maxima. The initial values of variables for the first iteration, $\mathbf{x}_1$, are taken from a reasonable range of the variables. At the start of the mth iteration, $1 < m \leq M$, $\mathbf{x}_m$ is computed by $\mathbf{x}_m = \mathbf{x}_m^{\mathrm{B}} + \mathbf{x}_m^{\mathrm{p}}$, where $\mathbf{x}_m^{\mathrm{B}}$ is the best solution among the first $m-1$ iterations and $\mathbf{x}_m^{\mathrm{p}}$ is a perturbation vector given by $\mathbf{x}_m^{\mathrm{p}} = \boldsymbol{\alpha}_m \odot \mathbf{x}_1$, with $\odot$ being an elementwise multiplication operator and each element of the vector $\boldsymbol{\alpha}_m$ being chosen independently from a uniform distribution on $[-a, a]$. At the end of the Mth iteration, this algorithm selects the best local optimal solution.

B. *Determining the Solution*

To determine the optimal solution of a given problem, we run the IOS algorithm with four to five different initial vectors $\mathbf{x}_1$ and three values of a for each initial $\mathbf{x}_1$, and then select the best solution. In our study, $M = 30$ and $a = 0.25$, 0.5, and 1.

V. Case Study—System Without Network Coding

In this section, we first study the cross-layer design problem with a single rate at all nodes for wireless mesh networks without network coding. A wireless mesh network is a multihop access network that contains a gateway node connected to the Internet, and all flows are either destined for the gateway (i.e., uplink flows) or generated by the gateway (i.e., downlink flows). To investigate the advantages of our joint design, we compare it to a default configuration. We also describe a simple heuristic to configure the routing and access probability parameters in a slotted-ALOHA-based wireless network that allows us to configure large-size networks. We then compare the jointly optimized cross-layer design, heuristic, default design, optimal MAC (OMAC)-only design, i.e., optimizing only the access probability parameters with default routing parameters, and optimal routing (ORouting)-only design, i.e., optimizing only the routing parameters with default access probability parameters. This shows that a simple heuristic can perform very well. Furthermore, we study the cross-layer design problem in multirate slotted ALOHA systems and compare the performance of multirate and single-rate systems.

We will show results for two 16-node mesh networks (Grid16 and Rand16) to compare the performance of joint, heuristic, and default designs. The positions of the nodes in the Rand16 network were drawn from a uniform random distribution. The two 16-node network topologies are shown in Fig. 2, where the gateway node is indicated by a rectangle in each figure. Note that we have studied several realizations (i.e., node placements) of Rand16 type networks and found similar trends and results. We will show results on the ensemble of these realizations later in the paper. The total number of flows in each network is set to be $2(N-1)$, with $N-1$ uplink flows to the gateway and $N-1$ downlink flows from the gateway. The weight of each uplink flow is 1, and the weight for each downlink flow is w, i.e., the traffic rate ratio of a downlink flow to an uplink flow is w. For simplicity only, we assume that the channel gain between nodes n_1 and n_2, $G_{n_1 n_2}$, is $(d_{n_1 n_2}/d_0)^{-\eta}$, where $d_{n_1 n_2}$ is the distance between the nodes, d_0 is a reference distance in the far field of the transmit antenna, and η is the path loss exponent. The physical-layer parameters are given in Table I, where the values of d_0 and η are taken from [36] and [37] assuming an outdoor environment.

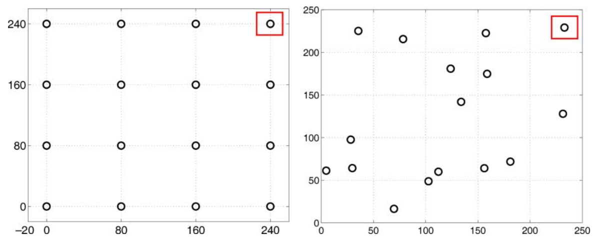

Fig. 2. Two 16-node networks: *(left)* Grid16 and *(right)* Rand16.

TABLE I
Physical-Layer Parameters

Parameter	Symbol	Value
Normalized rate	r	1
SINR threshold (dB)	$\gamma(r)$	6.4
Noise power (dBm)	N_0	-100
Path-loss exponent	η	3
Far-field crossover distance (m)	d_0	1

Since we are going to compare the results for the joint problems obtained via numerical computations to the results for a default and heuristic configurations obtained by simulation, we next describe these two configurations. We will then present and discuss our results.

