# Queuing Analysis for Radio Link Level Scheduling in a Multi-Rate TDMA Wireless Network

Long Bao Le, Ekram Hossain, and Attahiru Sule Alfa
University of Manitoba, Winnipeg, MB R3T 5V6, Canada.
Email: {long, ekram, alfa}@ee.umanitoba.ca.

***Abstract*—**

**We analyse the queuing performance of a radio link level round-robin scheduler for downlink data transmission in a multi-rate TDMA (Time Division Multiple Access) wireless network. One broadcast channel in the downlink is shared by multiple mobile users in a time mutiplexing fashion and a round-robin scheduler serves each user in exactly one time slot. The finite state Markov channel (FSMC) is used to capture different states of a slow Rayleigh fading channel. Depending on the channel condition, the modulation level at the transmitter is adapted and, therefore, one or multiple packets can be transmitted in one time slot. Using the matrix geometric method (MGM), the system is modeled as a quasi-birth and death (QBD) process and then the queue length and delay distributions are derived. We present typical numerical results and discuss their useful implications on system design.**

*Keywords*- Multi-rate transmission, adaptive modulation, finite state Markov channel (FSMC), round-robin scheduling, matrix geometric method (MGM), quasi-birth and death (QBD) process.

## I. Introduction

Link adaptation techniques which exploit the dynamics of the wireless channel to achieve high-speed wireless data transmission have attracted much research attention [1]. The data rate can be increased by adaptively adjusting the modulation level at the transmitter by using adaptive modulation ([2]) and a scheduling mechanism. In a cellular wireless system, to allocate the channel resources to multiple users in the downlink direction, a wireless scheduler is employed at the base station (BS). Data packets to be transmitted to different mobile users are buffered in separate queues before being transmitted. Queuing analyses for similar systems such as a polling system and a system with cylic service queues with Bernoulli schedules were performed in the past ([3]-[4]). These works, however, did not consider the multi-rate transmission (depending on the dynamics of the wireless channel) aspect at the radio link level, which is a key feature in 2.5G/3G wireless technologies (e.g., EGPRS). In this paper, we present a methodology for queuing analysis of downlink schedulers in a multi-rate transmission scenario taking the wireless channel dynamics into account. By formulating the problem as a quasi-birth and death (QBD) process, we use the well established results from Neuts [5] to obtain the probability distribution functions for the relevant performance measures.

## II. System Model and Assumptions

Suppose that there are $N$ separate radio link level queues at the BS which correspond to $N$ different mobiles. One common channel is shared by all mobiles in a time multiplexing fashion. For simplicity, we consider a *round-robin* scheduler at the BS for scheduling packets in downlink direction which serves one mobile in one time slot. Depending the channel condition, one or more packets can be transmitted during the assigned time slot. Scheduling of packets to a particular mobile can be modeled as a queue with vacation [6].

The multi-rate transmission is achieved by adaptive modulation where the modulation level at the transmitter can be adjusted depending on the channel condition. A finite state Markov channel (FSMC) with $(K+1)$ states is assumed to model a slow Rayleigh fading channel [7]. The received signal to noise ratio (SNR) $\gamma$ is patitioned into finite number of intervals. Let $\Gamma_0(=0) < \Gamma_1 < \Gamma_2 < ... < \Gamma_{K+1}(=\infty)$ be the thresholds of the received SNR for different states. The Rayleigh fading channel is said to be in state $k$ if $\Gamma_k < \gamma < \Gamma_{k+1}$ $(k = 0, 1, 2, 3, \cdots, K)$.

Suppose that only a finite number of modulation orders corresponding to $2^k$-QAM for $k = 1, 2, \cdots, K$, is allowed and the modulation order $k$ is used when the channel is in state $k$. Let the data rate coressponding to 2-QAM be $R_0$, the data rate corresponding to $2^k$-QAM $(k = 2, 3, \cdots, K)$ will be $kR_0$. We further assume that with basic rate $R_0$, one packet can be transmitted in one time slot and $k$ packets can be transmitted in one time slot when $2^k$-QAM is used.

