A Simple Asymptotically Optimal Joint Energy Allocation and Routing Scheme in Rechargeable Sensor Networks

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***Abstract*—In this paper, we investigate the utility maximization problem for a sensor network with energy replenishment. Each sensor node consumes energy in its battery to generate and deliver data to its destination via multihop communications. Although the battery can be replenished from renewable energy sources, the energy allocation should be carefully designed in order to maximize system performance, especially when the replenishment profile is unknown in advance. In this paper, we address the joint problem of energy allocation and routing to maximize the total system utility, without prior knowledge of the replenishment profile. We first characterize optimal throughput of a single node under general replenishment profile and extend our idea to the multihop network case. After characterizing the optimal network utility with an upper bound, we develop a low-complexity online solution that achieves asymptotic optimality. Focusing on long-term system performance, we can greatly simplify computational complexity while maintaining high performance. We also show that our solution can be approximated by a distributed algorithm using standard optimization techniques. In addition, we show that the required battery size is $O(\ln(1/\xi))$ to constrain the performance of our scheme within ξ-neighborhood of the optimum. Through simulations with replenishment profile traces for solar and wind energy, we numerically evaluate our solution, which outperforms a state-of-the-art scheme that is developed based on the Lyapunov optimization technique.**

***Index Terms*—Asymptotically optimal scheme, energy allocation, rechargeable sensor networks, routing.**

I. Introduction

Wireless sensor networks have been shown to be immensely useful for monitoring a wide range of environmental parameters, such as earthquake intensity, glacial movements, and water flow. Unattended operation of sensor networks for a long period is highly desirable due to typical remoteness and harshness of the environment. One of the main obstacles in developing long-lived networks is limited battery of sensor nodes. Energy harvesting from various natural sources, such as solar and vibration [1]–[3], has been shown to be effective in alleviating this problem by allowing sensor nodes to replenish their batteries. However, energy management still remains critical, in particular, when one cannot forecast the amount of energy replenishment. Keeping a high battery level may result in low network performance, while maintaining a low battery level increases risk of energy depletion.

There are several works that address the energy allocation problem in sensor networks with energy replenishment. In [5], a solution has been developed to maximize the total utility for a satellite with energy replenishment, based on the dynamic programming (DP) technique. In [6], the authors consider a network where nodes with and without replenishment coexist and propose two heuristic routing schemes to exploit renewable energy: One scheme looks for the path with minimum number of nodes without replenishment, and the other scheme allows one relaying node to deviate from the shortest path and forward packets opportunistically to nodes with energy replenishment. A battery recharging and discharging model has been developed in [7] for energy replenishment sensor networks. A threshold-based policy has been proven to guarantee at least 3/4 of the optimal performance. In [8], the authors have developed an energy-adaptive scheme that achieves order-optimal performance for a single node with energy replenishment. Lexicographically maximum rate assignment and routing for perpetual data collection has been studied in [9]. The authors have proposed a centralized solution, which can obtain the optimal lexicographic rate assignment, and a distributed solution, which reaches the optimum only in tree networks with predetermined routing paths. Task scheduling problem is considered for a single node with energy replenishment in [10]. The authors have developed two heuristic schemes that smooth the energy consumption over the running period. In [11], a power-aware routing policy has been developed. Computing a path with the least cost, the solution asymptotically achieves optimal competitive ratio as the network scales. Also, there are a few works that exploit the Lyapunov optimization technique to achieve asymptotic optimality [12], [13]. However, they require the replenishment processes to be i.i.d. or Markovian, which may not be true in practice due to fluky characteristics of renewable energy sources.

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In this paper, we are interested in developing low-complexity solutions that maximize the total user utility for a rechargeable sensor network, in particular, when future replenishment profile is unknown a priori. The problem can be formulated as a standard convex optimization problem with energy and routing constraints as in [4]. However, the solution requires centralized control and full knowledge of replenishment profiles in the future, which are hardly available in practice. In this paper, we characterize optimal performance and obtain insight into the asymptotical properties. Based on the time-invariant properties, we develop a low-complexity solution that is asymptotically optimal and can be approximated by a distributed algorithm. We summarize our main contributions as follows.

1) We characterize an upper bound for the utility performance of a sensor network with energy replenishment by constructing an infeasible scheme that outperforms the optimal scheme.
2) We develop a low-complexity online solution that jointly takes into account energy allocation and routing. Without advance knowledge of the future replenishment profile, our solution is provably efficient using estimation of replenishment rate and supply–demand mismatch. We show that the performance gap between our online solution and the infeasible solution for the upper bound diminishes as time tends to infinity.
3) We approximate our solution by a distributed algorithm and evaluate it through simulations based on replenishment profile traces for solar and wind energy. The results show that the solution performs close to the upper bound after a short time period and outperforms a state-of-the-art scheme that is developed based on the Lyapunov optimization technique.

Unlike the previous works, we consider a larger class of replenishment processes, which only require the existence of a mean value rather than assumptions of i.i.d. or Markovian. To the best of our knowledge, our solution is the first one that achieves asymptotic optimality under general replenishment profiles. Also note that although the solution in [4] achieves optimal performance by making use of fluctuations of the energy replenishment process, it requires future knowledge of the replenishment profile. In contrast, our online solution here does not require such knowledge and achieves asymptotic optimality by relying on long-term characteristic of the energy replenishment process. Through successfully removing time dependency in decisions, we significantly reduce the computational complexity.

Our paper is organized as follows: In Section II, we formulate our problem as a standard utility maximization problem. In Section III, we propose a simple solution that maximizes throughput for a single node. In Section IV, we extend our results to the network case, develop a low-complexity online solution that achieves asymptotic optimality, and approximate it by an even simpler distributed algorithm. After presenting simulation results in Section V, we conclude our paper in Section VI.

II. System Model

We consider a static sensor network, denoted by $G = (\mathcal{N}, \mathcal{L})$, where $\mathcal{N}$ is the set of nodes and $\mathcal{L}$ is the set of links. We assume a time-slotted system for a period of T time-slots. Each node has a battery whose size is assumed to be infinite. (We will relax the infinite-battery assumption in Section III-C.) Let $r_n(t)$ denote the amount of replenishment energy that arrives at node n in time-slot t, while $e_n(t)$ denotes the allocated energy of node n in time-slot t. Without loss of generality, we assume that the energy replenishment occurs at the beginning of each slot and the harvested energy is immediately stored in the battery. Let $B_n(t)$ denote the battery level of node n at the beginning of time-slot t, which is assumed to be initially empty for simplicity of exposition, i.e., $B_n(0) = 0$. The energy dynamics can be depicted as follows:

$$B_n(t+1) = \max\left\{B_n(t) + r_n(t) - e_n(t), 0\right\}. \quad (1)$$

We assume that the replenishment process has a finite mean value $\bar{r}_n$, i.e.,

$$\bar{r}_n \triangleq \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} r_n(t) \quad (2)$$

which is a mild assumption including a larger class of replenishment processes than those used in the prior works [12], [13], where $r_n(t)$ is assumed to be an i.i.d. process.

There are S flows in the network, and each flow s is associated with a source node f_s and a destination node d_s. Let $\mathcal{S}$ denote the set of the source nodes. During a time-slot, the data transmission of a node is characterized by a continuously nondecreasing and strictly concave rate-power function $\mu(P)$, satisfying $\mu(0) = 0$. Note that $\mu(P)$ represents the amount of data that can be transmitted using P units of energy in a time-slot under a given physical layer modulation and coding strategy (see [21] for details).

Let $x^s(t)$ be the amount of data that is delivered from the source f_s to the destination d_s in time-slot t over possibly multiple hops and multiple paths. Each user s is associated with a utility function $U_s(\bar{x}^s)$, which reflects the "satisfaction" of user s when it transmits at average data rate $\bar{x}^s \triangleq (1/T)\sum_{t=1}^{T} x^s(t)$. We assume that $U_s(\cdot)$ is a strictly concave, nondecreasing, and continuously differentiable function.

A. Problem Formulation

Our objective is to develop a low-complexity online solution to the joint problem of energy allocation and data routing to maximize aggregate utility for the rechargeable sensor network. Since the rate of energy replenishment is usually much slower than the rate of energy consumption, we assume that the reduction of energy is instantaneous for all the nodes along the path as in [11]. In our work, we do not explicitly consider wireless interference. Thus, our techniques can directly handle cases when adjacent nodes operate on orthogonal channels. An open question is whether one can develop a unified strategy that incorporates the simplicity of our scheme with the many excellent works in the literature that have focused on scheduling in the presence of interference, such as [14], [17], and the references therein. While this is beyond the scope of this work, it will form the basis of our future work.

We start with the definition of *rate region* for a node under energy replenishment profile $\vec{r}_n = (r_n(1), r_n(2), \cdots, r_n(T))$.