A. *Default and Heuristic Configurations*

To define precisely the default and heuristic configurations, we need to configure the following parameters and processes: a per-flow routing strategy that will be used to fill up the forwarding table in each node, the attempt probabilities π_n, and the flow selection criteria (i.e., how a node will select a flow, if it decides to transmit).

1) Routing: The simplicity of min-hop routing makes it a good candidate for a heuristic, even though it may be suboptimal. For comparison, both default and heuristic designs use the same single-path min-hop routing. Among all the min-hop paths for each flow, the one with the shortest distance (the sum of the physical distances of all links of the path) is chosen since the quality of a link often depends on the distance between the transmitter and the receiver. If the number of shortest-distance min-hop paths is more than one, e.g., in the grid network, the path yielding the maximum total traffic load is chosen to reduce collisions by decreasing traffic in the competing nodes. Hence, we do not claim that the min-hop path that we selected is the best among all the min-hop paths.

2) Medium Access Control: The default configuration uses the same attempt probability at all nodes, equal to $1/N$. For our heuristic, we first note that, once routes have been selected, it is possible to calculate the amount of traffic transmitted by each

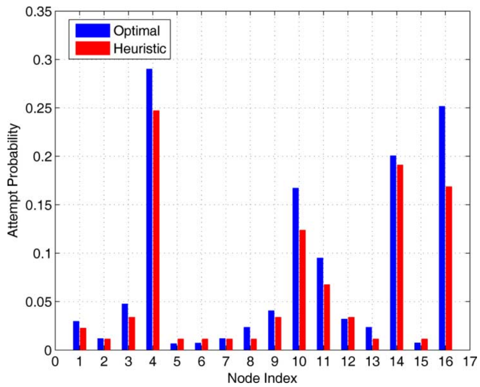

Fig. 3. Optimal and heuristic attempt probabilities in the Rand16 network at $P_{\mathrm{t}} = -34$ dBm for $w = 1$.

node assuming that each uplink flow has a throughput λ (and each downlink flow has a throughput $w\lambda$). The values of π_n clearly should depend on the traffic carried by node n as well as the traffic carried by the other nodes. We conjecture that a good approximation would be of the type

$$\pi_n = \frac{y_n}{\sum\limits_{n' \in \mathcal{N}} y_{n'}} \pi_0 \tag{38}$$

where y_n is the amount of traffic transmitted by node n and π_0 is an unknown factor depending on the network topology. Since it is not clear what is a suitable value of π_0, after some tests, we decided to set $\pi_0 = 1$. We validated our conjecture by comparing the heuristic values of π_n's determined using (38) for the optimal routing (i.e., by using the values of y_n's obtained by our numerical solution to the joint problem) with the values of π_n's obtained by our numerical solution. The optimal and heuristic values of π_n's are shown in Fig. 3 for the Rand16 network where the node index for the gateway is 16. These results are surprisingly close and show that $\pi_n \neq 1/N$. Note that node 4 is the second closest node to the gateway, and its transmission probability is high due to the large amount of traffic routed through it.

3) Flow and Link Selection: Once a node has decided to transmit, it needs to determine which flow to transmit. Since a single route has been selected for each flow, the link on which the selected packet will be transmitted is known. Thus, after a decision to transmit, a node needs to select a flow. In the default configuration, all the flows traversing a node are equally likely to be chosen. In the heuristic configuration, node n selects the carried flow f with probability $\frac{\lambda_f}{y_n}$.

We summarize the properties of the default and heuristic configurations in Table II.

4) Determining the Weighted Max-Min Throughput for These Configurations: The simulator is developed as custom code using the C++ programming language. We have not used commonly available network simulators as none of them provided the flexibility to tune the routing, random access, and network coding parameters, in addition to the physical-layer parameters, in the ways that were required for this study.

a) *Simulator setup*: The average rates of all the sources of the uplink flows are set to the same equal value (say, λ), the average rates at the gateway for all the downlink flows are set to $w\lambda$, and the traffic arrivals are assumed to be Poisson. The node decision to transmit or not and the selection of which flow on which link to transmit are implemented in the simulation as described above. Each node maintains a separate queue for each flow with a buffer of size 1000 packets. In the simulator, the number of packets in a queue is increased by one if a new packet arrives, decreased by one if a transmission is successful, and kept unchanged if a transmission is unsuccessful. Since a separate queue is maintained for each flow, this strategy is equivalent to the DFT retransmission strategy. When the source rate is low, a node may not always have a packet of the selected flow to transmit and, if so, the node does not transmit.