We determine the BER given that the channel is in state $k$, $BER_k$ $(k = 1, 2, 3, \cdots, K)$ based on the approximation of BER for $M$-QAM (for $M = 2^k$) given as follows [2]:

$$BER_{MQAM}(\gamma) \approx 0.2 \exp\left(\frac{-1.6\gamma}{M-1}\right). \tag{1}$$

Then

$$\begin{aligned} BER_k &= \frac{\int_{\Gamma_{k-1}}^{\Gamma_k} BER_{MQAM}(\gamma)\frac{1}{\Gamma}\exp\left(-\frac{\gamma}{\Gamma}\right)d\gamma}{p_k} \\ &\approx \frac{\frac{0.2}{\Gamma a}\left(\exp(-a\Gamma_{k-1}) - \exp(-a\Gamma_k)\right)}{p_k} \end{aligned} \tag{2}$$

where $M = 2^k$ and $a = \frac{1.6}{M-1} + \frac{1}{\Gamma}$.

Let the packet length be $L$ bits. Assuming independent bit errors, given that the channel is in state $k$, the packet error rate

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(PER) can be calculated as follows:

$$\theta_k = 1 - (1 - BER_k)^L. \tag{3}$$

When a transmitted packet is received correctly at the mobile terminal, an acknowledgment (ACK) is sent back to the BS, otherwise a negative acknowledgment (NACK) is sent asking for retransmission. We assume that the feedback channel is error-free and instantaneous. If a NACK is received, the erroneous packet is retransmitted until it is received correctly, i.e., the number of retransmissions is assumed to be unbounded.

## III. Formulation of the Queuing Model and Analysis

### A. Queuing Model

Since the queuing performances for all the mobile users are statistically the same, we focus on *user one* only. When the scheduler serves this target user, we say that it is in service. As it serves the other $N-1$ users, it is said to be on vacation. Therefore, the vacation period is exactly $N-1$ time slots. We analyse the system for $K=3$. Generalization of the analysis to any other value of $K$ is straightforward.

Exploiting the cyclic operation of the round-robin scheduling, we construct the following transition matrix to describe the system in the service and vacation periods:

$$\mathbf{C} = \begin{pmatrix} \mathbf{0} & \mathbf{T} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{T} & \ldots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{T} \\ \mathbf{T} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} \end{pmatrix} \tag{4}$$

where $\mathbf{T}$ is the probability transition matrix of the channel. Each block of rows of this matrix (containing actually $K+1$ rows because $\mathbf{T}$ is of size $(K+1)\times(K+1)$) describes the evolution of the channel when the service is switched among users. For example, block one represents service of user one; block two describes service of user two, and so on.

The probability transition matrix in vacation can be written as

$$\mathbf{C}_v = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{T} & \ldots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{T} \\ \mathbf{T} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} \end{pmatrix}. \tag{5}$$

The probability transition matrices during service time of user one in which the channel is in state $i$ can be described by $\mathbf{C}_i$ ($i = 0, 1, 2, 3$) as follows:

$$\mathbf{C}_i = \begin{pmatrix} \mathbf{0} & \mathbf{T}_i & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} \end{pmatrix} \tag{6}$$

where $\mathbf{T}_i$ is a matrix obtained from $\mathbf{T}$ by keeping the $i+1$st row and setting all other rows to $\mathbf{0}$.

The queuing problem is modeled in discrete time. We observe the system at epochs sequentially numbered $0, 1, 2, 3, \cdots$. All events which occur between epochs $t$ and $t+1$ are assumed to occur at epoch $t+1$. The arrival process is assumed to be Bernoulli with parameter $\alpha$.

