Analysis of a Cooperative Strategy for a Large Decentralized Wireless Network

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Abstract—This paper investigates the benefits of cooperation and proposes a relay activation strategy for a large wireless network with multiple transmitters. In this framework, some nodes cooperate with a nearby node that acts as a relay, using the decode-and-forward protocol, and others use direct transmission. The network is modeled as an independently marked Poisson point process, and the source nodes may choose their relays from the set of inactive nodes. Although cooperation can potentially lead to significant improvements in the performance of a communication pair, relaying causes additional interference in the network, increasing the average noise that other nodes see. We investigate how source nodes should balance cooperation versus interference to obtain reliable transmissions, and for this purpose, we study and optimize a relay activation strategy with respect to the outage probability. Surprisingly, in the high reliability regime, the optimized strategy consists on the activation of all the relays or none at all, depending on network parameters. We provide a simple closed-form expression that indicates when the relays should be active, and we introduce closed-form expressions that quantify the performance gains of this scheme with respect to a network that only uses direct transmission.

Index Terms—Cooperative communication, decode and forward (DF), interference, network management, outage probability (OP), Poisson point process (PPP), stochastic geometry.

I. INTRODUCTION

COOPERATIVE wireless networks in which relay nodes can be used to increase throughput and reliability have been studied in the past [1]. Although the capacity of the single-relay channel [2] remains unsolved and its optimal coding scheme unknown, there have been significant advances in quantifying the performance gain obtained through cooperation. However, finding capacity regions or analyzing the performance of large random wireless networks may be, if feasible, very hard. As an alternative, spatial models employing tools from stochastic geometry and graph theory provide a comprehensive framework for the analysis of large wireless networks with little interference management [3], [4].

The *outage probability* (OP) and its complement, the *success probability*, are useful metrics in decentralized networks [3]–[5] in which the users are assumed to be unaware of the instantaneous parameters of the network and cannot optimize their behavior to attain successful transmissions. Among other reasons, the relevance of the OP comes from the fact that, in an outage event, sent messages cannot be successfully transmitted, and hence the overall delay of the network is increased due to retransmissions. In this paper, we investigate the performance, in terms of OP, of a large decentralized wireless network in which transmitters may be aided by nearby relays. More precisely, we consider a network formed by two types of clusters: source–relay–destination clusters, which use the full-duplex *decode-and-forward* (DF) [2] scheme, and clusters with source–destination pairs that employ simple *direct transmission* (DT). These clusters could be interpreted as a single hop in a multihop transmission scheme or by themselves as single-hop communications. One of the central motivations behind this analysis is to provide an understanding of the limitations and benefits of cooperation in such decentralized scenarios. In fact, the advantage of cooperation among nodes for an individual source–destination link was widely studied in the past years, addressing both theoretical and practical issues [1], [6], [7]. In this paper, we analyze a scenario in which the communication impairments are caused by a network of users that are also attempting to achieve successful transmissions through cooperation and cause interference to each other. It is clear that relays can significantly improve the rate and reliability of a single source–destination pair. However, in a large wireless network, the nodes will observe an increase in their interference levels as more relays are activated. This means that while cooperation may be beneficial locally, globally its benefits may be reduced.

In this paper, the network is modeled as an independently marked homogeneous *Poisson point process* (PPP) [8], limited by the *signal-to-interference ratio* (SIR), where signal attenuation occurs both through path loss and slow fading [9]. The random distribution of the sources and their relays implies that, in addition to the random fading, averaging over all spatial positions is needed to derive the OP. We focus on the high reliability regime as defined in [5]. In that work, the authors study the outage behavior of general motion-invariant networks employing DT by resorting to an asymptotic analysis in which the density of interfering nodes goes to zero. In particular, they show that the OP using an arbitrary medium access scheme approaches $1 - \gamma \lambda_s^\kappa$ as the density of interferers $\lambda_s \to 0$, where
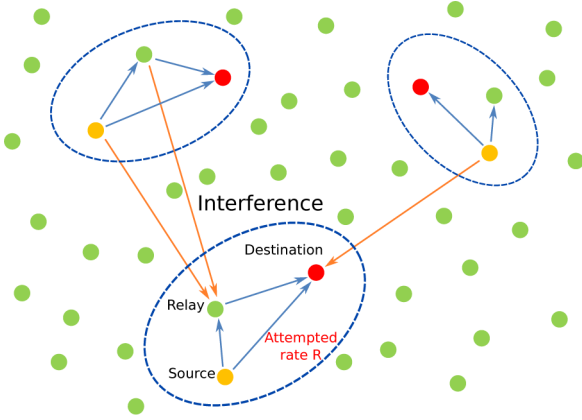
Fig. 1. The network is formed of clusters employing decode-and-forward or direct transmission.
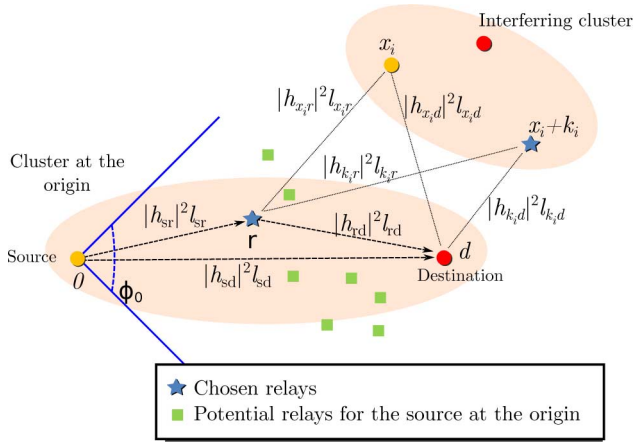

Fig. 2. The relay is chosen as the nearest neighbor of the source on a cone of aperture ϕ_0 with its axis aligned with the destination. Also, the power fading within the cluster at the origin and for the interference from other clusters are shown.

γ is the *spatial contention* parameter and κ is the *interference scaling exponent*. For the case of networks using the ALOHA access scheme, we have that $\kappa = 1$. The *high reliability regime* as defined in [5] refers to the operating regime in which the OP is small enough (close to zero) to guarantee that the asymptotic first-order approximation is a good representation of the network performance. This regime covers OP values of the order of 0.01, which are typical in wireless system designs [9]. An outage event is declared whenever the distribution of nodes and/or fading cause the chosen rate to be higher than the achievable rate of the transmission protocol of choice. Hence, the probability of these events (OP) is an upper bound on the asymptotic packet error probability of every pair of communicating nodes, which is a key metric of interest [11].

The transmission scheme of the network is a mixed one since some clusters will be using the DF scheme while others will employ DT (see Fig. 1). It is assumed that almost no *channel-state information* (CSI) is available at the transmitting nodes, which is often the case in decentralized wireless networks without feedback. Only a rough estimation of the position of nearby potential relays may be available, and hence it can be used for the relay selection. We assume that each source chooses its potential relay among the nodes that are not transmitting as its *nearest neighbor* (NN) on a cone with aperture angle ϕ_0, centered toward its destination (see Fig. 2). This scheme will increase the likelihood of finding a relay which is close to the source and at the same

time reduces the effect of the path loss on the relay–destination link. Notice that this effect is minimized if the relay, source and destination are aligned. As a special case of this scenario, the relay can be chosen as the NN of the source on the whole plane, requiring the least amount of CSI. The motivation behind choosing the NN as a relay comes from the fact that decode-and-forward is nearly optimal from an information theoretic point of view [1], [2], [12] when the relay is not far from the source. In this case, the probability of the relay not being able to decode the source message is minimized. A simple random relay activation scheme is introduced in which each candidate relay node decides whether to be active or not in a random manner, independently of each other, and of all network parameters. This simple activation scheme will act as a means of controlling the relay density in the network while still retaining a balance between interference generation and cooperation.

A. Related Work

Over the past years, the performance gains of cooperative communications in relay networks were widely studied from an information-theoretic perspective. Since the seminal work of Cover and El Gamal [2], several contributions have been published on the subject. More recently, the emphasis has been put on studying the performance of wireless relay channels where outage performance and ergodic rates of fading channels with Gaussian noise have been derived (see [1], [6], [7], [13], and the references therein). Among these valuable studies, the only impairments to the communication were due to additive Gaussian noise and fading, and very little attention was paid to the effect of the interference generated (or suffered) by other users. However, interference is probably the major impairment in wireless networks, especially in networks with little control and high mobility.

The study of the capacity of general wireless networks taking into account the interference generated by the different users was pioneered by the seminal work of Gupta and Kumar [14], where the concept of transport capacity and fundamental scaling laws on the network throughput were obtained considering only point-to-point coding. In [15], multiuser achievability regions were obtained, and it was shown that for some special wireless networks, significantly better scaling laws on the network throughput, with respect to the case in [14], are possible. Further progress was done in [16], where new scaling laws were derived using coherent multistage relaying with interference subtraction, and in [17], where extensions to fading channels were obtained.

Stochastic geometry and point processes [18], [19] are not only elegant mathematical frameworks, but also useful tools to deal with more realistic network models, where the spatial position of nodes and the effect of interference can be incorporated in a probabilistic manner [3]. Although several types of point processes can be used to model different kind of networks, it is the homogeneous PPP that has received the most attention. Although other types of point processes could provide more realistic models [20], the extended use of the homogeneous PPP comes from the possibility to obtain simple closed-form results in several cases of interest. The quantity called transmission capacity (TC) was introduced in [21] in order to include outage probability constraints in the scaling behavior. Several results have been obtained, through the use of the TC, for several practical situations, as multiple-input–multiple-output capable users

in wireless networks [22], decentralized power control [23], etc. (for a review of several other important results, please see [4] and the references therein).

B. Main Contributions

The main contribution of this paper is studying and optimizing the network management strategy for activating the relays in the proposed transmission scheme. The main conclusion is that in the high reliability regime, this optimized scheme consists on turning all the relays on or off simultaneously—that is, the optimal relay activation probability is either 0 or 1. To do this, we develop closed-form approximations for the OP of the network and study the interference–cooperation balance by finding the relay activation probability that minimizes the average OP. Moreover, the network parameter regions in which all the relays should be on are identified, and a simple relay activation scheme that is close to the optimal behavior is introduced. Finally, we provide simple expressions that quantify the performance gains in terms of OP for the scheme with the optimal relay activation scheme with respect to a network in which all users employ DT.