b) *Determining the weighted max-min throughput of a network configuration*: For a particular rate λ, the packet loss probability (PLP) of each queue is estimated from the ratio of the number of loss packets and the number of packets that arrived at the queue over a window of 1.0×10^8 slots after a network loading time of 10^6 slots. The total simulation time is then 1.01×10^8 slots. The PLPs of the queues are used to check system stability (see the Appendix). To determine the weighted max-min throughput with a small error, the rate λ is increased from a starting value λ_0 by increments of 0.0001 until the system becomes unstable. The system stability is checked at each step using the statistical test described in the Appendix. The largest value of λ for which the system is stable is the weighted max-min throughput.

TABLE II
SUMMARY OF THE DEFAULT AND HEURISTIC CONFIGURATIONS

Parameters	Default	Heuristic
Routing	Single path and min-hop with the shortest distance (the sum of the physical distance of all the links)	Single path and min-hop with the shortest distance (the sum of the physical distance of all the links)
Access Probability	Equal for all nodes $\frac{1}{N}$	According to the traffic carried by the nodes (38)
Flow selection probabilities at a node	Equal for each flow crossing the node	According to the traffic rate of the transmitted flows

B. Results

1) Joint Versus Default and Heuristic: We determine the weighted max-min per-node throughput, i.e., $\lambda^* + w\lambda^*$ (where λ^* is the solution to the joint problem), for the two 16-node mesh networks by solving the JRM-RA problem using the IOS technique. For the default and the heuristic configurations, the per-node throughputs are determined by simulation taking the minimum of the stable throughputs obtained over 10 simulation runs. The per-node throughputs achieved for the joint, heuristic, and default designs are shown in Fig. 4 for the two 16-node networks. It is seen that the throughput increases with transmission power for all configurations, and it is very sensitive to the transmission power for the joint and heuristic designs, but not for the default design. The max-min throughput obtained by the heuristic compares well to the throughput obtained via joint design and is significantly higher than the one obtained by the default configuration. Furthermore, the max-min node throughput with $w = 2$ is higher than with $w = 1$ for the joint configuration, while the opposite is observed for the default configuration. We

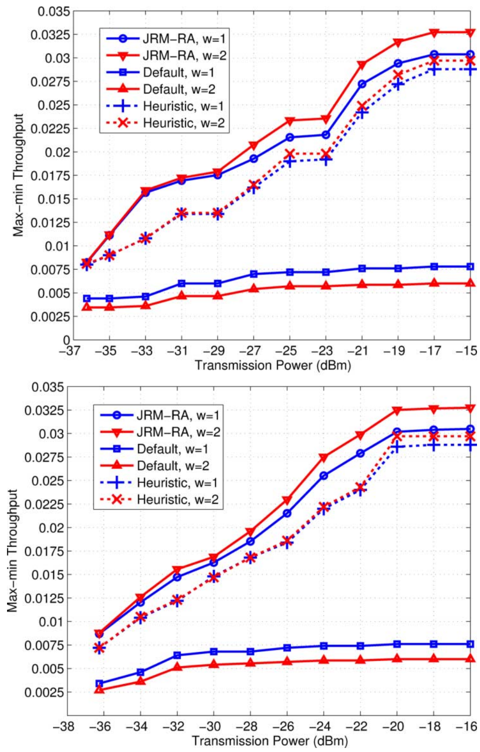

Fig. 4. Node throughput in the two 16-node networks without network coding: *(top)* Grid16 and *(bottom)* Rand16.

attribute this to the fact that uplink traffic to the gateway is a bottleneck due to contention, and by increasing w, less uplink traffic is required.

The results indicate that an 80 to 300% throughput gain can be achieved by the joint design with respect to the default design for the equal weighting case, i.e., $w = 1$. The throughput gain with $w = 2$ is in the range of 130 to 450%, higher than that with $w = 1$.

Remarks: 1) It should be mentioned that we have compared our numerical results, i.e., the optimal throughput obtained by solving the JRM-RA problem using the IOS technique, to the simulation results obtained by configuring the networks with the optimal parameters for several cases, and we found that the differences are negligible (see [1] for a comparison of numerical and simulation results).