The state space of the Markov chain, which describes the system is $\{(x_n, u_n, s_n), x_n \geq 0,\ 1 \leq a_n \leq N, 0 \leq s_n \leq K\ \}$, where $x_n$ is the number of packets in the system, $u_n$ is the user in service, and $s_n$ is the channel state at time $n$. The resulting transition matrix describing the number of packets in the queue is given by (7) (see **Appendix**).

Reblocking the transition matrix as being indicated, we obtain a QBD process as follows:

$$\mathbf{P} = \begin{pmatrix} \mathbf{B} & \mathbf{C} & & & & \\ \mathbf{E} & \mathbf{F} & \mathbf{D}_0 & & & \\ & \mathbf{D}_2 & \mathbf{D}_1 & \mathbf{D}_0 & & \\ & & \mathbf{D}_2 & \mathbf{D}_1 & \mathbf{D}_0 & \\ & & & \ddots & \ddots & \ddots \end{pmatrix} \tag{8}$$

where

$$\mathbf{C} = \begin{pmatrix} \mathbf{A}_{0,0} & \mathbf{0} & \mathbf{0} \end{pmatrix}$$

$$\mathbf{E} = \begin{pmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \\ \mathbf{B}_3 \end{pmatrix}$$

$$\mathbf{F} = \begin{pmatrix} \mathbf{A}_{1,1} & \mathbf{A}_{1,0} & \mathbf{0} \\ \mathbf{A}_{2,2} & \mathbf{A}_{2,1} & \mathbf{A}_{2,0} \\ \mathbf{A}_{3,3} & \mathbf{A}_{3,2} & \mathbf{A}_{3,1} \end{pmatrix}$$

$$\mathbf{D}_2 = \begin{pmatrix} \mathbf{A}_4 & \mathbf{A}_3 & \mathbf{A}_2 \\ \mathbf{0} & \mathbf{A}_4 & \mathbf{A}_3 \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_4 \end{pmatrix}$$

$$\mathbf{D}_1 = \begin{pmatrix} \mathbf{A}_1 & \mathbf{A}_0 & \mathbf{0} \\ \mathbf{A}_2 & \mathbf{A}_1 & \mathbf{A}_0 \\ \mathbf{A}_3 & \mathbf{A}_2 & \mathbf{A}_1 \end{pmatrix}$$

$$\mathbf{D}_0 = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{A}_0 & \mathbf{0} & \mathbf{0} \end{pmatrix}.$$

From the reblocked transition matrix in (8), we want to obtain the stationary probability $\pi = [\pi_\mathbf{0}\ \pi_\mathbf{1}\ \pi_\mathbf{2}\ \ldots]$ which satisfies $\pi = \pi\mathbf{P}$ and $\pi\mathbf{1} = 1$.

We know that there is a matrix $\mathbf{R}$ which is the minimal non-negative solution to the following matrix quadratic equation [5]:

$$\mathbf{R} = \mathbf{D}_0 + \mathbf{R}\mathbf{D}_1 + \mathbf{R}^2\mathbf{D}_2$$

such that $\pi_i = \pi_{i-1}\mathbf{R}$. There are several effiecient methods for calculating the $\mathbf{R}$ matrix [8]. Note that, because of the special structure of $\mathbf{D}_0$, $\mathbf{R}$ has the following structure [9]:

$$\mathbf{R} = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{R}_0 & \mathbf{R}_1 & \mathbf{R}_2 \end{pmatrix}.$$

