Pareto-Efficient and Goal-Driven Power Control in Wireless Networks: A Game-Theoretic Approach With a Novel Pricing Scheme

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***Abstract*—A Pareto-efficient, goal-driven, and distributed power control scheme for wireless networks is presented. We use a noncooperative game-theoretic approach to propose a novel pricing scheme that is linearly proportional to the signal-to-interference ratio (SIR) and analytically show that with a proper choice of prices (proportionality constants), the outcome of the noncooperative power control game is a unique and Pareto-efficient Nash equilibrium (NE). This can be utilized for constrained-power control to satisfy specific goals (such as fairness, aggregate throughput optimization, or trading off between these two goals). For each one of the above goals, the dynamic price for each user is also analytically obtained. In a centralized (base station) price setting, users should inform the base station of their path gains and their maximum transmit-powers. In a distributed price setting, for each goal, an algorithm for users to update their transmit-powers is also presented that converges to a unique fixed-point in which the corresponding goal is satisfied. Simulation results confirm our analytical developments.**

***Index Terms*—Distributed and goal-driven power control, game theory, Pareto efficiency, wireless networks.**

I. Introduction

ALLOCATION of radio resources is an important and challenging issue as the demand for wireless services increases. A fundamental component of radio resources is the transmit power. Two major objectives of power control in a wireless network are to extend users' battery life and to maintain an acceptable QoS in terms of the signal-to-interference ratio (SIR) for all users by minimizing interferences to users. Data services require a higher SIR as compared to the voice service because the latter is more tolerant to bit errors. In contrast to the voice service for which the QoS is measured by a step function of the SIR [1], the commonly used QoS measure for the data service is, in general, an increasing function of the SIR [2]–[7].

Power control in a single-service network is expected to provide each user with equal QoS in an optimum and Pareto-efficient manner. In situations where some users with very low path gains may impede the QoS provisioning to some other users, or when the number of users is high, it may be required to remove some users from the network in order to improve the QoS to the remaining users. Such removals may also be used to optimize the aggregate throughput (in terms of the aggregate SIR). A proper power control scheme should work well with such different goals.

Noncooperative game-theoretic schemes have been recently proposed for power control in [2]–[6], where each user chooses its own transmit power level and attempts to maximize its utility function. The game settles at a stable and predictable state called the Nash equilibrium (NE) (if one exists), at which no user has any incentive to unilaterally change its power level. Most of the existing game-theoretic approaches to power control are for single-service wireless networks.

In [2] and [3], a utility function is defined that depends on the bit error rate (BER) per unit of transmit power. A drawback of this is that the utility goes to infinity when the user transmits at zero power. To obtain zero utility at zero power, they modified the utility so that a unique but Pareto-inefficient NE exists. To improve the Pareto efficiency at the NE, a pricing-based utility function was introduced in [2] and [3] as a function of the BER per expended power unit minus a price that is a linear function of the transmit power. Then in [3], the strategy space of each user was modified so that the modified game became supermodular, confining all the NE to a set, and the smallest NE power vector represented a Pareto-dominant NE that is not yet Pareto-efficient.

In [4]–[6], an information theoretic approach is used to define the QoS. In [4] and [5], a logarithmic function of the SIR (proportional to the Gaussian channel capacity), and in [6] the Shannon capacity of a binary symmetric channel (BSC), are used as the QoS measures, both of which we will consider (in Section II) by adopting a general QoS function applicable to any channel model with average power constraint. In [5], a noncooperative power control game (NPCG) without power constraint was considered in which the utility was defined as the QoS minus a price that is a linear function of the transmit power. It was shown that at NE, some users are dropped from the system.

For a multirate code-division multiple-access (CDMA) network, a new pricing scheme was defined in [6] as a linear function of the ratio of received power to the total received power plus noise at the base station. It leads to the aggregate QoS optimization under fixed total transmit power constraint at NE, which may be a local (and not global) maxima over the whole users' transmit power space [6]. Besides, fairness to active users

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is not considered in [5] and [6]. In [7], a distributed power control without power constraint is formulated in which the utility is a sigmoid-like function, and a pricing function of the transmit power is used to improve system convergence by automatically decreasing the target SIR and even switching off some users at high traffic loads. In addition, a so called near-far fairness in [7] is provided by setting a lower price to farther users. In [8], a utility function is used that depends on the value of a parameter assigned by the base station to each user. Although the proposed NPCG converges to the near-optimal solution for the aggregate throughput, no method was proposed to obtain the value of the said parameter. Besides, multiple NE (which are not the global optimum) exist.

In [9], to optimize the sum of utilities of all users, a distributed power allocation algorithm in the downlink was presented, which consists of the mobile user selection stage and the power allocation stage. It was shown that it provides an asymptotically (in the number of mobile users) optimal power allocation. However, this needs iterative communication between the base station and users for the algorithm to converge, requiring the base station to broadcast at each iteration a dynamic price calculated using the difference of the sum of requested powers by users and the total available power and users to request their power levels based on the broadcast price.