The paper is organized as follows. In Section II, a general and a mathematical description of the network model are presented. We also discuss the DF scheme and its achievable rate in the assumed network model. In Section III, we introduce an expression for the OP for this network, deriving closed-form approximations to it. In Section IV, we study the performance of the network, finding the optimal relay activation probability, identifying the network parameters for which all the relays should be on or off, introducing the relay activation policy and comparing the performance of this scheme against DT. In Section V, we present some numerical simulations, and in Section VI, we provide some concluding remarks. Finally, long mathematical proofs are grouped together by section and deferred to the appendixes.

C. Notation

$\mathbb{R}$, $\mathbb{C}$, $\mathbb{R}^2$, and $\|\cdot\|$ denote the real and complex numbers, the real plane, and the canonical euclidean norm, respectively. $(\cdot)^*$ and $\Re(\cdot)$ denote complex conjugation and the real part of complex number. $\mathbb{E}_X[\cdot]$ denotes expectation with respect to the random variable X. We shall use the big O notation: $f(x) = O(g(x))$ as $x \to x_0$ if there exists $M > 0$ and such that $|f(x)| \leq M|g(x)|$ is some neighborhood of x_0. Finally, $\mathbb{1}(x \in A)$ denotes the indicator function, which is 1 if $x \in A$, and 0 otherwise.

II. GENERAL CONSIDERATIONS AND NETWORK MODEL

A. Model

We consider a spatial network model in $\mathbb{R}^2$ in which source nodes generate messages and attempt to transmit them to intended destinations, either through a direct link, in which case the destinations receive symbols from their sources only, or by using others nodes as relays. Every relay aids a single source node, acting as a secondary full-duplex transmitter sharing the same time-slots and frequency band. This setup allows the nodes to be grouped into *clusters* formed by a source–destination pair or by a source–relay–destination triplet, if the source has an associated relay, as shown in Fig. 1.

We start from a set of nodes Φ that we assume forms a homogeneous PPP of density λ. Some nodes from this set choose to access the network and become sources using slotted ALOHA [3] with transmit probability λ_s/λ. This splits the set Φ into two new independent homogeneous PPPs:
- Φ_s of sources of density λ_s;
- Φ_{in} of potential relay nodes of density $\lambda_{in} = \lambda - \lambda_s$;

such that $\Phi = \Phi_s \cup \Phi_{in}$. Notice that the proportion of sources and potential relays can be adjusted by the medium access probability.

Inactive nodes should then be assigned in a one-to-one fashion to each source such that cooperation is beneficial. To simplify the relay assignment strategy, we shall assume that the spatial density of the sources is much smaller than that of potential relays, i.e., $\lambda_s \ll \lambda_{in}$. Under this hypothesis, we will neglect the probability of two sources choosing the same inactive node as a relay since each source will have a rich selection of relay candidates in its vicinity ([23, Ex. 3]). Thus, we can simplify our model by including the position of the potential relay and its activation scheme as an independent mark to each source, obtaining the spatial distribution of the relay from the original homogeneous PPP Φ_{in} of intensity λ_{in} to which the relays are assumed to belong.

We consider the usual and realistic assumption that only little or no CSI is available, while nodes may have some estimation of the spatial position of neighboring nodes. For this reason, nodes cannot adjust their rates to achieve a reliable communication according to instantaneous conditions, but may use this spatial knowledge to select a relay.

Based on these considerations, the network is modeled as an independently marked PPP

$$\tilde{\Phi}_s = \{(x_i, (\varepsilon_{x_i}, k_i, \theta_{x_i}), h_{x_i r}, h_{x_i d}, h_{k_i r}, h_{k_i d},)\} \quad (1)$$

such that we have the following.
- The positions of the sources form the homogeneous PPP $\Phi_s = \{x_i\}$ of intensity λ_s.
- The triplet $(\varepsilon_{x_i}, k_i, \theta_{x_i})$ models the relay position and its state. The random variable (RV) θ_{x_i}, uniform in $[0, 2\pi)$, models the direction of each destination relative to its source, with $\theta = 0$ meaning that the destination is in the direction of the canonical vector $(1, 0)$ with respect to its source. k_i indicates the position of the potential relay relative to its source—that is, the potential relay for source x_i is located at $x_i + k_i$. According to what we mentioned earlier, the relay will be chosen as the NN of the source on a cone of aperture ϕ_0 with the destination on its axis. This means that the distribution of the potential relay k_i for a source at the origin conditioned on the direction of the destination θ_{x_i} will be (in polar coordinates) [24]

$$f_{k_i|\theta}(\rho, \phi) = \lambda_{in}\rho e^{-\lambda_{in}\phi_0\rho^2/2}\mathbb{1}\{|\phi - \theta| < \phi_0/2\}$$
$$\times \mathbb{1}\{0 \leq \phi \leq 2\pi, \rho \geq 0\}. \quad (2)$$

Using $\phi_0 = 2\pi$ means choosing the relay as the NN on the whole plane instead of a cone, independently of the direction of the destination and using the least CSI. Notice that in this case the distribution of the NN (2) becomes a bidimensional Gaussian RV of variance

$$\sigma_{in}^2 = \frac{1}{2\pi\lambda_{in}}. \quad (3)$$

Notice that we can parameterize the NN distribution (2) in terms of σ_{in} and for any cone aperture ϕ_0 as

$$f_{k_i|\theta}(\rho,\phi) = \frac{1}{2\sigma_{\text{in}}^2}\rho e^{-\frac{\phi_0 \rho^2}{4\pi\sigma_{\text{in}}^2}} \mathbb{1}\{|\phi - \theta| < \phi_0/2\}$$
$$\times \mathbb{1}\{0 \le \phi \le 2\pi, \rho \ge 0\}. \quad (4)$$

This implies that the effect of considering the nearest neighbor on a cone is simply restricting the NN on the plane distribution (Gaussian) to the cone and increasing the variance (by means of the ϕ_0 in the exponent). Thus, we can study the relay activation strategy in terms of the variance of the nearest neighbor on the whole plane (σ_{in}) and cone aperture ϕ_0 that the source uses. Additionally, notice that reducing the cone aperture allows the relay to be located toward the destination, but at the same time, the increased variance implies that the relay will be, on average, farther from the source than if we take $\phi_0 = 2\pi$. The RV ε_{x_i} indicates if the corresponding source uses a relay or not. In our case, we take it to be a *Bernoulli* RV with success probability p_{r}, independent of everything else. Notice that the parameter p_{r} allows the adjustment of the relay density and hence allows to control the additional interference introduced in the network, weighing the local and global effects of cooperation. In addition to a MAC access scheme, this parameter can also be used to model the unavailability of a relay for reasons that are out of the control of the relay itself (such as a malfunction or a depleted battery). In such a case, the independent occurrence of these events among the relays is a reasonable assumption.

- All nodes transmit with unit power, while the power received at y by a transmitter located at x is $|h_{xy}|^2 l_{xy}$, where $l_{xy} = \|x - y\|^{-\alpha}(\alpha > 2)$ is the usual path loss function and $|h_{xy}|^2$ is the power gain of Rayleigh fading with unit mean. This is equivalent to saying that h_{xy} are complex, circular [25], zero-mean Gaussian RVs.

- An additional source with the same marks as the others, independent of the point process $\tilde{\Phi}_{\text{s}}$ and with its destination at $d = (D,0)$, is added at the origin. The position of the relay for this source node will be r (with the same distribution as the $\{k_i\}$ RVs). The coefficients $|h_{\text{sr}}|^2$, $|h_{\text{rd}}|^2$ and $|h_{\text{sd}}|^2$ model the source–relay, relay–destination, and source–destination fading coefficients of this cluster, respectively.
  Slyvniak's Theorem [17], [18] guarantees that the study of this cluster's behavior will be representative of the behavior of any other similar cluster in the network, and hence it can be considered as a "typical cluster."

- $h_{x_i r}$ and $h_{k_i r}$ model the fading gains from each source and its relay to the relay of the source at the origin, while $h_{x_i d}$, $h_{k_i d}$ model the gains from each source and its relay to the destination of the source at the origin.

Please see Fig. 2 for a graphical representation of the key parameters in the model.

Remark 2.1: Notice that other schemes for activating and selecting the relays based on position can be studied by appropriately selecting the triplet $(\varepsilon_{x_i}, k_i, \theta_{x_i})$. For example, we could study the performance of choosing the nearest or the farthest neighbor on a cone of finite radius as a relay. The probability of activating a relay will be that of finding at least one potential

relay in the cone and the conditional distributions of the selected relay, given that the cone is not empty, can be found in [26].

It is assumed that during the transmission time all the positions of the nodes, fading coefficients and other network parameters encompassed in the marked PPP $\tilde{\Phi}_{\text{s}}$ remain constant—that is, there is no node mobility and a slow fading scenario is considered. Within a cluster, each source and its relay (if it is active) use Gaussian signaling, and their codebooks have correlation coefficient ρ. In addition, the codebooks between different clusters are independent. Destination and relay nodes in each cluster attempt to decode their messages while treating the interference from other clusters as noise. With these hypotheses, the following lemma can be proved.

Lemma 2.1: If $\alpha > 2$, then for almost all realizations of the point process $\tilde{\Phi}_{\text{s}}$, the aggregate interferences at the relay and destination of the typical cluster are zero-mean complex circular Gaussian variables whose variances conditioned on $\tilde{\Phi}_{\text{s}}$ are given by

$$I_{\text{r}} = \sum_{i:x_i \in \Phi_{\text{s}}} \left[\frac{|h_{x_i r}|^2}{\|x_i - r\|^\alpha} \right.$$
$$\left. + \varepsilon_{x_i}\left(\frac{|h_{k_i r}|^2}{\|x_i + k_i - r\|^\alpha} + \frac{2\Re\left\{h_{x_i r} h_{k_i r}^* \rho\right\}}{\|x_i - r\|^{\frac{\alpha}{2}}\|x_i + k_i - r\|^{\frac{\alpha}{2}}} \right) \right] \quad (5)$$

$$I_{\text{d}} = \sum_{i:x_i \in \Phi_{\text{s}}} \left[\frac{|h_{x_i d}|^2}{\|x_i - d\|^\alpha} \right.$$
$$\left. + \varepsilon_{x_i}\left(\frac{|h_{k_i d}|^2}{\|x_i + k_i - d\|^\alpha} + \frac{2\Re\left\{h_{x_i d} h_{k_i d}^* \rho\right\}}{\|x_i - d\|^{\frac{\alpha}{2}}\|x_i + k_i - d\|^{\frac{\alpha}{2}}} \right) \right].$$
$$(6)$$

Proof: See Appendix B. ∎

B. Achievable Rates

The main coding strategies for relay networks were introduced in the seminal work by Cover and El Gamal [2]. There have been three dominant relaying paradigms: DF, compress-and-forward (CF), and amplify-and-forward (AF). In AF, the relay simply repeats an amplified version of what it received without decoding the message. In CF, the relay chooses an appropriate sequence from a set that acts as a compressed version of what it received. In DF, the relay decodes the messages sent by the source, reencodes it, and forwards it to the destination, which decodes the message by using both the transmission from the source and from the relay. In general, DF will work best when the source–relay channel is good enough to avoid a bottleneck in the information flow with respect to a source–destination transmission. In a scenario in which the spatial distribution of nodes is considered, the quality of the source–destination and source–relay channels will be heavily influenced, through the path loss, by the distances between the nodes. This means that in order to avoid this bottleneck, the relay should be chosen so that on average the source–relay distance is smaller than the source–destination distance. Other variations of DF such as partial-decode-and-forward [2] relax the imposition of full-decoding at the relay. However, they require a careful optimization of the code at the encoder, which cannot be done in our setting due to the lack of CSI at the source.