Definition 1 (Rate Region): The rate region Λ_n of node n is defined as the set of all vectors $\vec{v}_n = (v_n(1), v_n(2), \cdots, v_n(T))$, such that for any $\vec{v}_n \in \Lambda_n$, there exists some energy allocation $\vec{e}_n$ that achieves $\vec{v}_n$, i.e., $v_n(t) = \mu(e_n(t))$, for all $t \in (1, \cdots T)$.

It has been shown that the rate region Λ_n of node n is convex (see [4, Lemma 4]). Let $w_{ij}^d(t)$ denote the amount of data on the outgoing link $(i,j) \in \mathcal{L}$ for destination node d in time-slot t, and we denote its vector as $\vec{w}_{ij} = (\sum_d w_{ij}^d(1), \sum_d w_{ij}^d(2), \cdots, \sum_d w_{ij}^d(T))$. We formulate the utility maximization problem as follows:

$$
\begin{aligned}
&\text{Problem } \mathbf{A}: \quad \max_{\vec{w}_{ij}, \vec{x}^s, \vec{e}_n} \sum_s U_s\left(\frac{1}{T}\sum_{t=1}^T x^s(t)\right)\\
&\text{subject to} \qquad w_{ij}^d(t) \geq 0 \qquad \forall t, \quad \forall d, \quad \forall (i,j) \in \mathcal{L}\\
&\sum_{t=1}^T \sum_j w_{ij}^d(t) - \sum_{t=1}^T \sum_j w_{ji}^d(t) - \sum_{t=1}^T \sum_{s: f_s = i, d_s = d} x^s(t) \geq 0,\\
&\qquad\qquad\qquad\qquad\qquad\qquad \forall d, \text{ and for all } i \neq d\\
&\sum_{j:(i,j)\in\mathcal{L}} \vec{w}_{ij} \in \Lambda_i, \qquad \text{for all node } i \in \mathcal{N}
\end{aligned} \tag{3}
$$

where the second constraint means that total amount of data for destination d into node i is less than or equal to total amount of data out of the node. If any node does not have enough data for a flow to send over all outgoing links, null bits are delivered.

The solution to Problem $\mathbf{A}$ will determine: 1) the amount of energy $e_n(t)$ that should be spent for each node $n \in \mathcal{N}$ in time-slot t; 2) the amount of data $x^s(t)$ that should be transmitted by each flow $s \in \mathcal{S}$ in time-slot t; and 3) routing decisions for each node i, i.e., choosing $w_{ij}^d(t)$ for each link (i,j) and each destination node d.

It has been shown in [4] that Problem $\mathbf{A}$ is a convex optimization problem and can be solved using the standard convex duality approach if full knowledge of the replenishment profile including for the future is provided. However, such knowledge is difficult to obtain in practice. Furthermore, even if such knowledge is assumed, this problem is computationally highly complex. The culprit is the "time coupling property," which is reflected in the last constraint $\sum_{j:(i,j)\in\mathcal{L}} \vec{w}_{ij} \in \Lambda_i$. In this paper, we show an upper bound on optimal performance that can be obtained by solving Problem $\mathbf{A}$. We also provide a low-complexity online solution, the performance of which forms a lower bound. Moreover, we show that the lower bound can get arbitrarily close to the upper bound, when T tends to infinity, which implies that our solution is asymptotically optimal.

III. Throughput Maximization: A Single-Node Case

We first investigate throughput performance of optimal energy allocation scheme for a single node. In this section, we omit the subscript n from all the notations defined in Section II since all results are for a single node n.

Let $\vec{e}^* = (e^*(1), e^*(2), \ldots, e^*(T))$ denote the optimal energy allocation that maximizes throughput of a single node under energy replenishment $\vec{r} = (r(1), r(2), \cdots, r(T))$. Let $J_{\text{one}}^*(T)$ denote the optimal throughput achieved by $\vec{e}^*$, that is

$$J_{\text{one}}^*(T) \triangleq \frac{1}{T}\sum_{t=1}^T \mu\left(e^*(t)\right). \tag{4}$$

In the following, we provide an upper and a lower bound for $J_{\text{one}}^*(T)$, whose difference can be arbitrarily small as T tends to infinity.

A. Upper Bound

Let $\bar{r}$ denote the average replenishment rate, defined as $\bar{r} \triangleq \lim_{T\to\infty}(1/T)\sum_{t=1}^T r(t)$.

Proposition 1: When T tends to infinity, $J_{\text{one}}^*(T)$ is upper-bounded by $\mu(\bar{r})$.

Proof: From (4) and Jensen's inequality with the concavity of $\mu(\cdot)$, we have that

$$J_{\text{one}}^*(T) = \frac{1}{T}\sum_{t=1}^T \mu(e^*(t)) \leq \mu\left(\frac{\sum_{t=1}^T e^*(t)}{T}\right) \leq \mu\left(\frac{\sum_{t=1}^T r(t)}{T}\right) \tag{5}$$

where the second inequality holds because the total allocated energy can be no greater than the total harvested energy. By taking the limsup on both sides, we can obtain that

$$\limsup_{T\to\infty} J_{\text{one}}^*(T) \leq \limsup_{T\to\infty} \mu\left(\frac{\sum_{t=1}^T r(t)}{T}\right) = \mu(\bar{r}). \tag{6}$$

Proposition 1 also implies that for any $\vec{v} \in \Lambda$, we have $\limsup_{T\to\infty}(1/T)\sum_{t=1}^T v(t) \leq \mu(\bar{r})$. Hence, for any $\epsilon > 0$, there exists T_0, such that for all $T > T_0$, we have

$$\frac{1}{T}\sum_{t=1}^T v(t) \leq \mu\left(\bar{r}(1+\epsilon)\right). \tag{7}$$

This equation will be used later in the proof of the network case.

B. Lower Bound

We consider the following energy allocation scheme, denoted by Scheme-LBONE:

- In each time-slot t, average harvested energy is estimated as follows:

$$\hat{r}(t) \triangleq \frac{1}{t}\sum_{\tau=1}^t r(\tau). \tag{8}$$

- Using the estimation, energy is allocated as

$$e(t) = \begin{cases} (1-\epsilon)\hat{r}(t), & \text{if } B(t) + r(t) \geq (1-\epsilon)\hat{r}(t)\\ B(t) + r(t), & \text{otherwise} \end{cases} \tag{9}$$

  where $\epsilon > 0$ is a system parameter that can be chosen to be arbitrarily small.

We denote the throughput of Scheme-LBONE by $J_{\text{one}}^{\text{lb}}(T) \triangleq (1/T)\sum_{t=1}^T \mathbb{E}[\mu(e(t))]$, where the expectation is taken with respect to the sample space of the replenishment process. We will obtain a lower bound for $J_{\text{one}}^*(T)$ by the following proposition.

Proposition 2: When T tends to infinity, $J^*_{\text{one}}(T)$ is lower-bounded by $\mu((1-\epsilon)^2\bar{r})$.

Proof: From (2), we have $\lim_{t\to\infty}\hat{r}(t)=\bar{r}$, which follows that for any $\epsilon>0$, there exists T_1, such that $|\hat{r}(t)-\bar{r}|<\epsilon\bar{r}$ holds for all $t>T_1$. Thus, we have that $(1-\epsilon)\bar{r}<\hat{r}(t)<(1+\epsilon)\bar{r}, \forall t>T_1$. It follows that

$$(1-\epsilon)\hat{r}(t)<(1+\epsilon)(1-\epsilon)\bar{r}<\bar{r} \qquad \forall t>T_1. \tag{10}$$

From (9), we consider the battery level $B(t)$ as a queue, and Scheme-LBONE as a work-conserving server with service rate $(1-\epsilon)\hat{r}(t)$, which is strictly less than the average arrival rate $\bar{r}$, for $t>T_1$. Hence, when T tends to infinity, the battery level will increase to infinity almost surely. This implies that the probability that the available energy is greater than $\bar{r}$ tends to one as t tends to infinity, i.e., $\lim_{t\to\infty}P(B(t)+r(t)\geq\bar{r})=1$. Combining with (10), we can obtain

$$\lim_{t\to\infty}P\left(B(t)+r(t)>(1-\epsilon)\hat{r}(t)\right)=1. \tag{11}$$

From (9), since $e(t)=\min\{(1-\epsilon)\hat{r}(t),B(t)+r(t)\}$, together with (11), together with $(1-\epsilon)\hat{r}(t)>(1-\epsilon)^2\bar{r}, \forall t>T_1$, we have that

$$\lim_{t\to\infty}P\left(e(t)>(1-\epsilon)^2\bar{r}\right)=1. \tag{12}$$

Equation (12) implies that the probability that the allocated energy is great than $(1-\epsilon)^2\bar{r}$ is one.