Here, we focus on the uplink power allocation in a distributed manner to dynamically set the price. Besides optimizing the aggregate throughput, we also address fairness and trading off between fairness and the aggregate throughput, which were not discussed in [9]. In contrast to [9], we require the base station only to broadcast the number of active users, and the base station's dynamic adjustment of the price by solving an optimization problem is not needed.

The existing game-theoretic approach to power control has no flexibility to work well in a Pareto-efficient manner for attaining different goals such as fairness, optimized aggregate throughput, and trading off between fairness and aggregate throughput. Furthermore, to the best of our knowledge, no distributed price setting (or equivalently distributed power control) algorithm exists that converges to the optimum fairness, the optimum aggregate throughput, or the trading off between fairness and aggregate throughput.

In this paper, we use a game-theoretic and distributed (user-based) approach to address the problem of constrained power control in a Pareto-efficient manner for attaining a given goal. Our main contributions in this paper are as follows. We propose a novel pricing scheme that is linearly proportional to the SIR and show that, with a proper choice of the price, the proposed pricing scheme can satisfy the fairness requirement in an optimum manner, can lead to the aggregate throughput (SIR) optimization, or is capable of trading off between fairness and aggregate throughput, while providing the Pareto efficiency at the NE. For each one of the above goals, we analytically obtain the optimal price for each user to be calculated at the base station (centralized setting). Furthermore, we present an algorithm for updating the transmit power as well as price setting in an iterative and distributed (decentralized) manner that converges to the Pareto-efficient NE in the centralized setting of the optimal prices.

The rest of this paper is organized as follows. We set up the system model in Section II. The problem is formulated in Section III. In Section IV, the regular (i.e., without pricing) power control game is introduced, and the NE and its properties are derived. The pricing scheme is proposed and discussed in Section V. In Section VI, we present distributed goal-driven power control algorithms. Section VII contains numerical results that confirm our analysis. The conclusions are presented in Section VIII.

II. System Model

We consider a single-cell wireless CDMA data network with M active users denoted by $\mathcal{M} = \{1, 2, \ldots, M\}$. Let p_i be the transmit power of user i. We assume the transmit power of each user is bounded, i.e., $0 \leq p_i \leq \overline{p}_i$ for all $i \in \mathcal{M}$, where $\overline{p}_i$ is the upper limit of the transmit power for user i. The received power at the base station is $\varphi_i = h_i p_i$, where h_i is the path gain from user i to the base station. The transmit power constraint imposes the received power to be bounded, i.e., $\varphi_i \leq \overline{\varphi}_i$ for all $i \in \mathcal{M}$, where $\overline{\varphi}_i = h_i \overline{p}_i$ is the upper bound on the received power. Without loss of generality, suppose that users are indexed such that $\overline{\varphi}_1 < \overline{\varphi}_2 < \cdots < \overline{\varphi}_M$. Each user has the same chip rate R_c (assumed to be equal to the spreading bandwidth, i.e., $W = R_\mathrm{c}$) and the same transmit data rate R. The processing gain is defined by $g = R_\mathrm{c}/R$. Noise is assumed to be additive white Gaussian whose power is σ^2 at the base station. The receiver is assumed to be a conventional matched filter. Thus, at the base station, the SIR of user i, denoted by γ_i, is $\gamma_i = g\varphi_i/I_i$, where $I_i = \sum_{k \neq i} \varphi_k + \sigma^2$ is the interference at the base station for user i. The transmit power and the SIR vectors are denoted by $\mathbf{p} = [p_1, p_2, \ldots, p_M]^\mathrm{T}$ and $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \ldots, \gamma_M]^\mathrm{T}$, respectively, where T denotes the transpose. Let $\overline{\mathbf{p}}$ denote the upper bound of the transmit power vector, whose components are all equal to the maximum possible value for all users. The transmit power vector for all users except user i is denoted by $\mathbf{p}_{-i}$. A transmit power vector is $\mathbf{p} = (p_i, \mathbf{p}_{-i})$.

There is a one-to-one relation between a transmit power vector and the achieved SIR vector ([6], [10]), which is

$$p_i = \frac{\gamma_i}{h_i(\gamma_i + g)} \times \frac{\sigma^2}{1 - \sum\limits_{k=1}^{M} \frac{\gamma_k}{\gamma_k + g}} \text{ for all } i \in \mathcal{M}. \tag{1}$$