There are several encoding and decoding techniques that implement the DF scheme, all of which are based on block-Markov

encoding. For our analysis of the error events, we consider either *regular encoding* and *sliding-window decoding* [28] at the destination or *regular encoding* and *backward decoding* [29], [30].[1] With any of these two schemes, conditioned on a particular realization of $\tilde{\Phi}_s$, the relay channel associated to the source located at the origin can achieve a rate [2]

$$R_{\mathrm{DF}} = \max_{\rho \in \mathbb{C}, |\rho| \leq 1} \min \left\{ \mathcal{C}\left(\frac{|h_{\mathrm{sr}}|^2 l_{\mathrm{sr}}(1 - |\rho|^2)}{I_{\mathrm{r}}} \right), \right.$$
$$\left. \mathcal{C}\left(\frac{|h_{\mathrm{sd}}|^2 l_{\mathrm{sd}} + |h_{\mathrm{rd}}|^2 l_{\mathrm{rd}} + 2\sqrt{l_{\mathrm{sd}} l_{\mathrm{rd}}} \Re\left(\rho h_{\mathrm{sd}} h_{\mathrm{rd}}^*\right)}{I_{\mathrm{d}}} \right) \right\} \quad (7)$$

where $\mathcal{C}(u) = \log_2(1 + u)$. The maximization with respect to ρ is considered because the correlation of the codebooks within a cluster affects the interference seen by other nodes and also the achievable rates for each cluster. This means that in general the value of ρ that maximizes the achievable rate could be selected [1], [2]. In this work, we shall consider the case $\rho = 0$, which simplifies the implementation of DF, as pointed out in [1, Remark 42] and [7], because symbol synchronization between the source and its corresponding relay is not strictly required. Although other choices of ρ could improve the outage behavior of the network, $\rho = 0$ is known to be the optimal value in the high reliability regime for a network in which only one source is allowed to use a relay [12]. Therefore, when the relay is present, we can define the outage event $\mathcal{A}(R) \cup \mathcal{B}(R)$ as

$$\mathcal{A}(R) = \left\{ |h_{\mathrm{sr}}|^2 l_{\mathrm{sr}} < T I_{\mathrm{r}} \right\}$$
$$\mathcal{B}(R) = \left\{ |h_{\mathrm{sd}}|^2 l_{\mathrm{sd}} + |h_{\mathrm{rd}}|^2 l_{\mathrm{rd}} < T I_{\mathrm{d}} \right\}$$

where R is the attempted rate by the source and $T = 2^R - 1$. The event $\mathcal{A}(R)$ means that the relay is in outage, while $\mathcal{B}(R)$ means that the destination is in outage while source and relay cooperate.

The DF scheme with backward or sliding-window decoding at the destination are *oblivious* [13] to the presence of the relay—that is, the source can use the same coding scheme for DF or DT without considering if the relay is present or not. This is very important since the relay can decide to activate itself (achieving the DF rate) or not (achieving the DT rate) without taking into account the source, which in both cases employs the same coding scheme. Only the destination knows if the relay is present and can adapt its decoding strategy suitably according to each case. Also, the rate R_{DF} does not depend on the correlation between the noises or interferences at the relay and the destination. This is true because the correlation between received signals at the relay and the destination becomes irrelevant when full decoding at the relay is imposed. As a matter of fact, this is not the case for the CF and AF schemes where the correlation between the noises can increase or decrease the corresponding achievable rate [31].

Finally, we also define the outage event for the case in which there is no relay, and thus the source simply uses DT [3]

$$\mathcal{A}_{\mathrm{DT}}(R) = \left\{ \frac{|h_{\mathrm{sd}}|^2 l_{\mathrm{sd}}}{I_{\mathrm{d}}} < T \right\}. \quad (8)$$

[1]Another alternative using *irregular encoding*, *random binning*, and *successive decoding* at the destination was introduced in [2], but it is not suited for our analysis since additional error events have to be considered.

The probability of this event is known to be [3] $\mathbb{P}_{\mathrm{out},DT}(R) = 1 - e^{-\lambda_s \delta D^2}$, where

$$\delta = CT^{2/\alpha} \quad (9)$$
$$C = \frac{2\pi}{\alpha} \Gamma\left(\frac{2}{\alpha}\right) \Gamma\left(1 - \frac{2}{\alpha}\right) \quad (10)$$

and $\Gamma(u) = \int_0^\infty t^{u-1} e^{-t} dt$ is the Gamma function. Using the asymptotic expansion of the OP, we can write $\mathbb{P}_{\mathrm{out},DT}(R) = \lambda_s \delta D^2 + O((\lambda_s \delta D^2)^2)$ as $(\lambda_s \delta D^2)^2 \to 0$. In the high reliability regime, when the success probability of the network is close to one, a reasonable approximation is to neglect the higher-order $O(\cdot)$ term and write $\mathbb{P}_{\mathrm{out},DT}(R) \approx \lambda_s \delta D^2$, meaning that the approximation will be good and that $\lambda_s \delta D^2$ will be small. In this expansion, we see that $\gamma = \delta D^2$ is the contention parameter of the network, as defined in Section I.

III. OUTAGE PROBABILITY OF THE NETWORK

In this section, we study the OP of the network as introduced in Section II. By conditioning on the fact that the cluster at the origin uses a relay or not, and on this relay position, we can see that the OP of the cluster at the origin (and hence of any other cluster) can be written as

$$\mathbb{P}_{\mathrm{out},\mathrm{mix}}(R) = \mathbb{P}\{\varepsilon_0 = 0\} \mathbb{P}\{\mathcal{A}_{\mathrm{DT}}(R) | \varepsilon_0 = 0\}$$
$$+ \mathbb{P}\{\varepsilon_0 = 1\} \mathbb{E}_r \left[\mathbb{P}\{\mathcal{A}(R) \cup \mathcal{B}(R) | r, \varepsilon_0 = 1\} \right]. \quad (11)$$

This expression can be evaluated in terms of the Laplace transform of the interference random variables I_r and I_d, as the following theorem states.

Theorem 3.1: The outage probability of the network $\mathbb{P}_{\mathrm{out},\mathrm{mix}}$ given by (11) can be written as.

$$\mathbb{P}_{\mathrm{out},\mathrm{mix}}(R) = \mathbb{P}\{\varepsilon_0 = 0\}\left[1 - \mathcal{L}_{I_d}\left(T/l_{\mathrm{sd}}\right)\right] + \mathbb{P}\{\varepsilon_0 = 1\}$$
$$\times \mathbb{E}_r \left[\frac{D^\alpha \mathcal{L}_{I_d, I_r}\left(\frac{T}{l_{\mathrm{rd}}}, \frac{T}{l_{\mathrm{sr}}}\right) - \|r-d\|^\alpha \mathcal{L}_{I_d, I_r}\left(\frac{T}{l_{\mathrm{sd}}}, \frac{T}{l_{\mathrm{sr}}}\right)}{D^\alpha - \|r-d\|^\alpha} \right]$$
$$(12)$$

where

$$\mathcal{L}_{I_d, I_r}(\omega_1, \omega_2) := \mathbb{E}_{\tilde{\Phi}_s}\left[e^{-(\omega_1 I_d + \omega_2 I_r)} \right], \quad \omega_1, \omega_2 \in \mathbb{C} \quad (13)$$

with $\Re\{\omega_1\}, \Re\{\omega_1\} > 0$ is the joint Laplace transform of the interference at the relay and at the destination. Additionally, setting $\omega_2 = 0$ in (13), we obtain $\mathcal{L}_{I_d}(\omega_1)$, the Laplace transform of the interference at the destination.

Proof: See Appendix C. ∎

The Laplace transforms of interference RVs are known in closed form in some special cases only, and in general they can only be expressed in terms of integrals in several dimensions (see [32] and the references therein). A brief review on them can be found in Appendix A. Now, using (35) the two-dimensional Laplace transform, $\mathcal{L}_{I_d, I_r}(\omega_1, \omega_2)$ can be evaluated as

$$\mathcal{L}_{I_d, I_r}(\omega_1, \omega_2) = \exp\left\{-\lambda_s p_r t(\omega_1, \omega_2, r, d)\right\}$$
$$\times \exp\left\{-\lambda_s(1 - p_r)\left[C(\omega_1^{2/\alpha} + \omega_2^{2/\alpha}) + f(\omega_1, \omega_2)\right]\right\}$$
$$(14)$$

where
$$f(\omega_1, \omega_2) = \int_{\mathbb{R}^2} \frac{\omega_1 \omega_2}{(\omega_1 + \|x - d\|^\alpha)(\omega_2 + \|x - r\|^\alpha)} dx$$
$$t(\omega_1, \omega_2, r, d) = \int_{\mathbb{R}^2} \mathbb{E}_k \left[1 - z(\omega_1, x, k, d) z(\omega_2, x, k, r) \right] dx$$

and C comes from (10). The expectation is with respect to the distribution k of the relay and $z(\omega, x, k, d)$ is given by

$$z(\omega, x, k, d) = \frac{1}{1 + \omega\|x - d\|^{-\alpha} + \omega\|k - d\|^{-\alpha}}. \quad (15)$$