Next, we will use epsilon-delta arguments to show that $\lim_{T\to\infty}(1/T)\sum_{t=1}^{T}P(e(t)>(1-\epsilon)^2\bar{r})=1$. According to (12), it follows that, for any $\phi>0$, there exists T_2, such that for all $t>T_2$, $|P(e(t)>(1-\epsilon)^2\bar{r})-1|<(\delta/2)$.

Let $T_3=(4T_2/\delta)$, now $\forall T>T_3$, we have

$$\begin{aligned}
&\left|\frac{1}{T}\sum_{t=1}^{T}P\left(e(t)>(1-\epsilon)^2\bar{r}\right)-1\right| \\
&\leq\frac{1}{T}\sum_{t=1}^{T_2}\left\{\left|P\left(e(t)>(1-\epsilon)^2\bar{r}\right)\right|+1\right\} \\
&\quad+\frac{1}{T}\sum_{t=T_2+1}^{T}\left|P\left(e(t)>(1-\epsilon)^2\bar{r}\right)-1\right| \\
&\leq\frac{2T_2}{T_3}+\frac{(T-T_2)}{T}\frac{\delta}{2} \\
&\leq\phi.
\end{aligned} \tag{13}$$

Therefore, according to epsilon-delta arguments, it follows that

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}P\left(e(t)>(1-\epsilon)^2\bar{r}\right)=1. \tag{14}$$

Now we can obtain the performance bound of Scheme-LBONE as follows:

$$\begin{aligned}
J^{\text{lb}}_{\text{one}}(T)&=\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\left[\mu\left(e(t)\right)\right] \\
&=\frac{1}{T}\sum_{t=1}^{T}\Big\{\mathbb{E}\left[\mu\left(e(t)\right)|e(t)>(1-\epsilon)^2\bar{r}\right] \\
&\qquad\cdot P\left(e(t)>(1-\epsilon)^2\bar{r}\right) \\
&\qquad+\mathbb{E}\left[\mu\left(e(t)\right)|e(t)\leq(1-\epsilon)^2\bar{r}\right] \\
&\qquad\cdot P\left(e(t)\leq(1-\epsilon)^2\bar{r}\right)\Big\} \qquad (15) \\
&\geq\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\left[\mu\left(e(t)\right)|e(t)>(1-\epsilon)^2\bar{r}\right] \\
&\qquad\cdot P\left(e(t)>(1-\epsilon)^2\bar{r}\right)>\mu\left((1-\epsilon)^2\bar{r}\right) \\
&\qquad\cdot\frac{1}{T}\sum_{t=1}^{T}P\left(e(t)>(1-\epsilon)^2\bar{r}\right). \qquad (16)
\end{aligned}$$

where (15) holds because of $\mathbb{E}[X]=\mathbb{E}[X|A]P(A)+\mathbb{E}[X|A^c]P(A^c)$. By taking liminf on both sides of (16), we can obtain from (14) that

$$\liminf_{T\to\infty}J^{\text{lb}}_{\text{one}}(T)\geq\mu\left((1-\epsilon)^2\bar{r}\right). \tag{17}$$

Since Scheme-LBONE is a feasible energy allocation scheme, we have that $\liminf_{T\to\infty}J^*_{\text{one}}(T)\geq\mu((1-\epsilon)^2\bar{r})$. ■

Comment: Note that Scheme-LBONE is an online scheme and does not require knowledge of the future replenishment profile. Hence, for a single-node case, Propositions 1 and 2 imply that Scheme-LBONE can achieve the performance arbitrarily close to the optimum by choosing ϵ sufficiently small.

C. Finite Battery Size

In the previous analysis, we assumed that the battery size is infinite, which is impossible in reality. In this section, we will first show that as long as the battery size is large enough, although finite, we can still guarantee that the performance of Scheme-LBONE is within ξ-neighborhood of the optimum. Furthermore, we show that the required battery size is $O(\ln(1/\xi))$.

Let M denote the battery size. From (16), we can see that the performance loss occurs when $B(t)+r(t)<(1-\epsilon)^2\bar{r}$. Also note that $B(t)+r(t)<(1-\epsilon)^2\bar{r}$ leads to $B(t+1)=0$. Thus, the probability of the energy outage event is given by

$$P^M_\omega=\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbf{1}_{\{B(t)=0\}}$$

where the subscript ω denotes the sample path, which the probability is a function of. We will show that $P^M_\omega\leq\delta$ holds almost surely, where δ is an arbitrary control parameter, when M is $O(\ln(1/\delta))$.

First, the battery can be viewed as a $G/G/1$ queue system with a finite buffer under fluid model, where the energy harvesting process $r(t)$ acts as the input and Scheme-LBONE works as a work-conserving server with service rate $(1-\epsilon)\hat{r}(t)$, as shown in the left figure of Fig. 1. Note that the queue length $B(t)$ evolves as

$$B(t+1)=\min\left\{\max\left\{B(t)+r(t)-e(t),0\right\},M\right\}. \tag{18}$$

Since the load intensity $\rho>1$, which is inconvenient to analyze, we will instead consider a "flipped" $G/G/1$ queue, where the input is $e(t)$ and the service rate is $r(t)$ as shown in the right figure of Fig. 1. Now the flipped queue has a load intensity $\rho<1$. We denote the queue length of the flipped queue

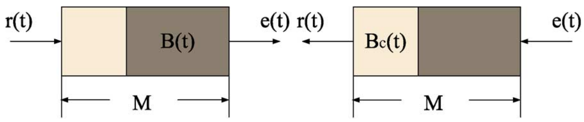

Fig. 1. Sensor node modeled by a G/G/1 queue with finite buffer size.

Fig. 2. $G/G/1$ queue with infinite buffer size.

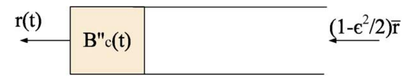

Fig. 3. $D/G/1$ queue with infinite buffer size.

as $B_c(t)$, which is initially assumed to be M. The queue length evolution of $B_c(t)$ is given by

$$B_c(t+1) = \min\left\{\max\left\{B_c(t)+e(t)-r(t), 0\right\}, M\right\}. \quad (19)$$

We claim that for any time-slot t, we always have $B_c(t) = M - B(t)$. We now use mathematical deduction to prove it.

- For t $= 1$, we have $B_c(1) = M = M - B(1)$.
- Assume that $B_c(\tau) = M - B(\tau)$ holds for time-slot τ.
- When $t = \tau + 1$, we have three cases.

Case 1) If $0 < B(\tau)+r(\tau)-e(\tau) < M$, from (18), we have $B(\tau+1) = B(\tau)+r(\tau)-e(\tau)$. On the other hand, we have

$$\begin{aligned}B_c(\tau+1) &= \min\left\{\max\left\{B_c(\tau)+e(\tau)-r(\tau), 0\right\}, M\right\}\\ &= \min\left\{\max\left\{M-B(\tau)+e(\tau)-r(\tau), 0\right\}, M\right\}\\ &= \min\left\{\max\left\{M-(B(\tau)+r(\tau)-e(\tau)), 0\right\}, M\right\}\\ &= M-(B(\tau)+r(\tau)-e(\tau))\\ &= M-B(\tau+1).\end{aligned}$$

Case 2) If $B(\tau)+r(\tau)-e(\tau) \le 0$, we have $B(\tau+1) = 0$.

$$\begin{aligned}B_c(\tau+1) &= \min\left\{\max\left\{B_c(\tau)+e(\tau)-r(\tau), 0\right\}, M\right\}\\ &= \min\left\{\max\left\{M-B(\tau)+e(\tau)-r(\tau), 0\right\}, M\right\}\\ &= \min\left\{\max\left\{M-(B(\tau)+r(\tau)-e(\tau)), 0\right\}, M\right\}\\ &= \min\left\{M-(B(\tau)+r(\tau)-e(\tau)), M\right\}\\ &= M = M-B(\tau+1).\end{aligned}$$

Case 3) If $B(\tau)+r(\tau)-e(\tau) \ge M$, we have $B(\tau+1) = M$. Similarly, we have $B_c(\tau+1) = 0 = M - B(\tau+1)$.

Therefore, we have shown that $B_c(t) = M - B(t)$. As a result, we have

$$P_\omega^M = \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbf{1}_{\{B_c(t)=M\}}. \quad (20)$$

Now, the problem has become how to find a bound on battery size M, such that $\lim_{T\to\infty}(1/T)\sum_{t=1}^{T}\mathbf{1}_{\{B_c(t)=M\}} \le \delta$ almost surely.