2) The time to solve the JRM-RA problem (as well as the JRM-BiNC-RA problem) is a few hours to 10 h for the 16-node networks depending on the transmit power, and hence our optimization tools are only applicable for static offline configurations.

2) Comparison to Optimal MAC and Optimal Routing Designs: To understand the gains brought by cross-layer design, we compute the throughputs obtained by OMAC and ORouting designs, i.e., we solve the JRM-RA problem using the IOS technique with the following modifications. For OMAC design, the flow conservation constraints of (12) are removed and the traffic rate of the links, i.e., $c_{f,l}$'s are calculated from the given default routing as a function of λ and replaced in (13). For ORouting design, we include constraints $\pi_n \leq 1/N$ for all n.

The per-node throughput achieved for the joint, heuristic, default, OMAC, and ORouting designs are shown in Fig. 5 for the Rand16 network with $w = 2$. The results indicate that a

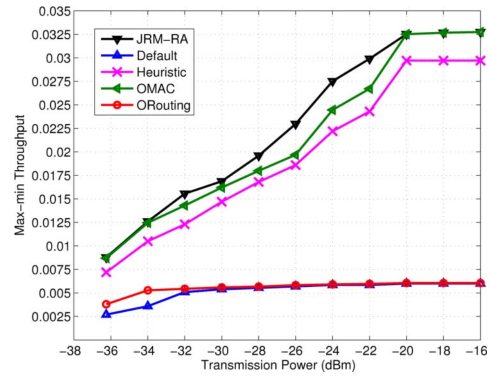

Fig. 5. Node throughput in the Rand16 network without network coding with $w = 2$ for different designs.

significant amount of throughput improvement can be obtained by OMAC design, while ORouting design yields a small amount of throughput improvement. The throughput gain by the jointly optimized design over the OMAC design is bounded by 20% in this scenario and in general will depend on network topology and transmit power. Hence, from a throughput point of view, configuring the MAC access probability parameters optimally is more desirable than configuring routing only. In the heuristic design, we configure the access probability parameters in combination with a default routing based on simple calculations. The heuristic design is found to provide throughput very close to the OMAC design. Hence, our simple heuristic is an attractive option with low complexity to configure wireless mesh networks.

3) Multirate Versus Single-Rate Systems: To compare the performance of multirate and single-rate slotted ALOHA systems, we determine the weighted max-min throughput of the two 16-node mesh networks by solving the JRM-RA problem using the IOS technique for the following cases.

- Each node uses only one modulation and coding scheme yielding a unit rate (the SINR threshold being 6.4 dB).
- Each node uses only one modulation and coding scheme yielding a rate of 2 (the SINR threshold being 9.4 dB).
- Each node uses two modulation and coding schemes respectively yielding rates 1 and 2 with the same two SINR thresholds as above. Results for the two 16-node networks with physical transmission rate 1 have already been presented in Fig. 4. Next, we show the results for physical transmission rate 2 for comparison.

The optimal per-node throughputs of these different cases in the two 16-node networks with $w = 2$ are shown in Fig. 6. We obtain similar results for the case $w = 1$.

Considering only the single-rate cases, clearly, a much higher throughput is achievable using a higher transmission rate. However, the network becomes connected at a higher transmission power for a higher transmission rate. The throughput improvement obtained by using two rates over the higher single-rate case is negligible for the 16-node grid network. In the case of our 16-node random network, the throughput improvement depends on the transmit power but is never very large. The same qualitative results were observed in the case of a scheduled network [5].

VI. Case Study—System With Network Coding

In this section, we study the cross-layer design problem with a single rate at all nodes for systems with network coding.

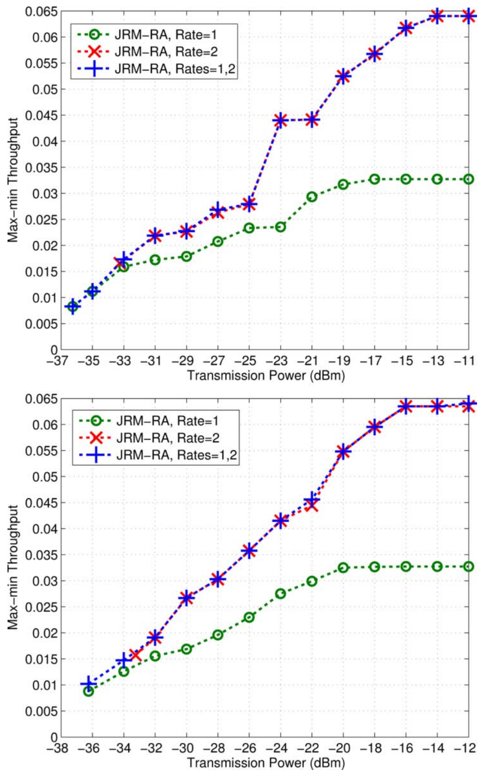

Fig. 6. Comparison of per-node throughput between single-rate and multirate cases with $w = 2$: *(top)* Grid16 and *(bottom)* Rand16.