For $z(\omega, x, k, r)$, a similar expression holds interchanging d with r. The complexity of these expressions is due mainly to the interferences (5) and (6), and it precludes closed-form computations. For this reason, we introduce the following far-field approximation for the path loss of the interfering clusters: The users within a cluster see the interference from other clusters as a point source of interference, meaning that

$$\|x_i - r\| \approx \|x_i + k_i - r\| \approx \|x_i + \tau k_i - r\| \quad (16)$$
$$\|x_i - d\| \approx \|x_i + k_i - d\| \approx \|x_i + \tau k_i - d\|. \quad (17)$$

The parameter τ allows to establish the far-field approximation using any point between each source and its relay. As we shall see, the results obtained are the same independently of its value. With this assumption, a single path loss will appear in the interferences, so (5) and (6) can be simplified as

$$\tilde{I}_{\mathrm{r}} = \sum_{i:x_i \in \Phi_{\mathrm{s}}} \frac{|h_{x_i r}|^2 + \varepsilon_{x_i} \left(|h_{k_i r}|^2 + 2\Re\{h_{x_i r} h_{k_i r}^* \rho\} \right)}{\|x_i + \tau k_i - r\|^\alpha} \quad (18)$$

$$\tilde{I}_{\mathrm{d}} = \sum_{i:x_i \in \Phi_{\mathrm{s}}} \frac{|h_{x_i d}|^2 + \varepsilon_{x_i} \left(|h_{k_i d}|^2 + 2\Re\{h_{x_i d} h_{k_i d}^* \rho\} \right)}{\|x_i + \tau k_i - d\|^\alpha}. \quad (19)$$

This approximation will be very good in the high reliability regime because the independent fading coefficients are conserved and the large-scale effect of path loss is still taken into account.

With these new interference expressions, we upper-bound the OP of the network by introducing the union bound on the outage events of DF

$$\mathbb{P}_{\mathrm{out,mix}}(R) \leq \mathbb{P}\{\varepsilon_0 = 0\} \mathbb{P}\{\mathcal{A}_{\mathrm{DT}}(R)|\varepsilon_0 = 0\} + \mathbb{P}\{\varepsilon_0 = 1\}$$
$$\times \mathbb{E}_{\mathrm{r}} \Big[\mathbb{P}\{\mathcal{A}(R)|r, \varepsilon_0 = 1\} + \mathbb{P}\{\mathcal{B}(R)|r, \varepsilon_0 = 1\} |\varepsilon_0 = 1 \Big]. \quad (20)$$

This will be a good approximation when the relay is not too far away from the source since, in that case, the event $\mathcal{B}(R)$ will be dominant and $\mathcal{A}(R)$ will have a relatively small probability of occurrence. Using the simplified interferences, we can evaluate this upper bound in closed form.

Theorem 3.2: The OP upper bound (20) for this network can be evaluated as

$$\mathbb{P}_{\mathrm{out,mix}}(R) \leq (1 - p_{\mathrm{r}}) \left[1 - e^{-\lambda_{\mathrm{s}}\Delta(p_{\mathrm{r}})D^2} \right]$$
$$+ p_{\mathrm{r}} \Bigg\{ 2 - \mathbb{E}_{\mathrm{r}} \left[e^{-\lambda_{\mathrm{s}}\Delta(p_{\mathrm{r}})\|r\|^2} \right] - e^{-\lambda_{\mathrm{s}}\Delta(p_{\mathrm{r}})D^2}$$
$$\times \left[1 + \lambda_{\mathrm{s}}\Delta(p_{\mathrm{r}})D^2 \left(1 + \frac{2 - \alpha}{\alpha D} \mathbb{E}_{\mathrm{r}}[\|r - d\|] \right) \right]$$
$$+ O\left((\lambda_{\mathrm{s}}\Delta(p_{\mathrm{r}})D^2)^2 \right) \Bigg\} \quad (21)$$

as $\lambda_{\mathrm{s}}\Delta(p_{\mathrm{r}})D^2 \equiv \lambda_{\mathrm{s}}\delta \left(1 + \frac{2p_{\mathrm{r}}}{\alpha} \right) D^2 \to 0$, with

$$\mathbb{E}_{\mathrm{r}} \left[e^{-\lambda_{\mathrm{s}}\Delta(p_{\mathrm{r}})\|r\|^2} \right] = \frac{\phi_0 \lambda_{\mathrm{in}}}{\phi_0 \lambda_{\mathrm{in}} + 2\lambda_{\mathrm{s}}\Delta(p_{\mathrm{r}})}. \quad (22)$$

When $\phi_0 = 2\pi$, we have a closed-form expression for the expectation

$$\mathbb{E}_{\mathrm{r}} [\|r - d\|] = \sigma_{\mathrm{in}} Q_{2,0}(D/\sigma_{\mathrm{in}}, 0) \quad (23)$$

where $Q_{2,0}$ is the $(2, 0)$ Nuttall Q-function [33]

$$Q_{2,0}(s, 0) = \sqrt{\frac{\pi}{8}} e^{-\frac{s^2}{4}} \left((s^2 + 2) I_0 \left(\frac{s^2}{4} \right) + s^2 I_1 \left(\frac{s^2}{4} \right) \right)$$

where I_0 and I_1 are the modified Bessel functions of the first kind of orders 0 and 1. In the general case, we have to find the expectation numerically, or we may use the following upper bound:

$$\mathbb{E}_{\mathrm{r}} [\|r - d\|] \leq D(1 + s\gamma(s, \phi_0)) \quad (24)$$

with $s = (\lambda_{\mathrm{in}}\phi_0 D^2)^{-1/2}$ and

$$\gamma(s, \phi_0) = \sqrt{\frac{\pi}{2}} \left\{ 1 + \left[\frac{8(1 - \cos(\phi_0/4))}{\phi_0} - 2 \right] \mathrm{erf} \left(\frac{1}{\sqrt{2}s} \right) \right\}.$$

■

Proof: See Appendix D.

IV. OPTIMAL RELAY ACTIVATION PROBABILITY

In Section III, we established an upper bound on the OP of the network choosing the relay as the NN in a cone, as a function of the relay activation probability p_{r} and the cone aperture ϕ_0. For a given network setup $(R, \alpha, \lambda_{\mathrm{s}}, \sigma_{\mathrm{in}})$, different values of p_{r} and ϕ_0 will yield different values of the OP: Increasing p_{r} will introduce additional interference in the network, while decreasing the cone aperture ϕ_0 will increase the average source–relay distance. If there is a high density of potential relays, the cone aperture can be used to balance the average source–relay and source–destination distances to optimize the performance of the network. For this reason, we should find the optimal values of p_{r} and ϕ_0, those which result in the smallest OP for each setup. In this section, we study the optimal value of p_{r} in terms of the OP and determine the gains that can be achieved in terms of OP by optimizing this parameter.

Optimization of the relay activation probability using standard methods is very involved due to the nonlinear nature of the expression of the OP. It would be expected that an optimal relay activation probability p_{r} would exist, which would optimally balance the effect of the added interference and the gains of activating additional relays.

Theorem 4.1 (Concavity of the OP): Neglecting the term $O\left((\lambda_{\mathrm{s}}\Delta(p_{\mathrm{r}})D^2)^2 \right)$ in (21), for each network setup $(\alpha, d, \phi_0, \lambda_{\mathrm{s}}, R)$ such that $\lambda_{\mathrm{s}}\delta D^2 < 0.38$, there is an interval of σ_{in}

$$0 \leq \sigma_{\mathrm{in}} \leq \sigma_{\mathrm{c}} \quad (25)$$

such that the OP upper bound is a concave function of p_{r}.

Proof: See Appendix E.

Lemma 4.1: Given a concave function $h(x)$ in a bounded and closed interval $[x_1, x_2]$, its minimum is attained at x_1 or x_2. ∎

Proof: See [34, Theorem 32.1]. ∎

Using Lemma 4.1 together with Theorem 4.1, we conclude that the best OP performance for any cluster in the network can be attained when all $(p_r = 1)$ or none $(p_r = 0)$ of the sources decide to use their associated relays. In one case, all the clusters will be using DF, and in the other case, all of them will be using DT. This is a somewhat surprising result in the sense that, in terms of the OP, the best performance can be obtained either by full cooperation or by not cooperating at all. There is no "optimal" density of used relays in the network or optimal mixed behavior in the sense that some clusters would enjoy the advantages of cooperation while others use DT in order to balance the generated interference. This interval clearly depends on the network setup parameters. However, working in the realistic high reliability regime, we can obtain a simple approximation of this condition that depends only on basic network parameters.

Corollary 4.1: In the high reliability regime, we approximate the concavity interval of Theorem 4.1 by finding the smallest positive solution to the following equation:

$$\frac{4\pi\alpha\sigma_c^2}{\phi_0 D^2} + (\alpha - 2)\frac{\mathbb{E}_r[\|r - d\|]}{D} - \alpha = 0. \quad (26)$$

Notice that the expectation also depends on σ_c, so the equation cannot be solved in closed form. By using (24) to upper-bound the expectation, the following sufficient condition for a concave OP concave is obtained:

$$\sigma_{in} \leq D\sqrt{\frac{\phi_0}{2\pi}}\left\{\left[\frac{1}{\alpha} + \varphi_c(\phi_0, \alpha)^2\right]^{1/2} - \varphi_c(\phi_0, \alpha)\right\} \quad (27)$$

where

$$\varphi_c(\phi_0, \alpha) = \frac{1}{4}\left(1 - \frac{2}{\alpha}\right)\gamma(1/\sqrt{2}, \phi_0). \quad (28)$$

Proof: See Appendix E. ∎

So far, we have established that there is a regime in which either $p_r = 1$ or $p_r = 0$ are the values that minimize the OP for a given network setup $(\alpha, d, \phi_0, \lambda_s, R)$, and in Corollary 4.1, we have determined conditions to find that interval. Now we wish to establish conditions under which we should activate all the relays, that is, when $p_r = 1$ will be the optimal choice.

Theorem 4.2 (Optimality Region of $p_r = 1$): Neglecting the term $O\left((\lambda_s\Delta(p_r)D^2)^2\right)$ in (21), for each network setup $(\alpha, d, \phi_0, \lambda_s, R)$ such that $\lambda_s\delta D^2 < 0.38$ there is an interval of σ_{in}

$$0 \leq \sigma_{in} \leq \sigma_t \quad (29)$$

such that the OP upper bound is minimized by activating all the relays.