Next, we will compare the finite-buffer $G/G/1$ queue with an infinite-buffer $G/G/1$ queue as shown in Fig. 2, for both of which the input process and server are exactly the same. We denote the queue length for the infinite-buffer queue $B_c'(t)$. From [15], we know that $B_c'(t) \ge B_c(t)$ for any sample path, which follows:

$$\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbf{1}_{\{B_c(t)=M\}} \le \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbf{1}_{\{B_c'(t)\ge M\}}. \quad (21)$$

Thus, if we have $\lim_{t\to\infty}(1/T)\sum_{t=1}^{T}\mathbf{1}_{\{B_c'(t)\ge M\}} \le \delta$, it follows that $\lim_{T\to\infty}(1/T)\sum_{t=1}^{T}\mathbf{1}_{\{B_c(t)=M\}} \le \delta$ almost surely.

Next, we compare the infinite-buffer $G/G/1$ queue with an infinite-buffer $D/G/1$ queue, as depicted in Fig. 3, where the input rate is a deterministic value $(1-(\epsilon^2/2))\bar{r}$. We denote the queue length of the infinite-buffer $D/G/1$ queue as $B_c''(t)$. From (10), we know that the $e(t)$ is always less than $(1-\epsilon^2)\bar{r}, \forall t > T_1$. This means that the input of the $G/G/1$ queue is always $(\epsilon^2/2)$ less than the input of the $D/G/1$ queue $\forall t > T_1$. Assuming that $r(t)$ is upper-bounded by $r_{\max}, \forall t$, it follows that $e(t) < r_{\max}, \forall t$, because $\hat{r}(t) \le r_{\max}$. Therefore, it can be seen that $B_c''(t) \ge B_c'(t)$ after the time $T_1 + (2r_{\max}T_1/\epsilon^2)$, which implies that $B_c''(t) \ge B_c'(t)$ always holds after some transient period.

Note that the load intensity for the $D/G/1$ queues is less than 1, and both input and output processes are stationary. Thus, the stationary distribution of the queue length exists. Hence, we have the stationary distribution of the $D/G/1$ queue forms an upper bound, that is

$$\begin{aligned}&\lim_{t\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbf{1}_{\{B_c'(t)\ge M\}}\\ &\le \lim_{t\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbf{1}_{\{B_c''(t)\ge M\}}\\ &= \lim_{t\to\infty} P\left(B_c''(t) \ge M\right) \qquad \text{(almost surely)} \quad (22)\end{aligned}$$

where $\lim_{t\to\infty} P(B_c''(t) \ge M)$ denotes the stationary probability of the event $B_c''(t) \ge M$.

Now our goal is to find a battery size M, such that $\lim_{t\to\infty} P(B_c''(t) \ge M) \le \delta$.

Note that if the replenishment process is Markovian or i.i.d., from [23], we have

$$\lim_{t\to\infty} P\left(B_c''(t) \ge M\right) \le \exp(-\gamma^* M) \quad (23)$$

where γ^* is a positive constant. It is worth pointing out that the i.i.d. case coincides with the well-known Kingman's Bound. In fact, (23) holds under more general replenishment processes [24].

By letting $\exp(-\gamma^* M) = \delta$, it follows that

$$M = \frac{\ln\left(\frac{1}{\delta}\right)}{\gamma^*}. \quad (24)$$

Hence, from (20)–(24), we have that when $M = (\log(1/\delta)/\gamma^*)$, the probability of the energy outage in the original queue is less than δ almost surely. This implies that $e(t) \geq (1-\epsilon)^2\bar{r}$ with probability greater than $1-\delta$.

By taking $\delta = \epsilon = (\xi/3)$, we have

$$\begin{aligned}\frac{\liminf_{T\to\infty} J_{\mathrm{one}}^{\mathrm{lb}}(T)}{\limsup_{T\to\infty} J_{\mathrm{one}}^{*}(T)} &\geq \frac{(1-\delta)\mu\left((1-\epsilon)^2\bar{r}\right)}{\mu(\bar{r})}\\ &\geq \frac{(1-\delta)(1-\epsilon)^2\mu(\bar{r})}{\mu(\bar{r})}\\ &= (1-\delta)(1-\epsilon)^2 = \left(1-\frac{\xi}{3}\right)^3\\ &> (1-\xi). \end{aligned} \tag{25}$$

Combining with (24), we can see that the required battery size under Scheme-LBONE is $(\ln(1/\xi)/\gamma^*) + (\ln 3/\gamma^*)$, i.e., $O(\ln(1/\xi))$, which is better than the bound $O(1/\xi)$ in [12] and [13].

IV. Utility Maximization: A Network Case

In this section, we investigate the problem of maximizing utility over the network with energy replenishment. In our formulation Problem **A**, we denote the achievable maximum utility by $J^*(T) \triangleq \max\sum_s U_s((1/T)\sum_{t=1}^{T} x^s(t))$. We first provide an upper bound on $J^*(T)$ using an infeasible scheme, and then propose a low-complexity online scheme that does not require future knowledge of replenishment profile. We show that the performance of our proposed scheme approaches the upper bound as time T tends to infinity.

A. Upper Bound

We consider a fictitious infeasible scheme, denoted by Scheme-UB, which not only knows in advance the average energy harvesting rate $\bar{r}_i$ for all $i \in \mathcal{N}$, but also can allocate more energy than the harvested energy. Scheme-UB works as follows.

- *Energy allocation:* Each node i spends a fixed amount of energy $\bar{r}_i(1+\epsilon)$ in all time-slots, i.e.,

$$e_i(t) = \bar{r}_i(1+\epsilon), \qquad \text{for all } i \text{ and } t. \tag{26}$$

  Clearly, this is more than the average replenishment rate and thus infeasible.
- *Routing:* The routing in each time-slot t is determined by solving the following strictly convex optimization problem:

$$\begin{aligned}&\max_{\vec{w}_{ij},\vec{x}^s} \sum_s U_s\left(x^s(t)\right)\\ &\text{subject to } w_{ij}^d(t) \geq 0 \qquad \forall d, \quad \forall (i,j)\in\mathcal{L}\\ &\quad \sum_{j:(i,j)\in\mathcal{L}} w_{ij}^d(t) - \sum_{j:(i,j)\in\mathcal{L}} w_{ji}^d(t) - \sum_{f_s=i,d_s=d} x^s(t) \geq 0,\\ &\hspace{20em} \forall d, \text{ and } \forall i \neq d\\ &\quad \sum_{j:(i,j)\in\mathcal{L}} \sum_d w_{ij}^d(t) \leq \mu\left(\bar{r}_i(1+\epsilon)\right) \qquad \forall i \in \mathcal{N}. \end{aligned} \tag{27}$$

In contrast to Problem **A**, the third constraint in the above problem is not coupled across time, which implies that routing decision in each time-slot t can be solved independently. We denote the unique solution to (27) by $\vec{x}_{\mathrm{ub}}(t) = [x_{\mathrm{ub}}^s(t)]$. Though Scheme-UB is an infeasible scheme, we will show that its performance, defined as $J^{\mathrm{ub}}(T) \triangleq \sum_s U_s((1/T)\sum_{t=1}^{T} x_{\mathrm{ub}}^s(t))$, dominates the optimal performance $J^*(T)$. Also, since the energy allocation and routing in Scheme-UB do not change over time, it follows that $x_{\mathrm{ub}}^s(t)$ is the same in all time-slots, which we denote as x_{ubc}^s. By denoting $J_c^{\mathrm{ub}} \triangleq \sum_s U_s(x_{\mathrm{ubc}}^s)$, we have $J^{\mathrm{ub}}(T) = J_c^{\mathrm{ub}}$.

Proposition 3: When T tends to infinity, $J^*(T)$ is upper-bounded by $J^{\mathrm{ub}}(T)$, and we have that $\limsup_{T\to\infty} J^*(T) \leq \limsup_{T\to\infty} J^{\mathrm{ub}}(T) = J_c^{\mathrm{ub}}$.

We refer to Appendix A for the proof.

B. Lower Bound

In this section, we propose a low-complexity online scheme, denoted by Scheme-LB, and show that its performance approaches the upper bound obtained in Section IV-A when T tends to infinity. We begin with the algorithm description of Scheme-LB.