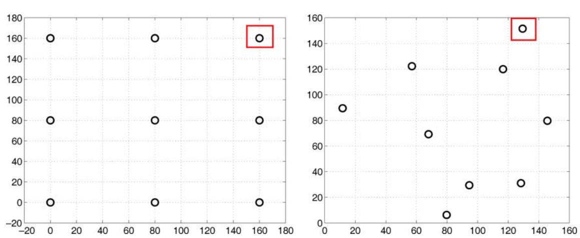

Fig. 7. Network topologies of 9-node networks: *(left)* Grid9 and *(right)* Rand9.

A. Bidirectional Network Coding Versus Full Network Coding

We use two 9-node networks (Grid9 and Rand9) to compare the performance of bidirectional network coding and full network coding as the computational complexity for full network coding for 16-node networks is too large. The two 9-node networks are shown in Fig. 7, where the gateway node is labeled by a rectangle. The physical-layer parameters are given in Table I. The total number of flows in each network is $2(N-1)$, with $N-1$ uplink flows to the gateway and the other $N-1$ downlink flows from the gateway.

We compute the relative throughput difference (in percentage) between the JRM-NC-RA and JRM-BiNC-RA designs for the Grid9 and Rand9 networks at $w=1$ and $w=2$ for different transmit power levels as

$$\%\ \text{Diff.} = \frac{\lambda_{\text{JRM-NC-RA}} \text{-} \lambda_{\text{JRM-BiNC-RA}}}{\lambda_{\text{JRM-NC-RA}}} \times 100$$

where $\lambda_{\text{JRM-NC-RA}}$ and $\lambda_{\text{JRM-BiNC-RA}}$ are the weighted max-min per-node throughputs for the JRM-NC-RA and JRM-BiNC-RA designs, respectively. We find that the maximum throughput difference is less than 1% and thus concluded that only a small amount of throughput is lost if the bidirectional network coding model is used instead of full network coding, for the networks under consideration and under the assumption that all the uplink flows (resp. downlink flows) have the same weight. We conjecture that even in a medium-size network the same is true to some extent. In the following, we use bidirectional network coding instead of full network coding to study two 16-node mesh networks.

B. Default and Heuristic Configurations

We now present how we have adapted our default and heuristic configurations to incorporate network coding.

1) Routing: Consider the same single-path min-hop routing for both heuristic and default configurations. With the restriction to bidirectional network coding, a node has two types of flows, the "local" ones (i.e., the one it generates and the one it receives) and the "relayed" ones (the number of relayed flows depends on the routing). To take full advantage of network coding, assume that the routing paths of corresponding downlink f_i and uplink $\overline{f}_i$ flows are the same (with the links directed in the opposite direction) and a node always attempts to network code a relayed flow with its bidirectional counterpart. Thus, only the paths of the uplink flows need to be determined. For each uplink flow, a min-hop path is chosen as discussed in Section V-A.1.

2) Medium Access Control: For the default design, the attempt probability of each node is set to $1/N$ as before. For the heuristic, we use the model in (38) by replacing the traffic load of the nodes (i.e., the y_n's) with the *effective traffic load* of the nodes described in the following.

Let M_n be the number of bidirectional flow pairs that node $n \in \mathcal{N} \setminus \{g\}$ relays, where g denotes the gateway. The amount of traffic transmitted by node $n \in \mathcal{N} \setminus \{g\}$ is $M_n(w\lambda + \lambda) + \lambda$, where $w\lambda + \lambda$ is the total rate of each bidirectional flow pair and λ is the rate of its own generated flow. On the other hand, the gateway transmits all the downlink flows without network coding, as it does not have any opportunity to network code since it does not relay any flow. The amount of traffic transmitted by the gateway is $(N-1)w\lambda$. Since node $n \in \mathcal{N} \setminus \{g\}$ is able to do network coding on each bidirectional flow pair that it relays, it can transmit all the uplink relaying traffic $M_n\lambda$ by network coding with the downlink relaying traffic $M_n w\lambda$ for $w \geq 1$. Thus, effectively, it needs to access the medium for transmitting an amount of traffic $M_n w\lambda + \lambda$. Let y_n denote the effective amount of traffic that node $n \in \mathcal{N} \setminus \{g\}$ needs to access the medium for, given by $y_n = M_n w\lambda + \lambda$. Since the gateway transmits all the traffic without any network coding, we set