For the high reliability regime, an approximation for σ_t is obtained by finding the smallest positive solution of the following equation:

$$\left[1 + \frac{2}{\alpha}\right]\left[\frac{4\pi\sigma_t^2}{\phi_0 D^2} + \left(1 - \frac{2}{\alpha}\right)\frac{\mathbb{E}_r[\|r - d\|]}{D}\right] = 1. \quad (30)$$

Notice that the expectation also depends on σ_t so the equation is coupled. By using (24) to upper-bound the expectation, the following sufficient condition for $p_r = 1$ to be optimal is obtained:

$$\sigma_{in} \leq D\sqrt{\frac{\phi_0}{2\pi}}$$
$$\times\left\{\left[\frac{2}{\alpha(\alpha + 2)} + \varphi_t(\phi_0, \alpha)^2\right]^{1/2} - \varphi_t(\phi_0, \alpha)\right\} \quad (31)$$

where

$$\varphi_t(\phi_0, \alpha) = \frac{1}{4}\left(1 - \frac{2}{\alpha}\right)\gamma(1/2, \phi_0). \quad (32)$$

Proof: See Appendix F. ∎

Using the previous theorems, we are able to state a relay activation scheme that optimizes the OP in a network operating in the high reliability regime: For a given value of ϕ_0, if σ_{in} is less than the solution of (30), then all the relays should be on. Otherwise, the relays should be turned off and DT should be employed. A computationally simpler alternative for turning the relays on would be using condition (31) instead. The value of ϕ_0 could additionally be chosen within this scheme to minimize the OP. Notice that $\sigma_t \equiv \sigma_t(\phi_0)$ is a function of ϕ_0. If for a network setup $(\alpha, d, \lambda_s, R)$ there is a value of ϕ_0 such that $p_r = 1$ is optimal, i.e., $\sigma_{in} \leq \sigma_t(\phi_0)$ holds, then there will be a range of values of ϕ_0 for which this condition will hold. We should therefore choose the value of ϕ_0 for which $\sigma_{in} < \sigma_t(\phi_0)$ holds and the OP is minimized. On the other hand, if there is no value of ϕ_0 such that $\sigma_{in} < \sigma_t$, we have that $p_r = 0$ is optimal, and hence DT should be employed.

As we shall observe in Section V, there will be scenarios in which setting $\phi_0 = 2\pi$ will yield approximately the same performance as optimizing the value of ϕ_0 in terms of the OP according to the previous observation. This means that in practical scenarios, this optimization may not always be of importance and the value of σ_t can be obtained by setting $\phi_0 = 2\pi$ in (30).

Finally, we want to compare the OP that can be achieved with the scheme defined in Theorem 4.2 to the one obtained using only DT. For each value of ϕ_0, if $p_r = 1$ minimizes the OP, then the scheme will exhibit gains with respect to DT, while if $p_r = 0$ is the optimum, the scheme reverts to DT, and no gains will be seen. The following theorem finds the approximate reduction of the OP of the scheme with respect to DT.

Theorem 4.3: In the high reliability regime, the relative decrease in OP of the activation scheme obtained by using (30) is

$$\frac{\mathbb{P}_{out,mix}}{\mathbb{P}_{out,DT}} \approx \begin{cases} \left(1 + \frac{2}{\alpha}\right)\left(\frac{4\pi\sigma_{in}^2}{\phi_0 D^2} + \left(1 - \frac{2}{\alpha}\right)\frac{\mathbb{E}_r[\|r-d\|]}{D}\right), & \sigma_{in} \leq \sigma_t \\ 1, & \text{otherwise.} \end{cases} \quad (33)$$

In addition, σ_t can be lower-bounded by (31).

Proof: See Appendix E. ∎

As we mentioned before, both σ_t and the actual reduction in OP are a function of ϕ_0. If ϕ_0 can be optimized, then for each network setup $(\alpha, d, \lambda_s, R)$, we have to determine (if they exist) the values of ϕ_0 such that $\sigma_{in} \leq \sigma_t$ (which ensure a gain with respect to DT) and from those values, the one that minimizes the OP.
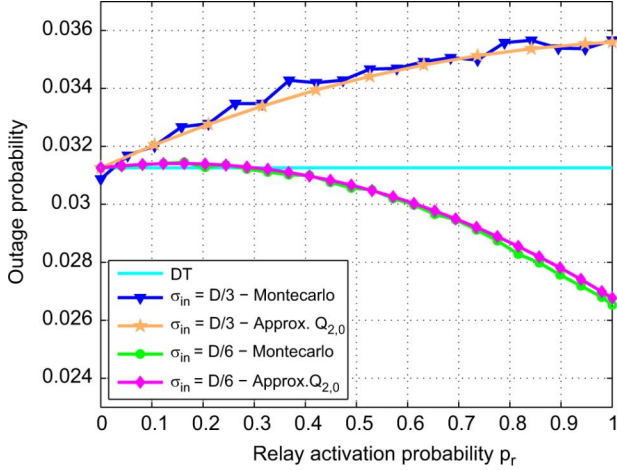
Fig. 3. Outage probability $\mathbb{P}_{\text{out,mix}}(R, p_r)$ as a function of p_r for values of σ_{in} showing optimality of $p_r = 0$ or $p_r = 1$. $d = (10, 0)$, $\lambda_s = 10^{-4}$, $R = 0.5$, $\alpha = 4$. Monte Carlo simulations are obtained by averaging 8×10^6 realizations of the PPP using (5) and (6). Approximations come from using (23) in (21).

Finally, it is interesting to mention that under certain conditions the OP is not a concave function of p_r—that is, the OP is minimized by choosing a value of p_r that is different from $p_r = 0$ or $p_r = 1$. However, in such scenarios, the network is well outside the high reliability regime and typical operating conditions.

V. NUMERICAL RESULTS

In this section, we present some simulations to study the behavior of the expressions we have introduced previously. In Fig. 3, the OP with respect to p_r is plotted for two different values of σ_{in}, one in which $p_r = 1$ is optimal and another one for which $p_r = 0$ is the optimal point, when the relay is selected as the nearest neighbor on the whole plane ($\phi_0 = 2\pi$). The theoretical expressions come from the upper bound (21) using (23), and they are compared to Monte Carlo simulations obtained by averaging 8×10^6 realizations of the PPP using the true interferences (5) and (6), taking $d = (10, 0)$, $\lambda_s = 10^{-4}$, $R = 0.5$ b/use, and $\alpha = 4$. We see that the approximations derived with the simplified interferences (18) and (19) are in excellent agreement with the actual OP derived with the more complex interferences.

In Fig. 4, we plot the optimal cone aperture ϕ_0 as a function of σ_{in}/D for different values of the path loss exponent α and for $d = (10, 0)$. To do this, we numerically find the value of ϕ_0 that maximizes the OP gain of the mixed protocol with respect to DT in (33) for each value of σ_{in}/D. It is interesting to note that as the path loss exponent decreases, the optimal cone aperture becomes $\phi_0 = 2\pi$ for a large range of values of σ_{in}/D. Only when the network of potential relays is very dense (small σ_{in}/D), a value of $\phi_0 < 2\pi$ should be chosen. This is because when the exponent diminishes, both the source–relay and the interference paths become stronger, but the effect of the increased interference is dominant. Thus, the diminished exponent creates an effect equivalent to increasing the average source–relay distance. The value of ϕ_0 must therefore become larger in order to decrease the average source–relay distance and compensate for this effect.

In Fig. 5, we study the maximum rate attainable for the on/off relaying strategy relative to the same rate of DT in percentage
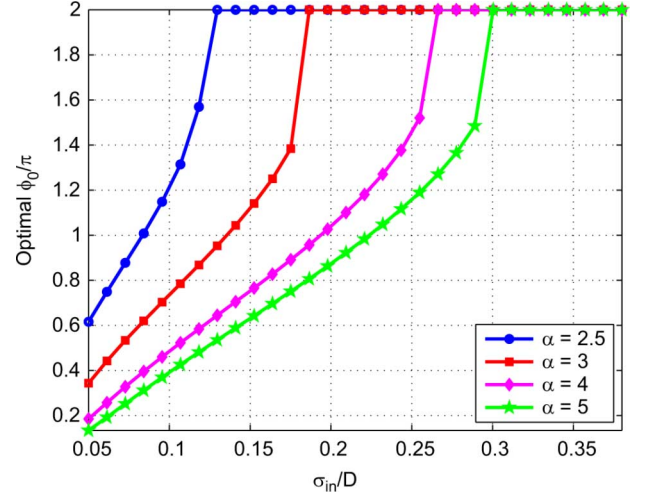

Fig. 4. Optimal cone aperture ϕ_0 as a function of σ_{in}/D obtained using (33) for different values of α. $\lambda_s = 10^{-4}$. $d = (10, 0)$.
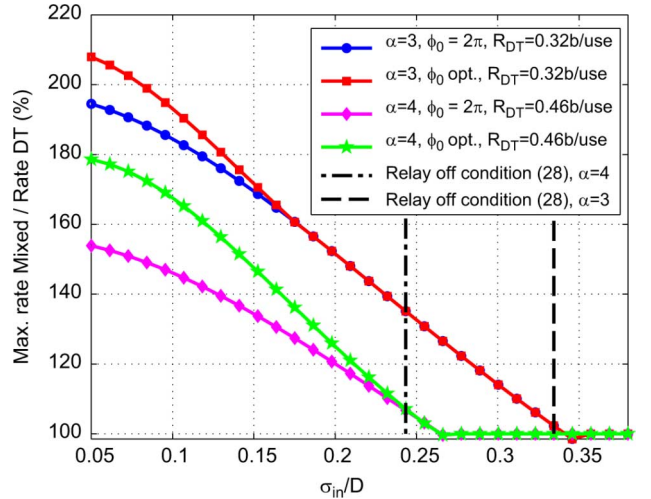

Fig. 5. Maximum rate attainable for the on/off relaying strategy relative to the same rate of DT for a given OP constraint. The mixed scheme rates are obtained by using (21) and the on/off condition from solving (30). We also plot the on/off condition (31). The optimal aperture angles come from Fig. 4. $\lambda_s = 10^{-4}$. $d = (10, 0)$.