- *Energy allocation:* As in Scheme-LBONE, in each time-slot t, each node estimates its average harvested energy as

$$\hat{r}_i(t) \triangleq \frac{1}{t}\sum_{\tau=1}^{t} r_i(\tau). \tag{28}$$

  Then, energy is allocated as

$$e_i(t) = \begin{cases}(1-\epsilon)\hat{r}_i(t), & \text{if } B_i(t)+r_i(t) \geq (1-\epsilon)\hat{r}_i(t),\\ B_i(t)+r_i(t), & \text{otherwise.}\end{cases} \tag{29}$$

- *Routing:* Routing in each time-slot t is determined by solving the following optimization problem:

$$\begin{aligned}&\max_{\vec{w}_{ij},\vec{x}^s} \sum_s U_s\left(x^s(t)\right)\\ &\text{subject to } w_{ij}^d(t) \geq 0 \qquad \forall d, \quad \forall (i,j)\in\mathcal{L}\\ &\quad \sum_{j:(i,j)\in\mathcal{L}} w_{ij}^d(t) - \sum_{j:(i,j)\in\mathcal{L}} w_{ji}^d(t) - \sum_{f_s=i,d_s=d} x^s(t) \geq 0,\\ &\hspace{20em} \forall d, \text{ and for } i \neq d\\ &\quad \sum_{j:(i,j)\in\mathcal{L}} \sum_d w_{ij}^d(t) \leq \mu\left(e_i(t)\right) \qquad \forall i \in \mathcal{N}. \end{aligned} \tag{30}$$

We denote the solution to (30) by $\vec{x}_{\mathrm{lb}}(t) = [x_{\mathrm{lb}}^s(t)]$. Note that the difference from Scheme-UB is the energy allocation, which is now based on the estimated average replenishment rate. Let $J^{\mathrm{lb}}(T) \triangleq \mathbb{E}[\sum_s U_s((1/T)\sum_{t=1}^{T} x_{\mathrm{lb}}^s(t))]$, where the expectation is taken over the sample space of the replenishment process. Also, let x_{lbc}^s denote the solution to (30) when $e_i(t) = (1-\epsilon)^2\bar{r}_i$ for all $i \in \mathcal{N}$ and $J_c^{\mathrm{lb}} \triangleq \sum_s U_s(x_{\mathrm{lbc}}^s)$. Then, we can obtain the following proposition.

Proposition 4: When T tends to infinity, $J^*(T)$ is lower-bounded by $J^{\mathrm{lb}}(T)$, and we have that $\liminf_{T\to\infty} J^*(T) \geq \liminf_{T\to\infty} J^{\mathrm{lb}}(T) \geq J_c^{\mathrm{lb}}$.

We refer to Appendix B for the proof.

Recall that both $J_c^{\rm ub}$ and $J_c^{\rm lb}$ are functions of ϵ. Next, we show via the following proposition that the lower bound $J_c^{\rm lb}$ can be arbitrarily close to the upper bound $J_c^{\rm ub}$ by setting ϵ sufficiently small.

Proposition 5: For any $\delta > 0$, there exists $\epsilon > 0$, such that $|J_c^{\rm ub} - J_c^{\rm lb}| < \delta$.

Proof: We define the ratio of two transmission rates

$$k \triangleq \min_{i\in\mathcal{N}} \frac{\mu\left((1-\epsilon)^2\bar{r}_i\right)}{\mu\left((1+\epsilon)\bar{r}_i\right)}. \tag{31}$$

Since $\mu(\cdot)$ is an increasing concave function, we have from Jensen's inequality that

$$\mu\left((1-\epsilon)^2\bar{r}_i\right) = \mu\left((1-\epsilon)^2\bar{r}_i + (2\epsilon-\epsilon^2)\times 0\right) \geq (1-\epsilon)^2\mu(\bar{r}_i)$$

and similarly we have $\mu((1+\epsilon)\bar{r}_i) \leq (1+\epsilon)\mu(\bar{r}_i)$ for all $i \in \mathcal{N}$. Hence, from the definition of k, it follows that

$$(1-\epsilon)^3 < \frac{(1-\epsilon)^2}{(1+\epsilon)} \leq k \leq 1. \tag{32}$$

Let $(\vec{w}_{ij}^{d*}, \vec{x}^{s*})$ denote an optimal solution to (27). Clearly, we have that $J_c^{\rm ub} = \sum_s U_s(x^{s*}(t))$. Then, we consider another vector $(k\vec{w}_{ij}^{d*}, k\vec{x}^{s*})$. Since $(\vec{w}_{ij}^{d*}, \vec{x}^{s*})$ is an optimal solution to (27), it satisfies all the constraints of (27). From the first and the second constraints of (27), we can easily show that the constant-mulitplied vector $(k\vec{w}_{ij}^{d*}, k\vec{x}^{s*})$ satisfies the first two constraints of (30). Also from the third constraint of (27) and the definition of k, we have that

$$\sum_j \sum_d k w_{ij}^{d*}(t) \leq k\mu\left(\bar{r}_i(1+\epsilon)\right) \leq \mu\left((1-\epsilon)^2\bar{r}_i\right).$$

Hence, the vector $(k\vec{w}_{ij}^{d*}, k\vec{x}^{s*})$ also satisfies the third constraint of (30) when $e_i(t) = (1-\epsilon)^2\bar{r}_i$ for all $i \in \mathcal{N}$. Since $J_c^{\rm lb}$ is the achievable maximum utility of (30) when $e_i(t) = (1-\epsilon)^2\bar{r}_i$ for all $i \in \mathcal{N}$, we have that

$$\begin{aligned} J_c^{\rm lb} &\geq \sum_s U_s\left(kx^{s*}(t)\right) \\ &\geq k\sum_s U_s\left(x^{s*}(t)\right) \qquad (33) \\ &= kJ_c^{\rm ub} \\ &\geq (1-\epsilon)^3 J_c^{\rm ub} \qquad (34) \end{aligned}$$

where (33) holds because $U_s(\cdot)$ is an increasing concave function and $k \leq 1$, and (34) comes from (32).

Therefore, we have $(1-\epsilon)^3 J_c^{\rm ub} \leq J_c^{\rm lb} \leq J_c^{\rm ub}$, where the latter inequality directly comes from Propositions 3 and 4. Thus, for any $\delta > 0$, we can find $\epsilon > 0$, such that $|J_c^{\rm ub} - J_c^{\rm lb}| < \delta$. ■

Proposition 5 implies that if ϵ is chosen to be sufficiently small, the performance of Scheme-LB approaches the optimal performance, as T tends to infinity. Hence, Scheme-LB is asymptotically optimal.

C. Distributed Algorithm Based on Duality

Note that Scheme-LB should solve a convex optimization problem, i.e., (30), in each time-slot t in a centralized manner. In this section, we extend our solution and develop a low-complexity distributed scheme that approximates Scheme-LB using the standard optimization technique of duality [16], [17].

From the dual counterpart to (30), we can obtain the following solution, denoted by *DualNet*, which can be implemented in a distributed manner. Since the technique is quite standard, we omit details and refer interested readers to our technical report [21].

- At each time t, source s generates data at rate $x^s(t)$ by solving

$$\max_{0\leq x^s(t)\leq x_{\max}} U_s\left(x^s(t)\right) - p_{f_s}^{d_s}(t)x^s(t) \tag{35}$$

  where $x_{\max}$ is a constant for the maximum data rate and $p_i^d(t)$ is the associated Lagrange multiplier for each second constraint of (30).
- Routing at each node i is determined by solving

$$\max_{0\leq\sum_j\sum_d w_{ij}^d(t)\leq\mu(e_i(t))} \sum_j\sum_{d\neq i} w_{ij}^d(t)\left(p_i^d(t) - p_j^d(t)\right). \tag{36}$$

- The Lagrange multipliers are updated as

$$p_i^d(t+1) = \left[p_i^d(t) - h\left(\sum_{j:(i,j)\in\mathcal{L}} w_{ij}^d(t) - \sum_{j:(i,j)\in\mathcal{L}} w_{ji}^d(t) - \sum_{f_s=i,d_s=d} x^s(t)\right)\right]^+ \tag{37}$$

  where h is a small step size.

It is worthwhile pointing out that (36) allocates energy for node i to transmit the data of commodity d to node j, where j and d are chosen for the largest $p_i^d(t) - p_j^d(t)$, which is similar to the well-known back-pressure scheme without interference constraint. Note that using the standard optimization technique, the performance of the dual solution gets closer to the optimal by increasing the number of iterations. Hence, the performance of *DualNet*, which performs a single iteration in each time-slot, will improve if we embed multiple iterations in each time-slot. Nevertheless, we show via simulations that *DualNet* with a single iteration still achieves good empirical performance that is close to the upper bound.

In addition, we know from the previous discussion that with probability one, the allocated energy of each node i in Scheme-LB tends to a static value, i.e., $(1-\epsilon)\hat{r}_i$. Therefore, the convergence of *DualNet* can be always guaranteed irrespective of the number of iterations per slot.