$$y_g = (N-1)w\lambda. \tag{39}$$

Then, in our heuristic, the attempt probability of node n is calculated as

$$\pi_n = \frac{y_n}{\sum_{n' \in \mathcal{N}} y'_n}.$$

To investigate how efficient our heuristic is in configuring the parameters π_n, we compute the optimal routing and the π_n's of the Rand16 network for the JRM-BiNC-RA design and then calculate the heuristic π_n's using (38). The optimal and heuristic values of π_n's are shown in Fig. 8 for the Rand16 network. The heuristic attempt probabilities are quite close to the optimal values. Note that node 11 is the closest node to the gateway, and

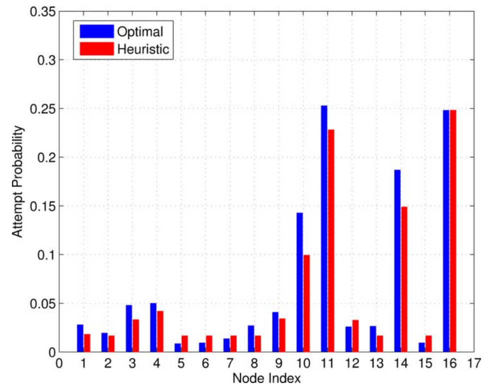

Fig. 8. Optimal and heuristic attempt probabilities in the Rand16 network at $w = 1, P_{\mathrm{t}} = -34$ dBm.

its transmission probability is high due to the large amount of traffic routed through it.

3) Flow(s) and Link(s) Selection: From the routing decision, each node knows the "local flow" and bidirectional flow pairs that it will transmit. The gateway transmits only $N-1$ "local" flows (i.e., downlink flows) while any of the other nodes can transmit one "local" (i.e., its own) as well as bidirectional flows. In the default design, once node $n \in \mathcal{N} \setminus \{g\}$ has decided to transmit, it selects either one of the bidirectional flow pairs that it relays or its own generated flow with equal probability $\frac{1}{M_n+1}$.

On the other hand, for the heuristic, node $n \in \mathcal{N} \setminus \{g\}$ selects one of the bidirectional flow pairs that it relays with probability $\frac{w}{M_n w+1}$ and its own flow with probability $\frac{1}{M_n w+1}$, given that the effective traffic of a bidirectional flow pair $w\lambda$ is and the effective traffic of its own flow is λ. In both designs, the gateway selects each of the downlink flows with equal probability $1/(N-1)$.

4) Simulation: For a system with network coding, we modify in the simulator the flow and link selection strategies described in Section V-A.4 according to the system configuration with network coding. A node records the incoming link of each packet arrival. When the source rate is low, a node may not always have a packet(s) of the selected flow(s) to transmit and, if so, the node does not transmit (or if only one packet is available when network coding is attempted, the packet is sent without network coding).

C. Results

We determine the weighted max-min per-node throughput for the two 16-node mesh networks by solving the JRM-BiNC-RA problem using the IOS technique. For the default and heuristic configurations, we obtained the results by simulation, taking the minimum of the stable throughputs obtained over 10 simulation runs. The per-node throughputs achieved for the joint, heuristic, and default designs are shown in Fig. 9 for the two 16-node networks. Similar results were observed for other network realizations. The results show that 100%–300% and 110%–450% throughput gains can be achieved by joint configuration with respect to the default configuration for $w = 1$ and $w = 2$, respectively. The heuristic is found to be efficient.