for a desired OP value of 0.03. The maximum rates are obtained by using (21). For the plots with $\phi_0 = 2\pi$, the rates are obtained by using (23), while in the other case, the expectations are computed numerically. For the plots with optimized cone aperture, we use the values of ϕ_0 from Fig. 4, taking $\lambda_s = 10^{-4}$ and $d = (10, 0)$. The on/off condition (which predicts when the rate of the mixed scheme reaches that of DT) is obtained by solving (30). We have also plotted as vertical lines the simpler on/off condition (31), which is in excellent agreement with the other one. We observe that optimizing the cone aperture can be helpful when the path loss exponent or the density of potential relays are large. In addition, as the path loss exponent decreases, we can achieve a lower maximum rate with DT for a given outage constraint; this implies that the benefits of a reduced exponent within the cluster are outmatched by the simultaneous increase in interference due also to the reduced exponent. The plot also shows that although the maximum rate for DT is smaller, the relative gains of the mixed scheme become larger. This means that the maximum achievable rate decreases slower for the mixed
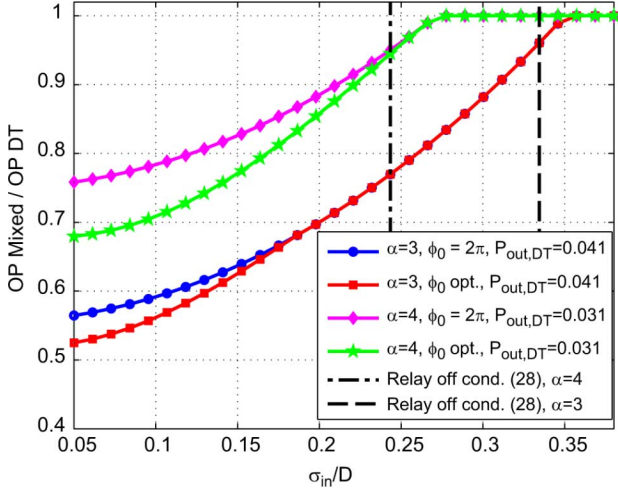
Fig. 6. Relative improvement in OP with respect to DT for the on/off scheme as a function of σ_{in}/D as predicted by Theorem 4.3. We have also plotted as vertical lines the on/off condition (31). $d = (10, 0)$.
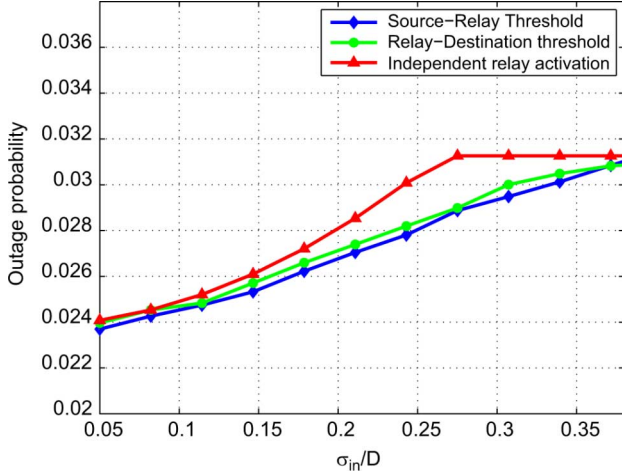

Fig. 7. Comparison between the OP attainable through independent relay activation and through the use of a threshold on the source–relay or source–destination channel. $\lambda_{\mathrm{s}} = 10^{-4}$, $R = 0.5$ b/use, $d = (10, 0)$, $\alpha = 4$. The independent relay curve comes from (21), and the rest from Monte Carlo simulations of the PPP. Thresholds are optimized numerically for best performance.

scheme than for DT as the path loss exponent decreases, which suggests that the increased interference is less damaging for the mixed scheme than for DT.

In Fig. 6, we plot the relative gain in OP with respect to DT as a function of σ_{in}/D using $\phi_0 = 2\pi$ and the optimal cone apertures from Fig. 4. The OP gains are obtained from Theorem 4.3. We also plotted as vertical lines the simpler condition (31), which is in excellent agreement with the other one.

Finally, in Fig. 7, we compare the performance of the proposed on/off strategy against two other simple relay activation schemes: one in which the relay is activated if the source–relay channel exceeds a threshold, and another one in which a threshold on the relay–destination channel is used. Both schemes make use of the available CSI. In the first case, the relay can determine if the threshold is exceeded, and in the second one, the destination, who has CSI on the relay–destination link, can send a bit (at negligible cost) indicating if the relay should transmit or not. In both cases, the path loss and

the corresponding fading coefficients are considered. The OP curves for these schemes are determined through Monte Carlo simulations of the point process, and for each point, the value of the threshold is numerically optimized to obtain the smallest OP possible. These curves are compared to the OP from the upper bound (21) and the on/off strategy. For these simulations, we use $\lambda_{\mathrm{s}} = 10^{-4}$, $R = 0.5$ b/use, $d = (10, 0)$, $\alpha = 4$. We observe that although these schemes employ available CSI that is not taken into account by the independent activation schemes, the performance is similar between the three strategies.

VI. Summary and Final Remarks

In this paper, we analyzed the performance of a large wireless network under a mixed cooperative randomized scheme that employs either DF or DT and obtained the optimal relay activation strategy for this network. When DF is used, the relays are chosen as the nearest neighbor within a cone, with its axis toward the destination. This is a natural assumption since DF is known to be near optimal when the relay is not too far from the source. At the same time, the effect of the path loss on the relay–destination link, which is very detrimental to the performance of the scheme, is reduced. The choice between DT and DF is done by the corresponding relay associated with each source via a randomized decision with probability p_{r} and without taking into account any additional knowledge the relays might have. This simple procedure, which is mathematically tractable, can be thought of as a MAC layer at the relays (in a similar fashion as the popular ALOHA protocol), with the objective of limiting the interference generation in the network. On the other hand, it could also model a situation in which the relays are unavailable due to conditions out of control of the source or the relay itself, such as, for example, a depleted battery. With this simple model, a balance between cooperation and interference generation can be established in the network. Surprisingly, for typical operating conditions, the optimal values of p_{r} are 0 or 1, revealing a binary behavior: All nodes in the network should use their relay or none at all. Following this conclusion, a relay activation strategy was introduced to achieve the optimal behavior. Even when cooperation is beneficial to all, the performance improvements may not be as large as in the typical fading relay channel with Gaussian noise. The reason for this comes from the fact that, in addition to fading, we have averaged over all possible node configurations, including many cases in which interference is very damaging.

It is interesting to mention that the model introduced and several results, such as Theorem 3.2, can be used to study other relay selection and activation algorithms based on position, such as choosing the relay as the nearest or farthest neighbor on a finite cone, and, with minor modifications, extend them to other cases involving additional CSI. Other protocols assuming higher degrees of CSI may yield better gains, but this may not be a realistic assumption in this context. A potential improvement could be obtained using more sophisticated cooperative transmission schemes that could take into account the impairments generated by the nearby interferers [35] (which introduce by far the most harmful interference). Basically, this could consist on decoding the messages sent by strong nearby interferers first, then subtracting them from the received signal, and finally attempting to decode the desired message. In such a situation, besides the intrinsic benefits of cooperation, the smart use of

the aggregate interference introduced in part by the cooperating nodes could ameliorate its harmful effect on the overall network. Another improvement could be obtained by using more advanced MAC schemes for the relays and the sources, such as CSMA, to avoid simultaneous nearby transmissions. In this context, a metric such as the transmission capacity [4] may be more appropriate for the analysis. Finally, the study of other co-operative schemes as AF and CF deserves full consideration. All these issues, as well as the effect of using several potential relays instead of only one, constitute important and interesting future work directions.

APPENDIX A
INTERFERENCE RVS AND THEIR LAPLACE TRANSFORMS

This Appendix is a simple review of the basic properties of the LT used in this work. For details, see [18] and [19]. Let $\tilde{\Phi} = \{(x_i, m)\}$ be an independently marked homogeneous PPP with $\Phi = \{x_i\}$ the homogeneous PPP in $\mathbb{R}^2$, and m a vector of marks on a subset of $\mathbb{R}^l$, $l \geq 1$. Define the interference RVs

$$I_d = \sum_{i:x_i \in \Phi_s} f_1(d, x_i, m_i), I_r = \sum_{i:x_i \in \Phi_s} f_2(r, x_i, m_i) \quad (34)$$

where f_1 and f_2 are real-valued nonnegative functions. The joint LT of the interference RVs at (ω_1, ω_2) is [8], [18]

$$\mathcal{L}_{I_d, I_r}(\omega_1, \omega_2)$$
$$= \exp\left\{-\lambda_s \int_{\mathbb{R}^2} \mathbb{E}_m\left[1 - e^{-\omega_1 f_1(d,x,m) - \omega_2 f_2(r,x,m)}\right] dx\right\}. \quad (35)$$

Taking $\omega_1 = 0$ or $\omega_2 = 0$, the single LT are obtained.

Lemma A.1: Suppose the marks of the HPPP are $m = \left(|h_1|^2, |h_2|^2, \varepsilon, k\right)$, with $|h_1|^2$ and $|h_2|^2$ unit mean independent exponential RVs, ε a Bernoulli RV with success probability p_r, and k an RV on $\mathbb{R}^2$. Let $f_1(d, x, m) = \left(|h_1|^2 + \varepsilon|h_2|^2\right) l(x + \tau k, d)$ with the path loss function $l(x, y) = \|x - y\|^{-\alpha}$ and $\tau \in [0, 1]$. Then, the LT is

$$\mathcal{L}_{I_d}(\omega_1) = \exp\left\{-\lambda_s C \omega_1^{2/\alpha}\left(1 + \frac{2p_r}{\alpha}\right)\right\}. \quad (36)$$

Proof: Taking $\omega_2 = 0$ in (35), writing the expectation with respect to the marks, and interchanging the integration order, we find that

$$\mathcal{L}_{I_d}(\omega_1) = \exp\left\{-\lambda_s p_r \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} 1 - \frac{1}{[1+\omega_1 l(x+\tau k, d)]^2} dx dF_k\right.$$
$$\left. -\lambda_s(1 - p_r) \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \frac{1}{1 + (\omega_1 l(x + \tau k, d))^{-1}} dx dF_k\right\}. \quad (37)$$

When the integrals with respect to x are computed, the result does not depend on k, so the distribution of k does not affect the final result. For the first integral, we have

$$\int_{\mathbb{R}^2} 1 - \frac{1}{[1+\omega_1 l(x+\tau k_i, d)]^2} dx = 2\pi \omega^{2/\alpha} \int_0^\infty \frac{1+2t^\alpha}{(1+t^\alpha)^2} t dt$$
$$= \omega_1^{2/\alpha}\left(1 + \frac{2}{\alpha}\right) C. \quad (38)$$