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$$
\mathbf{P} = \left(\begin{array}{c|ccc|ccc|ccc}
\mathbf{B} & \mathbf{A}_{0,0} & & & & & & & & \\
\hline
\mathbf{B}_1 & \mathbf{A}_{1,1} & \mathbf{A}_{1,0} & & & & & & & \\
\mathbf{B}_2 & \mathbf{A}_{2,2} & \mathbf{A}_{2,1} & \mathbf{A}_{2,0} & & & & & & \\
\mathbf{B}_3 & \mathbf{A}_{3,3} & \mathbf{A}_{3,2} & \mathbf{A}_{3,1} & \mathbf{A}_{3,0} & & & & & \\
\hline
 & \mathbf{A}_4 & \mathbf{A}_3 & \mathbf{A}_2 & \mathbf{A}_1 & \mathbf{A}_0 & & & & \\
 & & \mathbf{A}_4 & \mathbf{A}_3 & \mathbf{A}_2 & \mathbf{A}_1 & \mathbf{A}_0 & & & \\
 & & & \mathbf{A}_4 & \mathbf{A}_3 & \mathbf{A}_2 & \mathbf{A}_1 & \mathbf{A}_0 & & \\
\hline
 & & & & & & \ddots & \ddots & \ddots &
\end{array}\right) \quad (7)
$$

It can be shown that $\mathbf{R}_1 = \mathbf{R}_0^2,\ \mathbf{R}_2 = \mathbf{R}_0^3$, where $\mathbf{R}_0$ is the minimal non-negative solution to the following matrix equation:

$$\mathbf{R}_0 = \mathbf{A}_0 + \mathbf{R}_0\mathbf{A}_1 + \mathbf{R}_0^2\mathbf{A}_2 + \mathbf{R}_0^3\mathbf{A}_3 + \mathbf{R}_0^4\mathbf{A}_4.$$

We can find $\pi_{\mathbf{0}}$ and $\pi_{\mathbf{1}}$ from boundary and normalization conditions [5]. Other $\pi_i,\ i \geq 2$ are calculated as

$$\pi_i = \pi_{\mathbf{1}}\mathbf{R}^{i-1}. \quad (9)$$

**Stability condition:** The stability condition is $\mathbf{z}\mathbf{D}_0\mathbf{1} < \mathbf{z}\mathbf{D}_2\mathbf{1}$, where $\mathbf{z}$ is obtained from $\mathbf{z} = \mathbf{z}\mathbf{D}$ and $\mathbf{z}\mathbf{1} = 1$ for $\mathbf{D} = \mathbf{D}_0 + \mathbf{D}_1 + \mathbf{D}_2$ [5].

### *B. Average Queue Length*

The stationary distribution we obtained in the previous section is for the reblocked QBD process. The stationary distribution of the number of packets in the queue can be achieved by partitioning the obtained distribution. Let $\mathbf{f}_i = [\mathbf{0}\ \cdots\ \mathbf{I}_m\ \cdots\ \mathbf{0}]^T,\ i = 1, 2, \cdots, K$ ($T$ denotes the matrix transpose) be a matrix of size $n \times m$ where $\mathbf{I}_m$ is at $i$th position, $n = NK(K+1)$ and $m = N(K+1)$. These matrices are used to obtain the partitions of $\pi_i$.

The stationary probability of having $k$ packets in the queue can be written as

$$\mathbf{x_0} = \pi_{\mathbf{0}}\ , \quad \mathbf{x}_k = \pi_i\mathbf{f}_h, \quad k \geq 1 \quad (10)$$

where $(i-1)K < k \leq (i-1)K + K$ and $h = k - (i-1)K$

The average queue length can written as follows:[1]

$$L_q = \sum_{k=1}^{\infty} k\mathbf{x}_k\mathbf{1} = \sum_{i=1}^{\infty}\sum_{j=1}^{K}\left((i-1)K + j\right)\pi_i\mathbf{f}_j\mathbf{1}. \quad (11)$$

The average delay is given by

$$D_l = \frac{L_q}{\lambda}. \quad (12)$$

### *C. Delay Distribution*

In this section, we derive the delay distribution for an arbitrary arriving packet. Let the arrival slot be numbered as slot zero and it is not included in the delay calculation. If user $u$ is in service in slot one, the target user (user one in our analysis) is in service for the first time in slot $v$ which satisfies

[1] For calculation purposes, the queue length needs to be truncated.

$$v = \begin{cases} N - u + 2 \text{ modulo } N, & \text{if } N - u + 2 \text{ not divisible by } N \\ N, & \text{otherwise.} \end{cases} \quad (13)$$

Therefore, the delay of any arriving packet is at least one, which happens if user $N$ is in service during the arrival slot; the arriving packet and its head-of-line packets are transmitted successfully in slot one. From slot $v$ on, user one receives service once in exactly each $N$ slots. The transmission delay is the time for all packets ahead of the target packet (if any) and itself successfully leaving the queue.