Now, we study the throughput gains achieved by network coding when compared to a case without any network coding (recall that we only study XOR-based network coding without opportunistic listening). In Fig. 10, we present the relative throughput gain (in percentage) obtained by the JRM-BiNC-RA design with respect to the JRM-RA design for

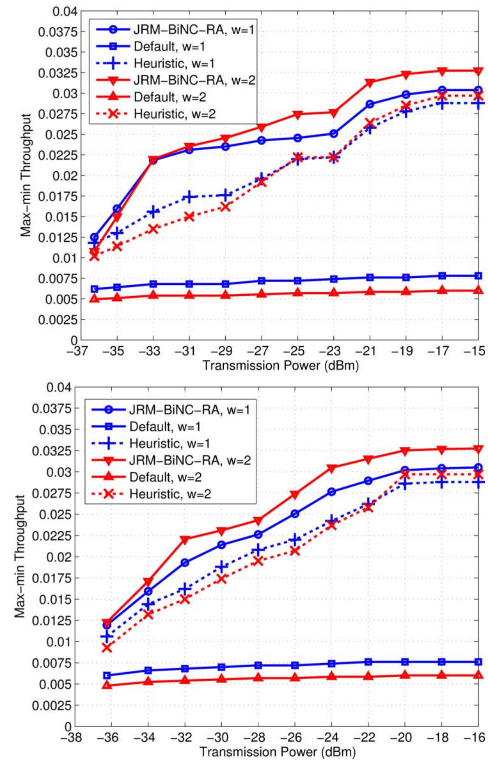

Fig. 9. Per-node throughput in the two 16-node networks with network coding: *(top)* Grid16 and *(bottom)* Rand16.

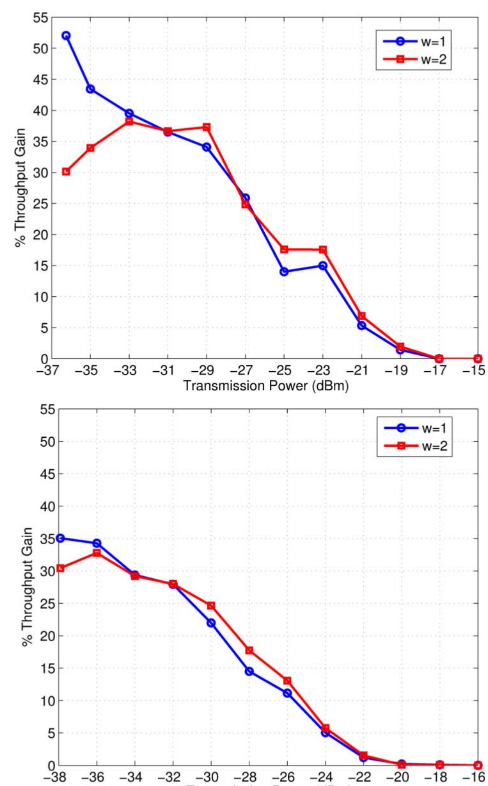

Fig. 10. Throughput gain of the JRM-BiNC-RA design with respect to the JRM-RA design: *(top)* Grid16 and *(bottom)* Rand16 (averaged over five realizations).

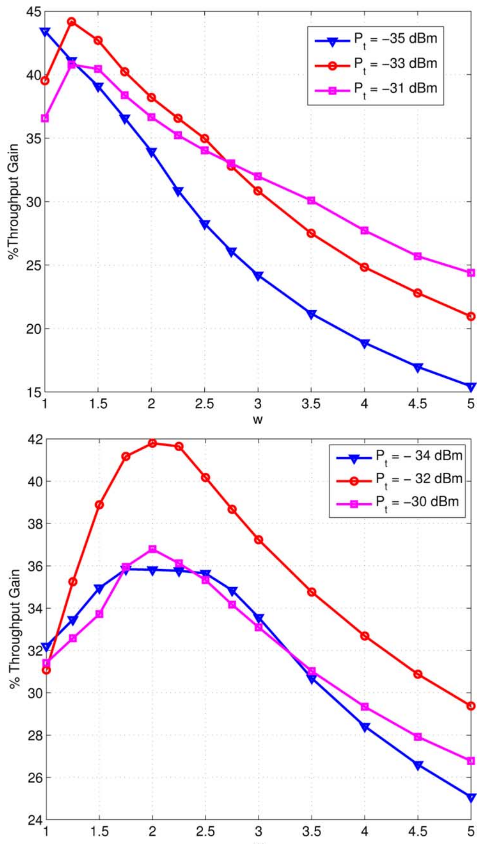

Fig. 11. Throughput gain versus w: *(top)* Grid16 and *(bottom)* Rand16.