For the last step, we integrate by parts, and C is defined in (10). The second integral is known from the DT case [3]

$$\int_{\mathbb{R}^2} \frac{1}{1 + (\omega_1 l(x, r))^{-1}} dx = C\omega_1^{2/\alpha}. \quad (39)$$

∎

APPENDIX B
PROOF OF LEMMA 2.1

The interference signals at the relay and the destination are

$$\tilde{Z}_r = \sum_{i:x_i \in \Phi_s}\left(\frac{h_{x_i r} X_{x_i}}{\|x_i - r\|^{\frac{\alpha}{2}}} + \varepsilon_{x_i} \frac{h_{k_i r} X_{k_i}}{\|x_i + k_i - r\|^{\frac{\alpha}{2}}}\right) \quad (40)$$

$$\tilde{Z}_d = \sum_{i:x_i \in \Phi_s}\left(\frac{h_{x_i d} X_{x_i}}{\|x_i - d\|^{\frac{\alpha}{2}}} + \varepsilon_{x_i} \frac{h_{k_i d} X_{k_i}}{\|x_i + k_i - d\|^{\frac{\alpha}{2}}}\right) \quad (41)$$

where (X_{x_i}, X_{k_i}) are the complex, circular, and zero-mean Gaussian signals with correlation coefficient ρ of each source and its relay (if it is active) [1]. The proof of the lemma follows from the fact that when $\alpha > 2$, I_d and I_r are finite for almost every realization of $\tilde{\Phi}_s$. This can be shown using the Laplace functional of $\tilde{\Phi}_s$[18], [36], and the following functions:

$$f(x, \varepsilon, k, h_1, h_2, h_3, h_4) = \frac{|h_1|^2}{\|x - r\|^\alpha}$$
$$+ \varepsilon\left(\frac{|h_3|^2}{\|x + k - r\|^\alpha} + \frac{2\Re\{h_1 h_3^* \rho\}}{\|x - r\|^{\frac{\alpha}{2}}\|x + k - r\|^{\frac{\alpha}{2}}}\right) \quad (42)$$

and $g(x, \varepsilon, k, h_1, h_2, h_3, h_4)$ defined in a similar form. Since signaling between clusters is correlated with correlation coefficient ρ within the cluster and independent between clusters, it can be shown that the partial sums (through a proper enumeration of the points of the particular realization of $\tilde{\Phi}_s$) in $\tilde{Z}_r$ and $\tilde{Z}_d$ are Gaussian with variances given by the corresponding partial sums in I_d and I_r. Thanks to the finiteness of I_d and I_r, and the *tightness* property ([36, Theorem 25.10]), we have the desired result.

APPENDIX C
PROOF OF THEOREM 3.1

Define $V := |h_{sd}|^2 l_{sd} + |h_{rd}|^2 l_{rd}$ and write

$$\mathbb{P}\left\{\mathcal{A}(R) \cup \mathcal{B}(R) | r, \varepsilon_0 = 1\right\}$$
$$= 1 - \mathbb{P}\left\{|h_{sr}|^2 \geq \frac{TI_r}{l_{sr}}, V \geq TI_d \middle| r, \varepsilon_0 = 1\right\}$$
$$= 1 - \mathbb{E}_{\tilde{\Phi}_s}\left[\bar{F}_{h_{sr}}(TI_r/l_{sr})\bar{F}_V(TI_d)\right] \quad (43)$$

where $\bar{F}_{h_{sr}}(u) = e^{-u}$ and $\bar{F}_V(\cdot)$ are the complementary cumulative distribution function (CCDF) of h_{sr} and V, respectively, and we used that h_{sr} and V are independent of each other and of $\tilde{\Phi}_s$. Since $\rho = 0$, when $\|r - d\| \neq D$, V is distributed as the sum of two independent exponential RVs with different means. In that case, its CCDF is

$$\bar{F}_V(u) = \frac{D^\alpha e^{-u\|r-d\|^\alpha} - \|r - d\|^\alpha e^{-uD^\alpha}}{D^\alpha - \|r - d\|^\alpha}. \quad (44)$$

When $\|r - d\| = D$, the means of the exponential RVs are the same, so V follows a Gamma distribution with 2 degrees of freedom. However this does not affect the average with respect to r that we need to carry out. Replacing both CCDFs in (43), we obtain $\mathbb{P}\{\mathcal{A}(R) \cup \mathcal{B}(R)|r, \varepsilon_0 = 1\}$ from (11). The term $\mathbb{P}\{\mathcal{A}_{\mathrm{DT}}(R)|\varepsilon_0 = 0\}$ is obtained in a similar fashion.

APPENDIX D
PROOF OF THEOREM 3.2

Analogously to the proof of Theorem 3.1, we can show that

$$\mathbb{P}\{\mathcal{A}_{\mathrm{DT}}(R)|\varepsilon_0 = 0\} = 1 - \mathcal{L}_{I_d}(T/l_{\mathrm{sd}}) \tag{45}$$

$$\mathbb{P}\{\mathcal{A}(R)|r, \varepsilon_0 = 1\} = 1 - \mathcal{L}_{I_r}(T/l_{\mathrm{sr}}) \tag{46}$$

$$\mathbb{P}\{\mathcal{B}(R)|r, \varepsilon_0 = 1\} = 1$$
$$- \frac{D^\alpha \mathcal{L}_{I_d}(T/l_{\mathrm{rd}}) - \|r-d\|^\alpha \mathcal{L}_{I_d}(T/l_{\mathrm{sd}})}{D^\alpha - \|r-d\|^\alpha}. \tag{47}$$

Using (36) from Lemma A.1, we can evaluate all the LTs. Taking expectation with respect to the relay position r, we obtain the general expression. To simplify the expectation of $\mathbb{P}\{\mathcal{B}(R)|r, \varepsilon_0 = 1\}$, we use (36) to evaluate the LT, and setting $u = \|r - d\|/D$, we write

$$\frac{D^\alpha \mathcal{L}_{I_d}(T/l_{\mathrm{rd}})}{D^\alpha - \|r-d\|^\alpha} - \frac{\|r-d\|^\alpha \mathcal{L}_{I_d}(T/l_{\mathrm{sd}})}{D^\alpha - \|r-d\|^\alpha}$$
$$= e^{-\lambda_s \Delta(p_r) D^2} \left(\frac{e^{\lambda_s \Delta(p_r) D^2 (1 - u^2)} - u^\alpha}{1 - u^\alpha} \right). \tag{48}$$

Using that as $\lambda_s \Delta D^2 (1 - u^2) \to 0$, we have $e^{\lambda_s \Delta D^2 (1 - u^2)} = 1 + \lambda_s \Delta D^2 (1 - u^2) + O((\lambda_s \Delta D^2 (1 - u^2))^2)$, that for $u > 0$

$$\frac{1 - u^2}{1 - u^\alpha} \geq 1 + \left(\frac{2}{\alpha} - 1 \right) u \tag{49}$$

and taking expectation with respect to u, we conclude that

$$\mathbb{E}_r \left[\frac{e^{\lambda_s \Delta(p_r) D^2 (1 - u^2)} - u^\alpha}{1 - u^\alpha} \right] \geq 1 + \lambda_s \Delta(p_r) D^2$$
$$\times \left[1 + \frac{2 - \alpha}{\alpha} \mathbb{E}_r[u] \right] + O\left((\lambda_s \Delta(p_r) D^2)^2 \right). \tag{50}$$

To find (23), start by writing

$$\mathbb{E}_r[\|r - d\|] = \int_{\mathbb{R}^2} \frac{1}{2\pi \sigma_{\mathrm{in}}^2} \|r - d\| e^{-\frac{\|r\|^2}{2\sigma_{\mathrm{in}}^2}} dr. \tag{51}$$

Now take $x = r - d$, change to polar coordinates to obtain

$$\mathbb{E}_r[\|r - d\|] = \int_0^\infty \left(\frac{u}{\sigma} \right)^2 e^{-\frac{u^2 + D^2}{2\sigma^2}} I_0 \left(\frac{Du}{\sigma^2} \right) du \tag{52}$$

$$= \sigma Q_{2,0} \left(\frac{D}{\sigma}, 0 \right). \tag{53}$$

In the first step, we used the definition of the modified Bessel function. For the actual value of $Q_{2,0}(u, 0)$, we use (91) and (60), both from [33].

To find (24), we first prove that

$$\|r - d\| \leq |\|d\| - \|r\|| + 2 \min(\|d\|, \|r\|) |\sin(\theta/2)| \tag{54}$$

where θ is the angle between r and d. We decompose $r - d$ as $r - d = u_1 + v_1 = u_2 + v_2$ with

$$u_1 = r - \frac{\|r\|}{\|d\|} d \quad u_2 = \frac{\|d\|}{\|r\|} r - d. \tag{55}$$

Then, we use that $\|u_1\| = 2\|r\| \sin(\theta/2)$, $\|u_2\| = 2\|d\| \sin(\theta/2)$, and $\|v_1\| = \|v_2\| = |\|r\| - \|d\||$, and the triangle inequality on both decompositions. By taking the expectation on both sides of (54) and solving the integrals, we finish the proof.