A SIR vector $\boldsymbol{\gamma}$ is feasible if a power vector $\mathbf{0} \leq \mathbf{p} \leq \overline{\mathbf{p}}$ that corresponds to that SIR vector exists. The power constraint $0 \leq p_i \leq \overline{p}_i$ for all $i \in \mathcal{M}$ can be stated by $\sum_{k=1}^{M} \frac{\gamma_k}{\gamma_k + g} \leq 1 - \sigma^2 (\frac{\frac{\gamma_i}{\gamma_i + g}}{h_i \overline{p}_i})$ for all $i \in \mathcal{M}$. Thus, a SIR vector is feasible if

$$\sum_{k=1}^{M} \frac{\gamma_k}{\gamma_k + g} \leq 1 - \sigma^2 \times \max_{i \in \mathcal{M}} \frac{\frac{\gamma_i}{\gamma_i + g}}{h_i \overline{p}_i}. \tag{2}$$

Similar to [4]–[6], we use an information theoretic approach to define the QoS function by the channel capacity as the highest rate at which user i's information can be sent through the channel with an arbitrary low probability of error [11]. We do not restrict ourselves to a specific channel model, modulation, and coding scheme. Generally, $q(\gamma_i)$ is an increasing and concave function of γ_i for every channel model with the average power constraint [12]; thus, the following two properties hold:

Property I: $q'(\gamma_i) > 0$ for all γ_i;

Property II: $q''(\gamma_i) < 0$ for all γ_i;

where $q'(\gamma_i)$ and $q''(\gamma_i)$ are the first and the second derivatives of $q(\gamma_i)$ with respect to γ_i, respectively. We rely only on these two general properties, and all of our results hold for any other QoS function that satisfies Properties I–II. Let

$$\beta = \lim_{\gamma_i \to \infty} q'(\gamma_i). \tag{3}$$

Since $q'(\gamma_i)$ is a strictly decreasing function of the SIR, and as $q'(\gamma_i) > 0$, we conclude that $\beta < q'(\gamma_i) \le q'(0)$ for all γ_i and $0 \le \beta < q'(0)$.

As two examples for the QoS function, consider a logarithmic function of the SIR, defined in [4] and [5], denoted by $q_{\mathrm{G}}(\gamma)$, and the channel capacity for a binary symmetric channel (BSC) as in [6], denoted by $q_{\mathrm{BSC}}(\gamma)$, and write

$$q_{\mathrm{G}}(\gamma) = k \log_2(1+\gamma) \text{ bits/s} \tag{4}$$

where k is a constant, and

$$q_{\mathrm{BSC}}(\gamma) = R\Big(1 + p_{\mathrm{e}}(\gamma)\log_2 p_{\mathrm{e}}(\gamma) + (1-p_{\mathrm{e}}(\gamma))\log_2(1-p_{\mathrm{e}}(\gamma))\Big) \text{ bits/s} \tag{5}$$

where $p_{\mathrm{e}}(\gamma)$ is the cross error probability defined in [11].

Note that these two examples for the QoS function satisfy Properties I-II, and in both cases, we have $\beta = 0$ [5], [6]. The logarithmic function (4) is the capacity of a Gaussian channel ($k = R/2$ for the discrete-time channel, and $k = R_{\mathrm{c}}$ for the continuous-time channel [11]), provided that the noise plus interference for each user is Gaussian [5].

III. Problem Formulation

A. Definitions

A NPCG $G = \langle \mathcal{M}, (P_i), (u_i) \rangle$ has three elements: a finite set of mobile users $\mathcal{M} = \{1, 2, \ldots, M\}$ as players, the strategy space $P_i = [0, \overline{p}_i]$ for each mobile user i is the interval that contains the transmit power choices, and a utility function u_i for each strategy profile $\mathbf{p} = [p_1, p_2, \ldots, p_M]^{\mathrm{T}}$. We assume that each user knows the number of players, the other users' utility functions, and the maximum received power of all users. In Section VI, we will introduce an algorithm that converges to the NE requiring none of above *a priori* information (except the number of users). A NPCG can be formally expressed [3] by $\max_{p_i \in P_i} u_i(p_i, \mathbf{p}_{-i})$ for all $i \in \mathcal{M}$.

The commonly used concept in solving game-theoretic problems is the NE at which no user can improve its utility by unilaterally changing its transmit power.

Definition 1: A transmit power vector $\mathbf{p}^* = [p_1^*, p_2^*, \ldots, p_M^*]^{\mathrm{T}}$ is the NE point for the NPCG $G = \langle \mathcal{M}, (P_i), (u_i) \rangle$ if, for every user i, $u_i\left(p_i^*, \mathbf{p}_{-i}^*\right) \ge u_i\left(p_i, \mathbf{p}_{-i}^*\right)$, for all $p_i \in P_i$.

The NE exists in game G if, for all $i \in \mathcal{M}$, P_i is a nonempty, convex, and compact subset of a Euclidean space $\mathcal{R}^M$, and $u_i(\mathbf{p})$ is continuous in $\mathbf{p}$ and quasi-concave in p_i [13].