Let us define the following matrices:

- $\mathbf{\Psi}_u(k,n)$ is a $(K+1) \times (K+1)$ matrix whose elements $(\mathbf{\Psi}_u(k,n))_{i,j}$ describe the probability that an arriving packet spends $n$ slots in the queue not including the arrival slot, given that it sees $k$ packets waiting in the queue and user $u$ is in service in slot one, channel state is $i$ in slot one and the target packet successfully leaves the queue in channel state $j$.
- $\mathbf{\Omega}(k,n)$ is a $(K+1) \times (K+1)$ matrix whose elements $(\mathbf{\Omega}(k,n))_{i,j}$ represent the probability that $k$ packets are successfully transmitted in $n$ slots counting from the end of the first service slot (slot $v$), starting in channel state $i$ and ending in channel state $j$.
- $\mathbf{S}_{l,k}^{(v)}$ is a $(K+1) \times (K+1)$ matrix whose elements $(\mathbf{S}_{l,k}^{v})_{i,j}$ represent the probability that $l$ packets are successfully transmitted in slot $v$ given that there were $k$ packets in slot one (there is no transmission from slot one to slot $v-1$), channel state is $i$ at the beginning of slot one and is $j$ at the end of slot $v$.

We have the following recursions

$$\mathbf{\Psi}_u(k, n+v) = \sum_{l=0}^{K}\mathbf{S}_{l,k}^{(v)}\mathbf{\Omega}(k-l+1, n) \quad (14)$$

$$\mathbf{\Omega}(k,n) = \sum_{l=0}^{K}\mathbf{S}_{l,k}^{(N)}\mathbf{\Omega}(k-l, n-N) \quad (15)$$

$$\mathbf{\Omega}(0,0) = \mathbf{I}_{K+1} \quad (16)$$

where $n$ is a multiple of $N$ because from slot $v$ on, the target queue is served once in $N$ slots until all packets ahead of the target packet (if any) and itself successfully leave the queue. Note that, $\mathbf{\Omega}(k,n) = \mathbf{0}$, for $k > nK/N$ since at most $K$ packets can be successfully transmitted in each interval of $N$ slots. Also, we have $\mathbf{\Psi}_u(k, n+v) = \mathbf{0}$ if $k > nK/N + K - 1$ because the first transmission after arrival (in slot $v$) can handle at most $K$ packets and all left packets including the target packet will be served in $n$ slots.

We now derive $\mathbf{S}_{l,k}^{(v)}$. Note that this matrix represents the fact that $l$ packets are sucessfully transmitted in slot $v$ given that there are $k$ packets before transmission (there is no transmission from slot 1 to slot $v-1$). Thus, the channel simply evolves in $v-1$ slots and the transmission occurs in slot $v$. Let $\mathbf{T}_k$ , $k = 0, 1, \cdots, K$ be a $(K+1)\times(K+1)$ matrix whose elements $(\mathbf{T}_k)_{i,j}$ describe the probability that the channel is in state $k$, starting in channel state $i$ and ending in channel state $j$ . $\mathbf{T}_k$ can be constructed by keeping only the $(k+1)$st row of $\mathbf{T}$ and setting all other rows to $\mathbf{0}$. Now, $\mathbf{S}_{l,k}^{(v)}$ can be calculated as follows:

$$\mathbf{S}_{l,k}^{(v)} = \mathbf{T}^{v-1}\left[\sum_{i=1}^{k} p_{l,i}^{(i)}\mathbf{T}_i + \sum_{i=k+1}^{K} p_{l,k}^{(i)}\mathbf{T}_i\right],\ l > 0 \tag{17}$$