APPENDIX E
PROOFS REGARDING THE CONCAVITY OF THE OP

Proof of Theorem 4.1: We rewrite (21) in terms of $\nu(p_r) := \lambda_s \Delta(p_r) D^2$ to obtain

$$\mathbb{P}_{\mathrm{out,mix}}(R) \leq \left[1 - \frac{\alpha(\nu(p_r) - \nu(0))}{2\nu(0)} \right] \left[1 - e^{-\nu(p_r)} \right]$$
$$+ \frac{\alpha(\nu(p_r) - \nu(0))}{2\nu(0)} \left\{ 1 + \frac{2\nu(p_r)}{\lambda_{\mathrm{in}} \phi_0 D^2 + 2\nu(p_r)} \right.$$
$$\left. - e^{-\nu(p_r)} \left[1 + \nu(p_r) \left(1 + \frac{2 - \alpha}{\alpha D} \mathbb{E}_r[\|r - d\|] \right) \right] \right\}. \tag{56}$$

Since $\nu(p_r)$ is linear in p_r, we can analyze the concavity of the OP with respect to $\nu(p_r)$ instead of p_r. We do this by studying when the second derivative of the OP upper bound (56) with respect to ν is negative. After differentiating twice with respect to ν and rearranging the terms, we obtain

$$\frac{d^2 \mathbb{P}_{\mathrm{out,mix}}}{d\nu^2} \leq \left\{ 2\alpha \lambda_{\mathrm{in}} \phi_0 D^2 e^{\nu(p_r)} \frac{2\nu(0) + \lambda_{\mathrm{in}} \phi_0 D^2}{(2\nu(p_r) + \lambda_{\mathrm{in}} \phi_0 D^2)^3} \right.$$
$$+ \frac{\alpha}{2} \left(1 - \frac{2}{\alpha} \right) p_c(\nu) \frac{\mathbb{E}_r[\|r - d\|]}{D}$$
$$\left. - \left(\frac{\alpha}{2} p_c(\nu) + \nu(0) \right) \right\} \frac{e^{-\nu(p_r)}}{\nu(0)} \tag{57}$$

with

$$p_c(\nu) = \nu^2 - (\nu(0) + 4)\nu + 2(1 + \nu(0)). \tag{58}$$

We study the derivative in the interval $0 \leq p_r \leq 1$, which maps to the interval $\nu(0) = \lambda_s \delta D^2 \leq \nu \leq \nu(1) = \lambda_s \delta D^2 (1 + 2/\alpha)$. Using standard arguments, it is straightforward to show that for $\alpha > 2$, we have $p_c(\nu) > 0$ in this interval whenever

$$\nu(0) = \lambda_s \delta D^2 \leq \frac{3 - \sqrt{5}}{2} \approx 0.38. \tag{59}$$

In addition, using that $\nu(p_r) \geq \nu(0) = \lambda_s \delta D^2 > 0$, we bound

$$\lambda_{\mathrm{in}} \phi_0 D^2 \frac{2\nu(0) + \lambda_{\mathrm{in}} \phi_0 D^2}{(2\nu(p_r) + \lambda_{\mathrm{in}} \phi_0 D^2)^3} \leq \frac{1}{\lambda_{\mathrm{in}} \phi_0 D^2} \tag{60}$$

in (57) to obtain

$$\frac{d^2 \mathbb{P}_{\mathrm{out,mix}}}{d\nu^2} \leq \frac{e^{-\nu(p_r)}}{\nu(0)} \left\{ \frac{4\pi \alpha e^{\nu(p_r)} \sigma_{\mathrm{in}}^2}{\phi_0 D^2} \right.$$
$$\left. + \left(\frac{\alpha}{2} - 1 \right) p_c(\nu) \frac{\mathbb{E}_r[\|r - d\|]}{D} - \left(\frac{\alpha}{2} p_c(\nu) + \nu(0) \right) \right\} \tag{61}$$

where we have used also that $\sigma_{\text{in}} = (2\pi\lambda_{\text{in}})^{-1}$. It is clear that the first term in (57) is positive, and that under (59), the second term also is, and the third one is negative. Notice also that for each ϕ_0, D, $\alpha > 2$ as $\sigma_{\text{in}} \to 0$, the first term goes to zero and $\mathbb{E}_{\text{r}}[\|r - d\|]/D \to 1$. Now using standard continuity arguments, it is straightforward to show that for each ϕ_0, D, $\alpha > 2$, and $\nu(0) = \lambda_{\text{s}}\delta D^2$ satisfying (59), if σ_{in} is small enough, then the third negative term will be greater that the other two, and hence the second derivative will become negative.

Proof of Corollary 4.1: In order to find the value of σ_{c}, we should find the smallest root of the second derivative of $\mathbb{P}_{\text{out,mix}}$. Since this cannot be done in closed form, we can find an approximate condition for concavity by finding the smallest root in σ_{in} of the upper bound (61) of the second derivative of $\mathbb{P}_{\text{out,mix}}$. For the high reliability regime, we can further approximate $\nu(0) = \lambda_{\text{s}}\delta D^2 \approx 0$, which leads to (26). To obtain condition (27), we upper-bound $\mathbb{E}[\|r - d\|]$ using (24) in (26), to obtain

$$2\alpha s^2 + (\alpha - 2)\gamma(s, \phi_0)s - 2 = 0. \tag{62}$$

This equation cannot be solved in closed form either due to the presence of the function γ, but it can be shown that an absolute upper bound of the smallest root is obtained from setting $\alpha = 2$ in the equation. In that case, the equation is independent of γ and can be solved in closed form to obtain the condition $s \le 1/\sqrt{2}$. It can be shown that setting $s = 1/\sqrt{2}$ in $\gamma(s, \phi_0)$ and solving (62) yields a lower bound on the smallest root for each value of α. Thus, (62) becomes a second degree polynomial in s which can be solved in closed form to obtain (27).

APPENDIX F
PROOF OF THEOREM 4.2

In Theorem 4.1, we showed that for each network setup such that (59) holds, there is an interval $\sigma_{\text{in}} \le \sigma_{\text{c}}$ in which the OP upper bound is concave in p_{r}. Now we show that under this condition, there is an interval in which $p_{\text{r}} = 1$ is optimal by finding conditions such that $\mathbb{P}_{\text{out,mix}}(p_{\text{r}} = 1) - \mathbb{P}_{\text{out,mix}}(p_{\text{r}} = 0) \le 0$. Setting $p_{\text{r}} = 0$ and $p_{\text{r}} = 1$, we can write

$$\mathbb{P}_{\text{out,mix}}(p_{\text{r}} = 1) - \mathbb{P}_{\text{out,mix}}(p_{\text{r}} = 0) \le e^{-\lambda_{\text{s}}\delta D^2} + \frac{2\lambda_{\text{s}}\Delta}{2\lambda_{\text{s}}\Delta + \phi_0\lambda_{\text{in}}}$$
$$- \left[1 + \lambda_{\text{s}}\Delta D^2\left(1 + \frac{2 - \alpha}{\alpha D}\mathbb{E}_{\text{r}}[\|r - d\|]\right)\right]e^{-\lambda_{\text{s}}\Delta D^2} \tag{63}$$

where we take $\Delta \equiv \Delta(1) = \delta\left(1 + \frac{2}{\alpha}\right)$. Now we upper-bound

$$\frac{2\lambda_{\text{s}}\Delta}{2\lambda_{\text{s}}\Delta + \phi_0\lambda_{\text{in}}} \le \frac{2\lambda_{\text{s}}\Delta}{\phi_0\lambda_{\text{in}}} = \frac{4\pi\lambda_{\text{s}}\Delta\sigma_{\text{in}}^2}{\phi_0} \tag{64}$$

and

$$e^{-\lambda_{\text{s}}\delta D^2} - e^{-\lambda_{\text{s}}\Delta D^2}\left(1 + \lambda_{\text{s}}\Delta D^2\right)$$
$$\le \lambda_{\text{s}}\delta D^2 e^{-\lambda_{\text{s}}\Delta D^2}\left(\frac{4}{\alpha^2}\lambda_{\text{s}}\delta D^2 - 1\right) \tag{65}$$

which is valid when (59) is met since then $e^{\nu(0)} \le 1 + \nu(0) + \nu^2(0)$. With this, we obtain

$$\mathbb{P}_{\text{out,mix}}(p_{\text{r}} = 1) - \mathbb{P}_{\text{out,mix}}(p_{\text{r}} = 0) \le \lambda_{\text{s}}\delta D^2 e^{-\lambda_{\text{s}}\Delta D^2}$$

$$\left\{\left[1 + \frac{2}{\alpha}\right]\left[\frac{4\pi\sigma_{\text{in}}^2}{\phi_0 D^2}e^{\lambda_{\text{s}}\Delta D^2} + \left(1 - \frac{2}{\alpha}\right)\frac{\mathbb{E}_{\text{r}}[\|r - d\|]}{D}\right]\right.$$
$$\left. + \frac{4\lambda_{\text{s}}\delta D^2}{\alpha^2} - 1\right\}. \tag{66}$$

Continuity arguments similar to those of Theorem 4.1 prove that if σ_{in} is small enough, then the right side of (66) will be negative and $p_{\text{r}} = 1$ will be optimal. To find and estimate for the maximum value of σ_{in}, we can find the roots of the right side of this expression, focusing on the terms between brackets. In the high reliability regime, the term $\lambda_{\text{s}}\delta D^2$ will be small (as shown at the end of Section II) so an approximate condition for concavity can be obtained by letting $\lambda_{\text{s}}\delta D^2 \to 0$, which leads to (30). The proof of the simpler condition (31) is obtained following the same arguments as in the proof of Corollary 4.1, except that in this case we can establish the condition $s \le 1/2$.

APPENDIX G
PROOF OF THEOREM 4.3

When $p_{\text{r}} = 1$ is optimal, we can compute the gains starting from (66), valid under (59), and noting that $\mathbb{P}_{\text{out,mix}}(p_{\text{r}} = 0) = \mathbb{P}_{\text{out},DT}$. Rearranging the terms, we obtain

$$\frac{\mathbb{P}_{\text{out,mix}}}{\mathbb{P}_{\text{out},DT}} \le 1 + \frac{\lambda_{\text{s}}\delta D^2}{\mathbb{P}_{\text{out},DT}}\left\{\left[\left(1 - \frac{2}{\alpha}\right)\frac{\mathbb{E}_{\text{r}}[\|r - d\|]}{D}e^{-\lambda_{\text{s}}\Delta D^2}\right.\right.$$
$$\left.\left. + \frac{4\pi\sigma_{\text{in}}^2}{\phi_0 D^2}\right] \times \left(1 + \frac{2}{\alpha}\right) + e^{-\lambda_{\text{s}}\Delta D^2}\left(\frac{4\lambda_{\text{s}}\delta D^2}{\alpha^2} - 1\right)\right\}. \tag{67}$$

By noting that when $p_{\text{r}} = 1$ is optimal, the term between brackets in the previous expression will be negative, we can upper-bound this by removing the term $(\lambda_{\text{s}}\delta D^2)/\mathbb{P}_{\text{out},DT}$ outside the brackets. To simplify the expression for the high reliability regime, we can take the approximation $\lambda_{\text{s}}\delta D^2 \approx 0$. Finally, when $p_{\text{r}} = 0$, the performance will be the same as DT, so the gain will be one.

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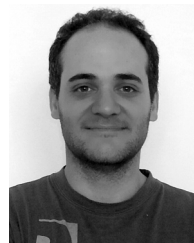
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