Another commonly used concept in game theory is the *best response function* for each player. Formally, the user i's best response function $b_i : P_{-i} \to P_i$, where P_{-i} is the Cartesian product of P_j for $j \ne i$ (i.e., $P_{-i} = \prod_{j \ne i} P_j$), is a set-valued function that assigns the set of the best power level in the utility sense to each interference power vector $\mathbf{p}_{-i} \in P_{-i}$, that is $b_i(\mathbf{p}_{-i}) = \{p_i \in P_i | u_i(p_i, \mathbf{p}_{-i}) \ge u_i(p_i', \mathbf{p}_{-i}), \text{ for all } p_i' \in P_i\}$. This alternative formulation can be used to find the NE by first calculating b_i for all i and then finding the $\mathbf{p}^*$ for which $p_i^* \in b_i\left(\mathbf{p}_{-i}^*\right)$ for all $i \in \mathcal{M}$. In other words, the NE is the fixed point of the best response function set, that is $\mathbf{p} = \mathbf{b}(\mathbf{p})$ where $\mathbf{b}(\mathbf{p}) = [b_1(\mathbf{p}), b_2(\mathbf{p}), \ldots, b_M(\mathbf{p})]^{\mathrm{T}}$. Note that $b_i(\mathbf{p})$ and $b_i(\mathbf{p}_{-i})$ are equivalent. If $b_i S$ are singleton-valued functions, we have M equations with M unknown $(p_i^*)_{i \in \mathcal{M}}$.

To compare the efficiency of two NE, the concept of Pareto dominance as defined below is used.

Definition 2: A transmit power vector $\mathbf{p}$ Pareto dominates another vector $\mathbf{p}'$ if, for all $i \in \mathcal{M}$, $q\left(\gamma_i\left(\mathbf{p}\right)\right) \ge q\left(\gamma_i\left(\mathbf{p}'\right)\right)$, and for some i, $q(\gamma_i(\mathbf{p})) > q(\gamma_i(\mathbf{p}'))$. A power vector $\mathbf{p}$ is Pareto efficient if there is no power vector that Pareto dominates $\mathbf{p}$.

B. Fairness (max-min SIR)

Fairness is an important notion in allocating resources in single-service wireless data networks. The criterion for fairness is highly application-dependent [14] and cannot be uniquely defined [15]. We consider two formulations of the fair power control problem, namely, the max-equal QoS and the max-min QoS. The max-equal QoS problem is to find the maximum achievable QoS that is the same for all users, and the max-min QoS problem is to find a transmit power vector so that the minimum achievable QoS is maximized. They are

$$\text{max-equal QoS}: \max_{\mathbf{0} \le \mathbf{p} \le \overline{\mathbf{p}}} \{q | q(\gamma_i = q, \text{ for all } i \in \mathcal{M}\} \tag{6}$$

$$\text{max-min QoS}: \max_{\mathbf{0} \le \mathbf{p} \le \overline{\mathbf{p}}} \min_i q(\gamma_i) \tag{7}$$

where $\mathbf{0} \le \mathbf{p} \le \overline{\mathbf{p}}$ means $0 \le p_i \le \overline{p}_i$ for all i.

In [16], it is assumed that all users operate with the same SIR. A similar notion called the near-far fairness is informally defined in [7]. A common formulation of the power control in [17]–[21] is the max-min SIR problem, i.e., $\max_{\mathbf{0} \le \mathbf{p} \le \overline{\mathbf{p}}} \min_i \gamma_i$, which is equivalent to (7) due to the fact that $q(\gamma_i)$ is an increasing function of γ_i. In Theorem 1, we introduce a unique and optimal solution to problems (6) and (7). The max-min is a well-known criterion for rate (congestion) control [22], which in this application is not equivalent to the max-equal criterion [23]. However, as the following will show, these two criteria are equivalent for power control.

Theorem 1 (Equivalence of the Max-Min QoS and the Max-Equal QoS for Power Control): For power control, the max-min QoS problem and the max-equal QoS problem have the same and unique optimal solution $p_i^* = \overline{\varphi}_1 / h_i$ for all $i \in \mathcal{M}$.

Proof: See Appendix I. ■

Definition 3: We call the received power, the SIR, and the QoS achieved by the optimal solution to the two equivalent problems (6) and (7), in which they are optimally the same for all users in the set $\mathcal{M}$, as the optimum-fair received power (OF-RP) denoted by φ_{of}, the optimum-fair SIR (OF-SIR) denoted by γ_{of}, and the optimum-fair QoS (OF-QoS) denoted by q_{of} for all users in the set $\mathcal{M}$, respectively. Thus

$$\varphi_{\text{of}} = \min_{i\in\mathcal{M}}(\overline{\varphi}_i), \tag{8}$$

$$\gamma_{\text{of}} = g\frac{\varphi_{\text{of}}}{(M-1)\varphi_{\text{of}}+\sigma^2}, \tag{9}$$

$$q_{\text{of}} = q(\gamma_{\text{of}}). \tag{10}$$

C. Optimization of the Aggregate Throughput

From a system point of view, the power control goal is to optimize the aggregate throughput subject to the peak transmit power constraint (O-AT), as defined in [24] by

$$\text{O-AT} : \max_{\mathbf{0}\le\mathbf{p}\le\overline{\mathbf{p}}} \sum_i \gamma_i. \tag{11}$$

In [25], the aggregate throughput is defined as the aggregate of the variable transmission rates for a given SIR, which is equivalent to O-AT (11) [24]. The following theorem is proved in [26] (Propositions 2–3).