$$\mathbf{S}_{0,k}^{(v)} = \mathbf{T}^{v-1}\left[\mathbf{T}_0^{(v)} + \sum_{i=1}^{k} p_{0,i}^{(i)}\mathbf{T}_i + \sum_{i=k+1}^{K} p_{0,k}^{(i)}\mathbf{T}_i\right] \tag{18}$$

for $k < K$

$$\mathbf{S}_{l,k}^{(v)} = \mathbf{T}^{v-1}\left[\sum_{i=1}^{K} p_{l,i}^{(i)}\mathbf{T}_i\right],\ l > 0 \tag{19}$$

$$\mathbf{S}_{0,k}^{(v)} = \mathbf{T}^{v-1}\left[\mathbf{T}_0 + \sum_{i=1}^{K} p_{0,i}^{(i)}\mathbf{T}_i\right] \tag{20}$$

for $k \geq K$, where $p_{i,j}^{(k)}$ is the probability that $i$ packets are correctly received given that $j$ packets were transmitted in channel state $k$ (see (24) of **Appendix**). Note that, $\mathbf{S}_{l,k}^{v} = \mathbf{0}$ if $l > k$ or $l < 0$.

Now we calculate the probability that the delay is $D$ slots (excluding the arrival slot). Recall that in $D$ slots, the first transmission takes place in slot $v \leq N$ from the arrival slot and other $J$ transmissions occur in each $N$ slots from slot $v$. We calculate $J$ as follows:

$$J = \begin{cases} \lfloor D/N \rfloor - 1, & \text{if } D \text{ is divisible by } N \\ \lfloor D/N \rfloor, & \text{otherwise.} \end{cases} \tag{21}$$

where $\lfloor z \rfloor$ picks the largest interger which is smaller or equal to $z$.

Slot $v$ when the first transmission since arrival takes place can be calculated as $v = D - JN$. We can calculate the corresponding $u$ from (13). Now, let $\mathbf{x}_{i,u}$ be a $(K+1)$-dimensional row vector whose elements $\mathbf{x}_{i,u}(k)$ are the probability that an arriving packet see $i$ packets in the queue, user $u$ is in service and the channel state is $k$. Due to the memeryless property of Bernoulli arrival process, these can be obtained from the stationary distribution in (10). Note that we can express $\mathbf{x}_i$ as follows:

$$\mathbf{x}_i = [\mathbf{x}_{i,1},\ \mathbf{x}_{i,2},\ \mathbf{x}_{i,3}, \cdots, \mathbf{x}_{i,N}].$$

Let $\mathbf{g}_u = [\mathbf{0}\ \cdots\ \mathbf{I}_{K+1}\ \cdots\ \mathbf{0}]^T,\ u = 1, 2, \cdots, N$ be a matrix of size $(K+1)N \times (K+1)$ where $\mathbf{I}_{K+1}$ is at the $u$th position. Then,

$$\mathbf{x}_{i,u} = \mathbf{x}_i \mathbf{g}_u \tag{22}$$

The probability that the delay is $D$ slots (not including the arrival slot) can be written as

$$P_D = \sum_{i=0}^{K_0} \mathbf{x}_{i,u}\psi_u(i, D)\mathbf{1} \tag{23}$$

where $K_0 = JK + K - 1$. In (23), the limit of the sum is $K_0$ since at most $K$ packets can be successfully transmitted in one time slot.

## IV. Numerical Results and Discussions

The arrival process is Bernoulli with parameter $\alpha$ = 0.3 and the number of users ($N$) is assumed to be 5. The packet length is 100 bits, the slot time is $0.5\ ms$. The received SNR thresholds are chosen such that $BER < 10^{-3}$, and these are as follows: $\Gamma_0 = 0$, $\Gamma_1 = 5.2\ dB$, $\Gamma_2 = 9.97\ dB$, $\Gamma_3 = 13.65\ dB$, $\Gamma_4 = \infty$. The resulting PER in different states are $\theta_1 = 0.0102$, $\theta_2 = 0.0162$, $\theta_3 = 0.0121$ with average SNR to be $\overline{\Gamma} = 15\ dB$.