D. Finite Battery Size for the Network Case

In Section III-C, we showed that with a finite battery size of $O(\ln(1/\xi))$, the performance of Scheme-LBONE is within ξ-neighborhood of the optimum. In this section, we will extend this idea to the network case.

i) Proposition 3 still holds, and $J_c^{\rm ub}$ is an upper bound.
ii) From (47) in the proof of Proposition 4, we have $\lim_{t\to\infty} P(e_i(t) \geq (1-\epsilon)^2\bar{r}_i, \forall i \in \mathcal{N}) = 1$ under Scheme-LB, which, however, does not hold any longer if the battery size is finite. Extending the one-node analysis

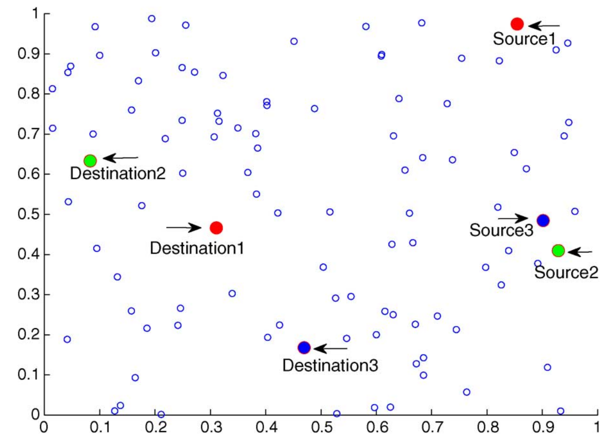

Fig. 4. Sensor network with 100 nodes.

in Section III-C, we can find a battery size M such that, for any $\delta > 0$, $\lim_{t\to\infty} P^M(e(t) > (1-\epsilon)^2\bar{r}) > 1-\delta$, for all the nodes. Hence, we have

$$\lim_{t\to\infty} P(e_i(t) \geq (1-\epsilon)^2\bar{r}_i \qquad \forall i \in \mathcal{N}) > (1-\delta)^{|\mathcal{N}|} > 1-|\mathcal{N}|\delta \tag{38}$$

where $|\mathcal{N}|$ is the number of nodes in the network.

iii) Following the same lines as the proof of Proposition 4, we can obtain that $\liminf_{T\to\infty} J^*(T) \geq (1-|\mathcal{N}|\delta)J_c^{\mathrm{lb}}$. Since $|\mathcal{N}|$ is bounded, the lower bound $(1-|\mathcal{N}|\delta)J_c^{\mathrm{lb}}$, and the bound on the required battery size is given by

$$M = \frac{\ln 3 + \ln|\mathcal{N}|}{\gamma^*} + \frac{\ln\left(\frac{1}{\xi}\right)}{\gamma^*} \tag{39}$$

which remains $O(\ln(1/\xi))$.

V. Numerical Evaluation

We evaluate our schemes through simulations. We consider a network with 100 nodes, which are randomly deployed in a 1×1 field, as shown in Fig. 4. We connect each pair of nodes within distance 0.2 by a link. We set three flows in the network, where the source and the destination for each flow are marked with the same color in the figure. We compare the performance of *DualNet* to a state-of-the-art scheme called *ESA* [13], which achieves asymptotic optimality under i.i.d. energy replenishment profiles. We assume that the rate-power function follows $\mu(P) = \ln(1+10P)$ (bits/s), and the utility function is given as $U_s(x_s) = \ln(1+x_s)$. We set the parameter ϵ to 10^{-4}. The battery sizes are assumed to be infinite.

We simulate the schemes with two different types of renewable energy: solar and wind. We adopt raw data collected at the National Renewable Energy Laboratory [20] for a period of one month (June 5–July 5, 2011) and set each time-slot to 1 min. Fig. 5 illustrates the two types of replenishment profiles during the month. The solar energy data set (Global 40-South LI-200) measures solar resource for collectors tilted 40° from the horizontal and optimized for year-round performance. From the data, we can obtain the replenishment profile for the solar energy, assuming that each node is equipped with a solar panel of dimension 20×20 mm^2. For the wind resource, the data is measured using sensors placed 2 m from the ground. The power

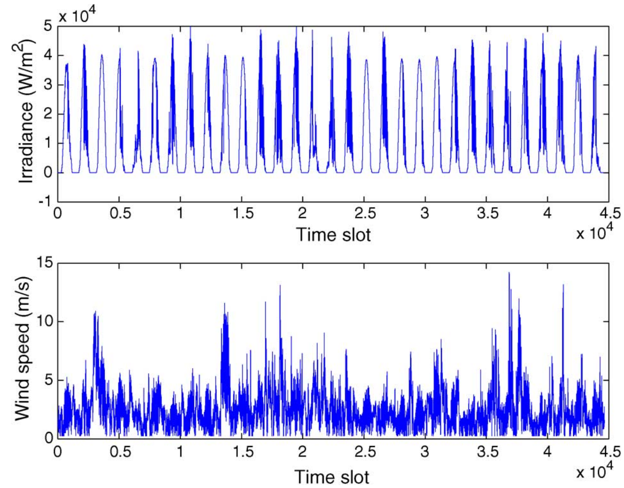

Fig. 5. Measurement for solar and wind energy.

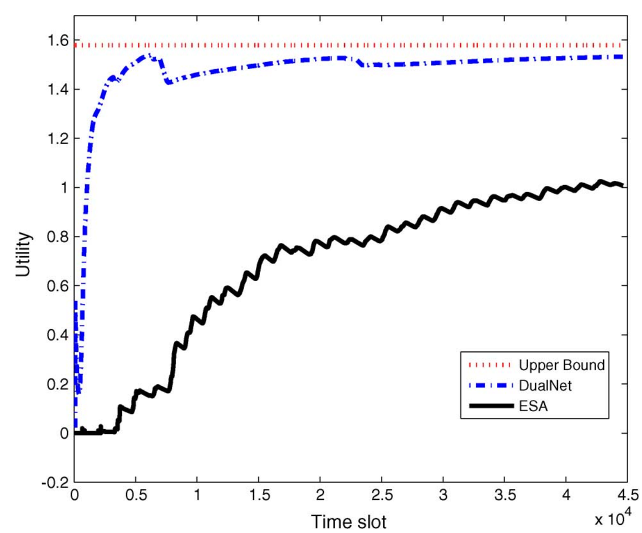

Fig. 6. Utility performance for solar energy.

can be calculated from the measured wind speed V as in [22]: $P_{\mathrm{wind}} = 0.5 \times \rho \times A \times V^3$, where ρ denotes the air density set to $\rho = 1.23$ (kg/m^3), and A is the swept area of the wind turbine set to $A = 50 \times 50$ mm^2.

Figs. 6 and 7 show the simulation results for the solar energy and the wind energy, respectively. The dotted curve represents the upper bound J_c^{ub} that is obtained by solving (27) for the given ϵ. It can be considered as the utility achieved by the infeasible scheme Scheme-UB. The dashed curve represents the utility achieved by *DualNet*. For both energy sources, the performance of *DualNet* approaches the upper bound as time increases. Also, an interesting observation in both results is that the performance achieved by *DualNet* has been once close to the upper bound when time is fairly small. This phenomenon occurs because the estimated average harvested energy at that time is greater than the actual (long-term) average. The results also show that *DualNet* outperforms *ESA*, and the performance differences are significant even after a long time period. This is because the Lyapunov optimization technique adopted by *ESA* requires an assumption that the replenishment energy in each time-slot is either i.i.d. or Markovian. In contrast, our solution

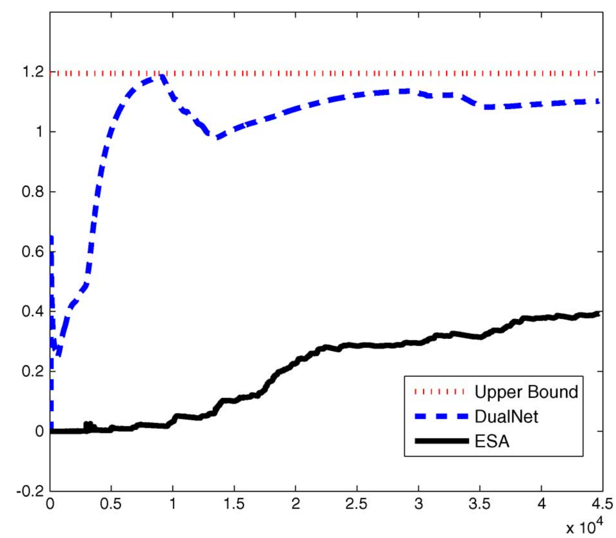

Fig. 7. Utility performance for wind energy.