Theorem 2: If $\frac{\overline{\varphi}_M}{\sigma^2} \ge 1$, then the optimal solution to the O-AT problem is $p_M^* = \overline{p}_M$ and $p_i^* = 0$ for $i = 1, 2, \ldots, M-1$.

Note that $\frac{\overline{\varphi}_M}{\sigma^2} \ge 1$ is usually satisfied, which we assume here as well. Thus for O-AT, only the user with the highest maximum-received-power transmits at its maximum power while the remaining users do not transmit at all. Although this strategy maximizes the aggregate throughput, it may be extremely unfair to users with low maximum-received-power who may never get a satisfactory SIR (QoS). Thus, in general, the O-AT (11) and the max-min QoS (7) do not have the same solution. Usually, a higher aggregate throughput is achieved at the expense of fairness and vice versa.

D. Limited Fairness (γ^{th}-max-min SIR)

The OF-SIR (9) depends on the min-max received power (8) and the number of users for a given spreading gain and noise power. Thus, the OF-SIR (and consequently the OF-QoS (10)) is very low in the presence of a high number of users and/or when some users encounter strong fading or are located at very far distances from the base station. In these cases, it is not reasonable to limit the SIR (QoS) to a low level for all users for the sake of fairness, as it would force all users to experience a low QoS (i.e., all users are punished). It also heavily degrades the aggregate throughput. In such cases, it is useful to drop those users whose channels are very bad and the strict fairness constraint to be relaxed in order to improve the QoS for the remaining active users. This motivates us to define the γ^{th}-max-min SIR as follows. If the OF-SIR for all users is lower than a threshold γ^{th}, those users with the lowest path gains are switched off one-by-one until the OF-SIR for the remaining users becomes equal to or higher than γ^{th}. The threshold value γ^{th} may also be chosen for trading off between fairness and aggregate throughput. Using Theorem 1, we immediately have the following theorem.

Theorem 3: Define

$$\widetilde{\Gamma}_i = g\frac{\overline{\varphi}_i}{(M-i)\overline{\varphi}_i+\sigma^2}; \quad i = 1, 2, \ldots, M. \tag{12}$$

Note that $\widetilde{\Gamma}_i$ is the SIR achieved by each user i to M when users 1 to $i-1$ switch off and users i to M transmit at a level so that their received power at the base station is $\overline{\varphi}_i$. We have $\widetilde{\Gamma}_i < \widetilde{\Gamma}_j$ for all $i < j$. If $\gamma^{\text{th}} > \widetilde{\Gamma}_M$, then the γ^{th}-max-min SIR has no solution, and if $\gamma^{\text{th}} \le \widetilde{\Gamma}_1$ then γ^{th}-max-min SIR and max-min SIR are equivalent, i.e., γ^{th}-max-min SIR is equal to $\widetilde{\Gamma}_1$. If $\widetilde{\Gamma}_1 < \gamma^{\text{th}} \le \widetilde{\Gamma}_M$ then the γ^{th}-max-min SIR is $\widetilde{\Gamma}_k$ where $k \in \{2, \ldots, M\}$ for which $\widetilde{\Gamma}_{k-1} < \gamma^{\text{th}} \le \widetilde{\Gamma}_k$.

E. Pareto-Efficient and Goal-Driven Power Control

As stated in Sections B–D above, a power control scheme may serve different goals such as fairness, limited fairness, or aggregate throughput optimization. Sometimes it is required to trade off between fairness and aggregate throughput. In summary, we wish to devise a Pareto-efficient and distributed power control for satisfying any one of the following goals:

— max-min SIR,
— γ^{th}
— optimizing the aggregate throughput, or
— trading off between fairness and aggregate throughput optimization.

In what follows, we show that satisfying any one of the above goals results in Pareto efficiency.

Theorem 4: The solutions to the max-min SIR, the γ^{th}-max-min SIR, or the O-AT problems are Pareto efficient.