The probability distributions for queue length and delay are shown in Fig. 1 and Fig. 2, respectively, for Doppler shift $f_d = 50$Hz. These can be used for system design purposes. For example, the buffer size can be chosen such that the dropping probability due to buffer overflow remains below the desired value.

Based on the delay distribution, the probability that an arriving packet violates the delay limit can be determined. More interestingly, an admission control policy can be implemented to guarantee the delay requirement as follows. Given the channel parameters, we obtain the delay distributions for different packet arrival rate. Then the maximum packet arrival rate can be determined such that the probability that the delay is greater than certain value $D_{max}$ is below the desired value (e.g., 0.02). If the channel condition does not change very fast, the calculation of the maximum packet arrival rate can be done online to update the admission control parameter. Otherwise, these maximum packet arrival rates can be determined offline and the admission control parameter update can be done by table lookup.

In Fig. 3, we show typical variations in delay with Doppler shift. We show both average delay and "90% delay" (meaning that the actual delay is less than the "90% delay" value for 90% of time). We observe that the "90% delay" is much greater than the average delay. Also, faster fading (higher Doppler shift) results in smaller delay.

### Appendix

We derive the components of the transition matrix in (7). Let $\theta_k$ the probability of unsuccessful transmission when the channel is in state $k$ (see (3)). Assuming that the event of successful transmission of consecutive packets is independent, the probability that $i$ packets are correctly received given that $j$ packets were transmitted when the channel state is $k$ can be written as follows:

$$p_{i,j}^{(k)} = \binom{j}{i}\theta_k^{j-i}(1-\theta_k)^i. \tag{24}$$

Let us define the following matrices:

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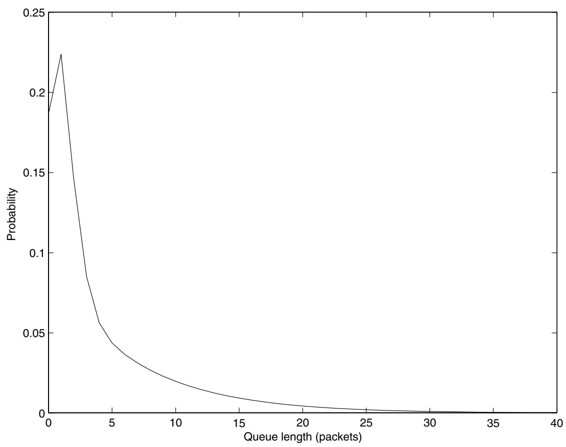


Fig. 1. Probability distribution for queue length (for $f_d$ = 50 $Hz$).

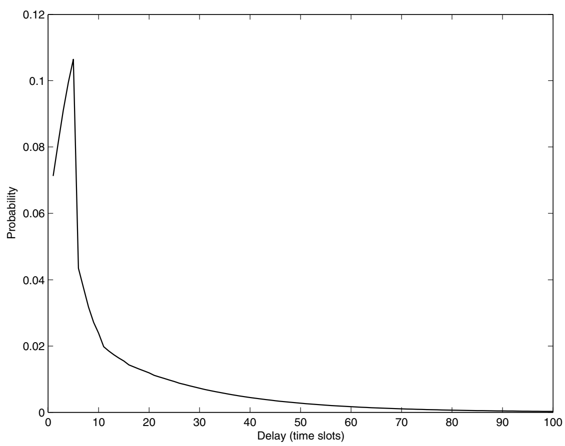


Fig. 2. Probability distribution for transmission delay (for $f_d$ = 50 $Hz$).