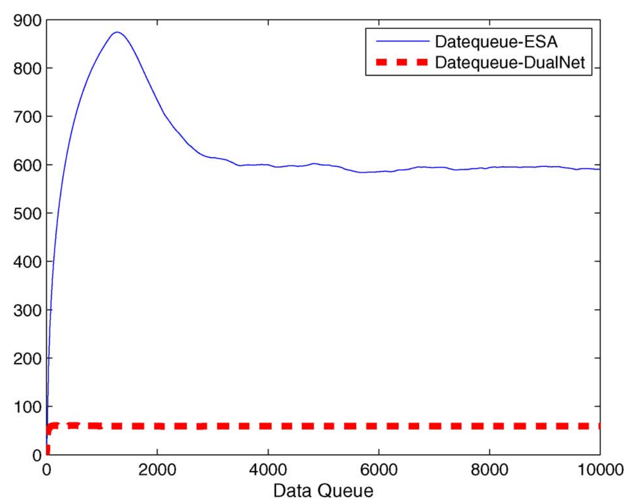

Fig. 8. Data queue for one-node case.

is developed under a mild assumption requiring only the existence of mean replenishment rate.

A. Discussion

To better demonstrate the reason for the difference, we consider the simplest network with one source and one destination. We simulate both schemes assuming that the energy arrival process is an i.i.d Poisson process with parameter $\lambda = 1$. Fig. 8 illustrates the data queue evolution, and Fig. 9 shows the energy queue evolution. From Fig. 8, we can see that our scheme *DualNet* has a shorter queue length, that is, a better delay performance, than *ESA*. Also, from Fig. 9, we can observe that *DualNet* performs well with much smaller battery size, which is set to be 100 units compared to 1800 in *ESA*.

Note that under *ESA* or other schemes using Lyapunov optimization technique, the allocated energy in each time-slot is a function of its current queue length and current energy level, i.e., $e(t) = f(Queue(t), B(t))$. From our analytical results, we have seen that the optimal utility can be achieved by a static energy allocation close to the average harvesting rate $\bar{r}$. From this,

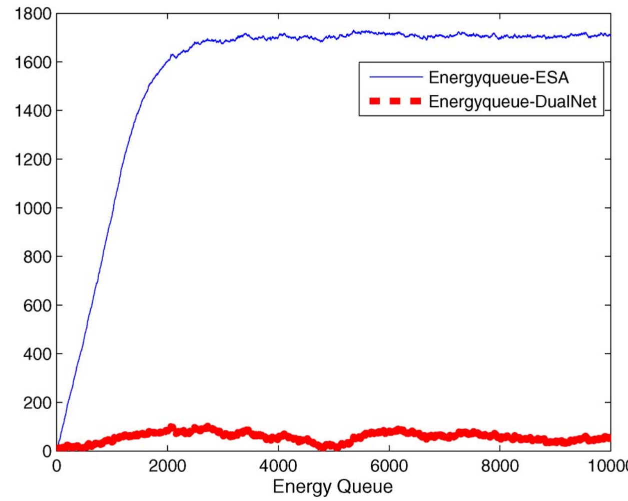

Fig. 9. Energy queue for one-node case.

we can infer that *ESA* will start performing well when the energy allocation becomes static, in other words, when $Queue(t)$ and $B(t)$ increase to some high levels such that their variations at each time-slot is relatively small. Since it will take long to reach a large queue length and a high battery level, we can see a fairly long transient period before it converges in Figs. 8 and 9. Similar phenomena occur in other contexts, such as the poor delay performance of CSMA-based scheduler [18].

In contrast, in our scheme, the allocated energy $e(t)$ converges to $\bar{r}$ in a more straightforward way without causing the data queues or the energy levels to build up. Therefore, our scheme has a better delay performance as well as smaller battery size requirement.

VI. Conclusion

In this paper, we study the joint problem of energy allocation and routing to maximize total user utility in a sensor network with energy replenishment. Under general replenishment profiles with finite mean value, we develop a low-complexity online solution that is asymptotically optimal. Characterizing the optimal performance by an upper bound achieved by an infeasible solution, we show that the long-term performance of our online solution approaches the upper bound. To the best knowledge of the authors, this is the first result that achieves asymptotic optimality in multihop networks with general energy replenishment profiles. Also, by removing time coupling properties between controls, our online solution achieves low complexity and can be approximated by a distributed algorithm. Moreover, we show that the required battery size is $O(\ln(1/\xi))$ to constrain the performance of our scheme within ξ-neighborhood of the optimum. Through simulations based on traces from two different types of energy source, we evaluate our solutions and show that it outperforms a state-of-the-art scheme and achieves the performance close to the optimal. An important question that remains unanswered is whether one can develop such simple asymptotically optimal schemes for networks with replenishment that also take into account interference. This is an interesting and important question that we plan to pursue for future work in this area.

Appendix A
Proof of Proposition 3

Proof: From the stationary property of the problem, we have $x^s_{\text{ub}}(t) = x^s_{\text{ubc}}$ for all t. Thus, we have that

$$\limsup_{T\to\infty} J^{\text{ub}}(T) = \limsup_{T\to\infty} \sum_s U_s\left(x^s_{\text{ubc}}\right) = J^{\text{ub}}_c.$$

We will prove that $\limsup_{T\to\infty} J^*(T) \le J^{\text{ub}}_c$ by showing that the achievable maximum utility of (27) is no smaller than that of a solution to Problem $\mathbf{A}$.

We first consider the following problem, where the difference from Problem $\mathbf{A}$ is the last constraint:

$$\begin{aligned}
&\max \sum_s U_s\left(1T\sum_{t=1}^{T} x^s(t)\right)\\
&\text{subject to} \quad w^d_{ij}(t) \ge 0 \qquad \forall t, \quad \forall d, \quad \forall (i,j)\in\mathcal{L}\\
&\sum_{t=1}^{T}\sum_j w^d_{ij}(t) - \sum_{t=1}^{T}\sum_j w^d_{ji}(t) - \sum_{t=1}^{T}\sum_{s:f_s=i,d_s=d} x^s(t) \ge 0,\\
&\qquad\qquad \forall d, \text{ and for } i\ne d\\
&\sum_{t=1}^{T}\sum_{j:(i,j)\in\mathcal{L}}\sum_d w^d_{ij}(t) \le T\mu\left(\bar{r}_i(1+\epsilon)\right) \qquad \forall i\in\mathcal{N}.
\end{aligned} \tag{40}$$

From (7), it is clear that the last constraint in Problem $\mathbf{A}$, i.e., $\sum_{j:(i,j)\in\mathcal{L}} \vec{w}_{ij} \in \Lambda_i$, is stricter than the last constraint of (40), $\sum_t\sum_{j:(i,j)\in\mathcal{L}}\sum_d w^d_{ij}(t) \le T\mu(\bar{r}_i(1+\epsilon))$, when T is sufficiently large. Hence, by letting $J^B(T) \triangleq \max\sum_s U_s((1/T)\sum_{t=1}^{T} x^s(t))$ denote the achievable maximum utility of (40), we have that

$$\limsup_{T\to\infty} J^*(T) \le \limsup_{T\to\infty} J^B(T). \tag{41}$$

We also consider another strictly convex optimization problem with the same objective function and show that its solution is also the solution to (40), which implies that both optimization problems have the same maximum utility

$$\begin{aligned}
&\max \sum_s U_s\left(\frac{1}{T}\sum_{t=1}^{T} x^s(t)\right)\\
&\text{subject to} \quad w^d_{ij}(t) \ge 0 \qquad \forall t, \quad \forall d, \quad \forall (i,j)\in\mathcal{L}\\
&\sum_{j:(i,j)\in\mathcal{L}} w^d_{ij}(t) - \sum_{j:(i,j)\in\mathcal{L}} w^d_{ji}(t) - \sum_{f_s=i,d_s=d} x^s(t) \ge 0,\\
&\qquad\qquad \forall t, \quad \forall d, \text{ and for } \subset\ne d\\
&\sum_{j:(i,j)\in\mathcal{L}}\sum_d w^d_{ij}(t) \le \mu\left(\bar{r}_i(1+\epsilon)\right) \qquad \forall t, \quad \forall i\in\mathcal{N}.
\end{aligned} \tag{42}$$

Note that the difference from (40) is the last two constraints, where now we do not have summation over time. The solution space of (40) includes the solution space of (42) since it can be easily shown that if $(\vec{w}^d_{ij}, \vec{x}^s)$ satisfies the constraints of (42), it also satisfies the constraints of (40). This implies that if the optimal solution to (40) also satisfies all the constraints of (42), then it is also the optimal solution to (42).

Let $(\vec{w}^{d'}_{ij}, \vec{x}^{s'})$ denote the optimal solution to (40). Also, we define two constants $w^{d''}_{ij} \triangleq (1/T)\sum_{t=1}^{T} w^{d'}_{ij}(t)$ and $x^{s''} \triangleq (1/T)\sum_{t=1}^{T} x^{s'}(t)$. We consider a time-invariant vector $(\vec{w}^{d''}_{ij}, \vec{x}^{s''})$, where $w^{d''}_{ij}(t) = w^{d''}_{ij}$ and $x^{s''}(t) = x^{s''}$ for all time-slots. We will show that this time-invariant vector is a common optimal solution to both (40) and (42).