Proof: This theorem is easily proved by contradiction. If the optimal transmit power vector $\mathbf{p}^*$, corresponding to each goal is not Pareto efficient, then there exists a different transmit power vector $\mathbf{p}'$ such that $\gamma_i(\mathbf{p}') \ge \gamma_i(\mathbf{p}^*)$ for all $i \in \mathcal{M}$, and $\gamma_i(\mathbf{p}') > \gamma_i(\mathbf{p}^*)$ for some i, thus resulting in a higher value for that goal's criterion (i.e., higher values for the max-min SIR, the γ^{th}-max-min SIR, or the O-AT problems, respectively) which contradicts the fact that $\mathbf{p}^*$ is the optimal solution. ■

IV. Regular Noncooperative Power Control Game: The Game Without Pricing

In a regular noncooperative power control game (R-NPCG), no *pricing* is applied and each user maximizes its own QoS in a distributed manner by choosing an appropriate transmit power. Indeed, users compete for the QoS. Thus, the utility function of user i in a R-NPCG is

$$u_i(p_i, \mathbf{p}_{-i}) = q(\gamma_i). \tag{13}$$

Note that γ_i is a function of p_i and $\mathbf{p}_{-i}$.

Theorem 5: There exists a unique NE in a R-NPCG $G_r = \langle \mathcal{M}, (P_i), (u_i) \rangle$ at which the power of each user is set to its maximum value. In addition, the NE is Pareto efficient.

Proof: It is evident that P_i is a nonempty, convex, and compact subset of a Euclidean space $\mathcal{R}^M$. One can easily see that $u_i(\mathbf{p})$ is continuous in $\mathbf{p}$. From Property II (defined in Section II), we note that $\frac{\partial^2 u_i}{\partial p_i^2} = \left(\frac{gh_i}{I_i}\right)^2 q''(\gamma_i) < 0$, and thus the utility function is quasi-concave in p_i. One can use these conditions and easily prove that the NE exists [13]. Since $q(\gamma_i)$ is a strictly increasing function of p_i for any given $\mathbf{p}_{-i}$ (see Fig. 1), each user transmits at its maximum power, independent of others, i.e., $b_i(\mathbf{p}_{-i}) = \overline{p}_i$ for all $i \in \mathcal{M}$. Thus, there is a

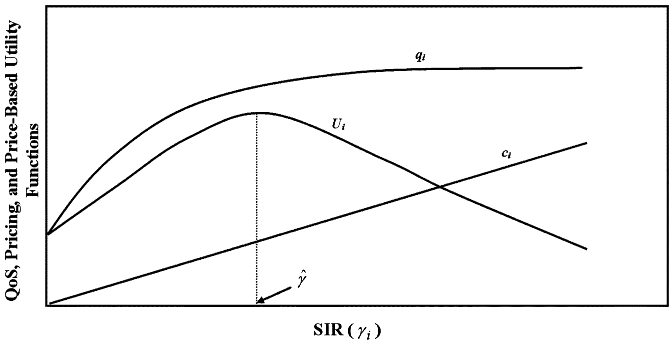

Fig. 1. The QoS, the pricing, and the price-based utility functions vs. SIR.

unique NE $\mathbf{p}^*$ at which the transmit powers of all users are set to their maximum values. If $\mathbf{p}^* = \overline{\mathbf{p}}$ is not Pareto efficient, then there exists another transmit power vector $\mathbf{p}'$ that Pareto dominates $\mathbf{p}^*$, i.e., $q(\gamma_i') \geq (\gamma_i^*)$ for all $i \in \mathcal{M}$, and $q(\gamma_i') > q(\gamma_i^*)$ for some i, where γ_i' and γ_i^* are the corresponding SIRs achieved by user i for the transmit power vectors $\mathbf{p}'$ and $\mathbf{p}^*$, respectively. As the QoS is an increasing function of SIR, the actual SIR is either increased or is kept fixed, i.e., $\gamma_i' = \gamma_i^*$ for some users and $\gamma_i' > \gamma_i^*$ for others. This is not possible unless the transmit power by each user is increasing (see (1) and note that as $\gamma_i/(\gamma_i + g)$ is an increasing function of γ_i, a fixed or an increasing γ_i results in increasing the transmit power for all users), which contradicts the fact that users are transmitting at their maximum power. ■

For a R-NPCG, the NE is Pareto efficient, but results in the maximum transmit power, which means that attaining any of the goals stated in Section III-E is not possible. However, in what follows, we present a novel pricing scheme to achieve any of the given goals in a Pareto-efficient manner.

V. Proposed Pricing Scheme

A. *Pricing Based Noncooperative Power Control Game (P-NPCG)*

When a user transmits in a shared medium, that user should pay a price (cost) for receiving the service and for causing undesirable interferences to others. It is well established that the pricing scheme could affect the individual user's decision in such a way that Pareto efficiency [3], aggregate QoS [6], or fairness [7] is improved. To the best of our knowledge, none of the existing pricing schemes is adequate for a goal-driven and Pareto-efficient power control.