- $\mathbf{H}_{m,n}$ is a $(K+1)N \times (K+1)N$ matrix which has the same structure as matrix $\mathbf{C}$ in (4) describes the fact that $m$ packets are successfully transmitted given that there were $n$ packets in the queue before transmission ($n < K$).
- $\mathbf{H}_{m,K}$ is a $(K+1)N \times (K+1)N$ matrix which has the same structure as matrix $\mathbf{C}$ in (4) describes the fact that $m$ packets are successfully transmitted given that there were $K$ or more packets in the queue before transmission.

These matrices can be calculated as follows:

$$\mathbf{H}_{m,n} = \sum_{i=1}^{n} p_{m,i}^{(i)} \mathbf{C}_i + \sum_{i=n+1}^{K} p_{m,n}^{(i)} \mathbf{C}_i, \ m > 0 \tag{25}$$

$$\mathbf{H}_{0,n} = \mathbf{C}_v + \mathbf{C}_0 + \sum_{i=1}^{n} p_{0,i}^{(i)} \mathbf{C}_i + \sum_{i=n+1}^{K} p_{0,n}^{(i)} \mathbf{C}_i \tag{26}$$

$$\mathbf{H}_{m,K} = \sum_{i=1}^{K} p_{m,i}^{(i)} \mathbf{C}_i, \ m > 0 \tag{27}$$

$$\mathbf{H}_{0,K} = \mathbf{C}_v + \mathbf{C}_0 + \sum_{i=1}^{K} p_{0,i}^{(i)} \mathbf{C}_i. \tag{28}$$

The arrival process is Bernoulli with parameter $\alpha$. Let $\alpha' = 1 - \alpha$, the components of the transition matrix in (7) can be

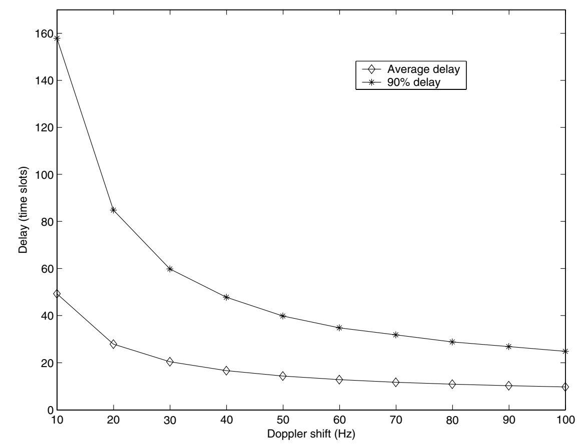


Fig. 3. Typical variations in transmission delay with Doppler shift.

written as follows:

$$\mathbf{B} = \alpha' \mathbf{C}, \quad \mathbf{C}_{0,0} = \alpha \mathbf{C}$$

$$\mathbf{B}_m = \alpha' \mathbf{H}_{m,m}$$

$$\mathbf{A}_{n,m} = \alpha \mathbf{H}_{m,n} + \alpha' \mathbf{H}_{m-1,n}$$

$$\mathbf{A}_0 = \mathbf{A}_{3,0} \ , \ \mathbf{A}_1 = \mathbf{A}_{3,1}$$

$$\mathbf{A}_2 = \mathbf{A}_{3,2} \ , \ \mathbf{A}_3 = \mathbf{A}_{3,3} \ , \ \mathbf{A}_4 = \mathbf{B}_3.$$

We recall that $\mathbf{C}_v$ denotes channel transition probability during vacation and $\mathbf{C}_i$ represents the fact that channel is in state $i$ during the service slot. Remember that if the channel is in state zero, there is no transmission; $i$ packets are transmitted if the channel state is $i$ for $i = 1, 2, 3, ..., K$. As an example, $\mathbf{A}_{1,3}$ describes the fact that the number of packets is three (unchanged) in one time slot. This event occurs in two cases. The first case is that there is one arrival and one packet is successfully transmitted. The second case is that there is no arrival and no packet is successfully transmitted.

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