We first show that it is an optimal solution to (40). Since $(\vec{w}^{d'}_{ij}, \vec{x}^{s'})$ is a solution to (40), it satisfies the constraints and we have that $\sum_{t=1}^{T}\sum_j w^{d'}_{ij}(t) - \sum_{t=1}^{T}\sum_j w^{d'}_{ji}(t) - \sum_{t=1}^{T}\sum_{s:f_s=i,d_s=d} x^{s'}(t) \ge 0$. Dividing by T, we obtain that

$$\sum_j w^{d''}_{ij}(t) - \sum_j w^{d''}_{ji}(t) - \sum_{s:f_s=i,d_s=d} x^{s''}(t) \ge 0 \tag{43}$$

for all $t\in[1,T]$, since $(w^{d''}_{ij}(t), x^{s''}(t))$ are equal over time. Hence, the inequality is also true when summing from $t=1$ to T. Hence, $(\vec{w}^{d''}_{ij}, \vec{x}^{s''})$ satisfies the second constraint of (40). Similarly, since we have $\sum_{t=1}^{T}\sum_{j:(i,j)\in\mathcal{L}}\sum_d w^{d'}_{ij}(t) \le T\mu(\bar{r}_i(1+\epsilon))$, dividing by T, we have that

$$\sum_{j:(i,j)\in\mathcal{L}}\sum_d w^{d''}_{ij}(t) \le \mu\left(\bar{r}_i(1+\epsilon)\right). \tag{44}$$

By taking the summation from $t=1$ to T, it yields that $\sum_{t=1}^{T}\sum_{j:(i,j)\in\mathcal{L}}\sum_d w^{d''}_{ij}(t) \le T\mu(\bar{r}_i(1+\epsilon))$. Therefore, $(\vec{w}^{d''}_{ij}, \vec{x}^{s''})$ satisfies all the constraints of (40). Also, we have that

$$\sum_{t=1}^{T} x^{s'}(t) = T\cdot\frac{1}{T}\sum_{t=1}^{T} x^{s'}(t) = \sum_{t=1}^{T} x^{s''}(t).$$

This means that $(\vec{w}^{d''}_{ij}, \vec{x}^{s''})$ achieves the same utility value as the optimal solution $(\vec{w}^{d'}_{ij}, \vec{x}^{s'})$, which implies that it is another optimal solution to (40).

We next show that $(\vec{w}^{d''}_{ij}, \vec{x}^{s''})$ is also an optimal solution to (42). Note that from our earlier statement on the solution spaces of (40) and (42), it suffices to show that $(\vec{w}^{d''}_{ij}, \vec{x}^{s''})$ satisfies all the constraints of (42), which has already been obtained from (43) and (44). Hence, $(\vec{w}^{d''}_{ij}, \vec{x}^{s''})$ is an optimal solution to (42).

Let $J^C(T)$ denote the achievable optimal utility of (42). Since both optimization problem (40) and (42) have an identical objective function and share at least a common maximizer, the achievable optimal utility should be equal, i.e.,

$$J^B(T) = J^C(T). \tag{45}$$

Furthermore, from our development of the common solution, we can always find an optimal solution to (42) that is time-invariant, and thus we can reduce the solution space to time-invariant vectors without affecting the achievable maximum utility. Next, we will prove that $J^C(T) = J^{\text{ub}}_c$, which is the achievable maximum utility of the optimal solution to (27).

First, note that the time-invariant solution $(\vec{w}^{d''}_{ij}, \vec{x}^{s''})$ to (42) satisfies the constraints of (27) since the constraints of both equations are the same. This implies that $J^{\text{ub}}_c \ge \sum_s U_s(x^{s''}) = \sum_s U_s((1/T)\sum_{t=1}^{T} x^{s''}(t)) = J^C(T)$.

On the other hand, let $(\vec{w}^{d*}_{ij}, \vec{x}^{s*})$ represent one solution to (27). Thus, we have $J^{\text{ub}}_c = \sum_s U_s(x^{s*})$. Consider the time-invariant vector $(\vec{w}^{d*}_{ij}, \vec{x}^{s*})$, where $w^{d*}_{ij}(t) = w^{d*}_{ij}$ and $x^{s*}(t) = x^{s*}$ for all time-slots. Note that $(\vec{w}^{d*}_{ij}, \vec{x}^{s*})$ satisfies all the constraints of (42) and thus leads to a suboptimal value, i.e., $J^C(T) \geq \sum_s U_s((1/T)\sum_{t=1}^{T} x^{s*}(t)) = \sum_s U_s(x^{s*}) = J_c^{\mathrm{ub}}$. Thus, we have proved that

$$J^C(T) = J_c^{\mathrm{ub}}. \tag{46}$$

Therefore, we have that from (41), (45), and (46)

$$\limsup_{T\to\infty} J^*(T) \leq \limsup_{T\to\infty} J^B(T) = \limsup_{T\to\infty} J^C(T) = J_c^{\mathrm{ub}}.$$

■

Appendix B
Proof of Proposition 4

Proof: Since Scheme-LB is a feasible scheme, we have $J^*(T) \geq J^{\mathrm{lb}}(T)$ by definition.

The energy allocation component of Scheme-LB is exactly the same as Scheme-LBONE for the single node case, thus all the results in Section III-B also hold. Let A_i denote the event $e_i(t) \geq (1-\epsilon)^2\bar{r}_i$. From (12), we have that $\lim_{t\to\infty} P(A_i) = 1$ for each i. Given a finite number of nodes in the network, we can obtain that

$$\lim_{t\to\infty} P\left(e_i(t) \geq (1-\epsilon)^2\bar{r}_i \qquad \forall i \in \mathcal{N}\right) = 1 \tag{47}$$

which immediately implies [as in (13)]

$$\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} P\left(e_i(t) \geq (1-\epsilon)^2\bar{r}_i \qquad \forall i \in \mathcal{N}\right) = 1. \tag{48}$$

Then, we can obtain that

$$\begin{aligned}
&J^{\mathrm{lb}}(T)\\
&= \mathbb{E}\left[\sum_s U_s\left(\frac{1}{T}\sum_{t=1}^{T} x_{\mathrm{lb}}^s(t)\right)\right]\\
&\geq \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}\left[\sum_s U_s\left(x_{\mathrm{lb}}^s(t)\right)\right]\\
&\geq \frac{1}{T}\sum_{t=1}^{T}\Bigg\{\mathbb{E}\Bigg[\sum_s U_s\left(x_{\mathrm{lb}}^s(t)\right)|e_i(t) \geq (1-\epsilon)^2\bar{r}_i, \forall i \in \mathcal{N}\Bigg]\\
&\quad \cdot P\Big(e_i(t) \geq (1-\epsilon)^2\bar{r}_i, \quad \forall i \in \mathcal{N}\Big)\\
&\quad + \mathbb{E}\Bigg[\sum_s U_s\left(x_{\mathrm{lb}}^s(t)\right)|e_i(t) < (1-\epsilon)^2\bar{r}_i, \text{for some } i \in \mathcal{N}\Bigg]\\
&\quad \cdot P\left(e_i(t) < (1-\epsilon)^2\bar{r}_i, \text{for some } i \in \mathcal{N}\right)\Bigg\}\\
&\geq \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}\left[\sum_s U_s\left(x_{\mathrm{lb}}^s(t)\right)|e_i(t) \geq (1-\epsilon)^2\bar{r}_i, \quad \forall i \in \mathcal{N}\right]\\
&\quad \cdot P\left(e_i(t) \geq (1-\epsilon)^2\bar{r}_i, \quad \forall i \in \mathcal{N}\right)\\
&\geq \frac{1}{T}\sum_{t=1}^{T} J_c^{\mathrm{lb}} \cdot P\left(e_i(t) \geq (1-\epsilon)^2\bar{r}_i, \quad \forall i \in \mathcal{N}\right)
\end{aligned} \tag{49}$$

where the first inequality holds due to Jensen's Inequality as well as the concavity of $U(\cdot)$, the second inequality holds because of $\mathbb{E}[X] = \mathbb{E}[X|A]P(A) + \mathbb{E}[X|A^c]P(A^c)$, and the last inequality holds since J_c^{lb} is achieved when $e_i(t) = (1-\epsilon)^2\bar{r}_i, \forall i \in \mathcal{N}$. Taking liminf on the both sides and from (48), we can obtain that

$$\liminf_{T\to\infty} J^*(T) \geq \liminf_{T\to\infty} J^{\mathrm{lb}}(T) \geq J_c^{\mathrm{lb}}.$$

■

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