Unlike the existing pricing schemes, our proposed pricing scheme is an increasing function of the SIR. In its simplest form, it could be a linear function of the SIR. Let $c_i(\gamma_i)$ be the pricing function of user i for γ_i at the base station, and the pricing-based utility function for user i be

$$U_i(p_i, \mathbf{p}_{-i}) = q(\gamma_i) - c_i(\gamma_i) \tag{14}$$

whose simplest form is

$$U_i(p_i, \mathbf{p}_{-i}) = q(\gamma_i) - \alpha_i \gamma_i \tag{15}$$

where $\alpha_i \geq 0$ is the price per unit of the actual SIR at the base station for user i. The price α_i is declared to user i by the base station. We will use (15) as the price-based utility function of the P-NPCG, but the results can be extended to the general case of $c_i(\gamma_i)$. In this case, the cost to each user is proportional to its SIR. We will show that this pricing scheme enables us to adequately influence the best response function of each user so that all users reach a unique NE, at which the Pareto efficiency and any one of the goals (enumerated in Section III-E) can be satisfied simultaneously, each by a proper choice of the price. This is different from [1], [3], [6], and [7], in which a pricing mechanism is employed either for just improving the Pareto efficiency, optimizing the aggregate QoS for a fixed total transmit power, optimizing the aggregate utility, for fairness, or for improving the system convergence under no peak transmit power constraint.

We begin by using the single-pricing scheme, in which the price is the same for all users, and obtain the corresponding NE. We also dynamically obtain the optimal single price so that its corresponding NE leads to the OF-QoS. Then, we extend it to the binary-pricing scheme in which the prices for users are either of the two different values. The binary-pricing scheme can be used for satisfying γ^{th}-max-min SIR or O-AT goals at the NE. The dynamic values of binary-prices for attaining each of these goals are also obtained.

B. *Single-Pricing Noncooperative Power Control Game (SP-NPCG)*

Define the SP-NPCG $G_s = \langle \mathcal{M}, (P_i), (U_i) \rangle$, where $U_i(p_i, \mathbf{p}_{-i}) = q(\gamma_i) - \alpha\gamma_i$ for all $i \in \mathcal{M}$ and $\alpha \geq 0$. We show that for the SP-NPCG a unique and Pareto-efficient NE exists for any $\alpha \geq 0$. We also show that there is a unique α in a SP-NPCG that results in fairness (OF-QoS) and Pareto efficiency at the same time.

Theorem 6: In the SP-NPCG, the best response of user $i \in \mathcal{M}$ to a given interference power vector $\mathbf{p}_{-i}$ is (16) at the bottom of the page, where q'^{-1} is the inverse function of the first derivative of the QoS function and β is defined in (3).

$$b_i(\mathbf{p}_{-i}) = \begin{cases} \overline{p}_i, & \text{if } 0 \leq \alpha \leq \beta \\ \min\left\{\overline{p}_i, \frac{q'^{-1}(\alpha)}{gh_i}\left(\sum_{j\neq i} p_j h_j + \sigma^2\right)\right\}, & \text{if } \beta' \alpha \leq q'(0) \\ 0, & \text{if } \alpha > q'(0) \end{cases} \tag{16}$$

Proof: To obtain $b_i(\mathbf{p}_{-i})$, we use the first and the second derivatives of the price-based utility with respect to p_i

$$\frac{\partial U_i}{\partial p_i} = \frac{gh_i}{I_i}\left(q'(\gamma_i) - \alpha\right) \tag{17}$$

$$\frac{\partial^2 U_i}{\partial p_i^2} = \left(\frac{gh_i}{I_i}\right)^2 q''(\gamma_i). \tag{18}$$

We know that $q'(\gamma_i)$ is a strictly decreasing function of γ_i and $\beta < q'(\gamma_i) \leq q'(0)$. Hence, for $0 \leq \alpha \leq \beta$, we have $\partial U_i/\partial p_i > 0$, and thus U_i is an increasing function of p_i. In this case, similar to the R-NPCG, the best response for user i is to transmit at its maximum power, i.e., for $0 \leq \alpha \leq \beta$, $b_i(\mathbf{p}_{-i}) = \overline{p}_i$ for all $i \in \mathcal{M}$. For $\beta < \alpha \leq q'(0)$, the equation $\partial U_i/\partial p_i = 0$, or equivalently $q'(\gamma_i) = \alpha$, has the same unique solution $\widehat{\gamma} = q'^{-1}(\alpha)$ for all $i \in \mathcal{M}$. Note that as $q'(\gamma_i)$ is a strictly decreasing function, its inverse exists, and that $\widehat{\gamma}$ is a decreasing function of α. As $q''(\gamma_i) < 0$ for all γ_i, and hence $\partial^2 U_i/\partial p_i^2 < 0$, the roots of (17) maximize U_i for a given interference $I_i = \sum_{j \neq i} p_j h_j + \sigma^2$, for $\beta < \alpha \leq q'(0)$ (see Fig. 1). For a fixed interference I_i, a one-to-one relation exists between the SIR and the transmit power, and thus the best transmit power in response to $\mathbf{p}_{-i}$ that maximizes U_i is also unique and is equal to $\widehat{p}_i = \widehat{\gamma} I_i/(gh_i)$ for all $i \in \mathcal{M}$. If $\widehat{p}_i > \overline{p}_i$, user i cannot transmit at power $\widehat{p}_i$. In this case, since $\widehat{p}_i$ is the unique maximizer of U_i, then U_i is an increasing function of p_i in $p_i \leq \overline{p}_i < \widehat{p}_i$ for a fixed interference. Therefore, the best response to $\mathbf{p}_{-i}$ is the maximum value of the transmit power, i.e., $b_i(\mathbf{p}_{-i}) = \overline{p}_i$. This implies that for $\beta < \alpha \leq q'(0)$, $b_i(\mathbf{p}_{-i}) = \min\left\{\overline{p}_i, \frac{q'^{-1}(\alpha)}{gh_i}\left(\sum_{j \neq i} p_j h_j + \sigma^2\right)\right\}$ for all $i \in \mathcal{M}$. For $\alpha > q'(0)$, we have $\partial U_i/\partial p_i < 0$, thus U_i is a decreasing function of p_i. In this case, the best response for user i is no transmission, i.e., for $\alpha > q'(0)$, $b_i(\mathbf{p}_{-i}) = 0$ for all $i \in \mathcal{M}$. ■