Grid16 and 16-node random networks with the gateway at the corner. Note that the gain is averaged over five realizations for the case of 16-node random network. The results show that, at low transmission power, network coding can provide a significant throughput gain, in the range of 25%–50%. At higher transmission power, network coding becomes less attractive as there are more and more single-hop paths to the gateway.

Interestingly, except at very low transmission power, the throughput gain for a downlink/uplink ratio of $w = 2$ is higher than for a ratio of $w = 1$, especially for the Rand16-type networks. We attribute this to the fact that, in a network coding pair, the downlink link has a higher successful transmission probability than the uplink link due to congestion as traffic increases for the nodes close to the gateway and the gateway node itself generates a large amount of traffic. Although the traffic rate is balanced on a network coding link pair at $w = 1$, differences in the successful transmission probabilities on the two links for a network-coded packet create an imbalance in offered traffic due to retransmissions, and hence the number of network coding opportunities is significantly reduced. On the other hand, at $w = 2$, while there is traffic imbalance on a network-coded link pair, due to a high retransmission rate on the lower traffic uplink link and a low retransmission rate on the higher traffic downlink link, offered traffic on a network-coded link pair is in fact more balanced. As a result, a higher throughput gain is obtained at $w = 2$.

In Fig. 11, given a transmission power, the throughput gains for different values of w are presented for the two networks. Although the value of w at which the highest throughput gain is obtained differs from one network to another, for all the networks we have studied, we found that the typical value of w for the highest throughput gain is in the range of 1–3. Since typical values of w for Internet traffic are around 2, these results show that the typical imbalance of downlink and uplink traffic rates increases network coding opportunities. We also study the cross-layer design problem with rates 1 and 2 in the two 16-node networks and compare the performance of the single-rate and multirate systems. The insights for the slotted ALOHA systems without network coding (in Section V-B.3) remain the same even when network coding is enabled. Due to space limitation, we do not present the results here.

VII. Conclusion

In this paper, we have studied the joint configuration of routing, access probability, and transmission rate parameters in slotted ALOHA wireless mesh networks. We have formulated and solved several optimization problems for several wireless mesh network scenarios. The studies for the single-rate systems show that: 1) compared to a default configuration, the optimal joint configuration of network parameters can improve throughput performance significantly; 2) in terms of throughput, it is better to optimize the MAC access probability parameters than the routing; 3) throughput gains with optimized cross-layer design can be as high as 20% when compared to a design that only optimizes the MAC access probabilities. In addition: 4) we have proposed a heuristic configuration of the transmission probabilities based on the traffic load of the nodes that performs very well; and 5) at low transmit power, a simple XOR network coding without opportunistic listening can yield nonnegligible throughput gains.

We have also compared the throughput performance of single-rate and multirate systems. The throughput improvement when using two rates with respect to the case with one rate (i.e., the highest of the two) depends on the network topology and node transmit power, but is found to be not very significant.

Appendix
Statistical Test of Stability

The max-min throughput of a network is the maximum traffic rate that can be injected in each source such that the network remains stable. We consider that a network is stable if *all* its queues are stable. The problem is then to estimate whether a queue is stable for a given load. This is a complex problem for which we do not have a rigorous solution. Instead, we use a simple statistical test that can be justified as follows.

The test is based on the behavior of M/M/1/K queues (note that the same argument can be done using M/D/1/K queues). Note that the loss probability P_K in an M/M/1/K is given by

$$P_K = \left(\frac{1-\rho}{1-\rho^{K+1}}\right)\rho^K \tag{40}$$

where ρ is the server utilization. When K is large, if $\rho < 1$, we have $P_K \simeq (1-\rho)\rho^K$, which is the standard formula for the M/M/1/∞ queue. This value approaches zero rather quickly as K gets large, so that the loss probability is very small unless ρ is very close to 1.

If $\rho > 1$, we have for a large K that $P_K \simeq (\rho - 1)/\rho$, which is a pure fluid model. If $\rho = 1$, we get $P_K = \frac{1}{K+1}$. In other words, the buffer loss probability is a very powerful test for the stability of a queue. It gets close to 0 very quickly when $\rho < 1$ and increases reasonably fast when $\rho > 1$.

To determine the stability of a network for a particular source rate, we consider that the buffer size of each queue is K instead of infinity, and assume that the system is unstable if P_K of any queue exceeds $1/(K+1)$. By increasing the source rate from a low value in several steps and checking the stability of each queue at each step by simulation, the maximum source rate yielding stability of all queues can be determined for a given network configuration.

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