Note that the best response function here is a continuous and onto function of α. As we will show, this enables us to adjust each user's transmit power by dynamically setting the price for attaining the given goal.

Theorem 7: The NE of SP-NPCG G_s exists and is unique.

Proof: The proposed pricing scheme forces the utility (the QoS minus the cost) function to be quasi-concave [see (18)]. Similar to Theorem 5 for the R-NPCG, one can show that the NE exists for the SP-NPCG. By definition, the NE is the fixed point in the best response function set that satisfies $\mathbf{p} = \mathbf{b}(\mathbf{p})$. For the two cases where $0 \leq \alpha \leq \beta$ and $\alpha > q'(0)$, the fixed point of the best response function set is unique and corresponds to the maximum transmit power and to no transmission by all users, respectively. For the case where $\beta < \alpha \leq q'(0)$, we use the concept of standard functions to prove the uniqueness of the NE. A function $\mathbf{b}(\mathbf{p})$ is standard if for all $\mathbf{0} \leq \mathbf{p} \leq \overline{\mathbf{p}}$, the following properties hold [27]:

- Positivity: $\mathbf{b}(\mathbf{p}) > \mathbf{0}$
- Monotonicity: if $\mathbf{p} \geq \mathbf{p}'$ then $\mathbf{b}(\mathbf{p}) \geq \mathbf{b}(\mathbf{p}')$, and
- Scalability: for all $c > 1$, $c\mathbf{b}(\mathbf{p}) > \mathbf{b}(c\mathbf{p})$.

It has been shown in [27] that the fixed point (if it exists) in a standard function is unique. Similar to [3], it can easily be shown that $\mathbf{b}(\mathbf{p})$ for $\beta < \alpha \leq q'(0)$ obtained in (16) is a standard function, and thus the unique NE exists. ■

Theorem 7 guarantees the uniqueness of the NE for the SP-NPCG, and Theorem 8 obtains this unique value. Let $\mathbf{p}^* = [p_1^*, p_2^*, \ldots, p_M^*]^{\mathrm{T}}$ denote the transmit power vector at the NE for the SP-NPCG G_s. Define

$$\Gamma_i = g\frac{\overline{\varphi}_i}{\sum\limits_{j=1}^{i-1} \overline{\varphi}_j + (M-i)\overline{\varphi}_i + \sigma^2}, \quad i = 1, 2, \ldots, M. \tag{19}$$

Note that Γ_i is the SIR achieved by user i when users 1 to $i-1$ transmit at their maximum power, and users i to M transmit at a level so that their received power at the base station is $\overline{\varphi}_i$. We have $\Gamma_i \leq \Gamma_j$ and, $q'(\Gamma_i) \geq q'(\Gamma_j)$ for all $i < j$.

Theorem 8: In the SP-NPCG, there is the unique NE at which for various values of α, the transmit power is given by (20) at the bottom of the page.

Proof: See Appendix II. ■

In the following corollary, we derive some interesting properties of the NE for the SP-NPCG, which we later use.

Corollary 1 (Some Properties of the Ne in the SP-NPCG): At the NE for the SP-NPCG with the price α

a) if $0 \leq \alpha < q'(\Gamma_M)$, each user transmits at its maximum power;
b) if $\alpha = q'(\Gamma_1)$, then at the NE, OF-QoS (OF-RP and OF SIR) is satisfied, i.e., the NE satisfies max-equal QoS (6) and max-min QoS (7), and so it is Pareto efficient; and
c) if $\alpha > q'(0)$, all users are switched off.

Proof: The statements a) and c) are immediate from (20). If $\alpha = q'(\Gamma_1)$, then from (19) and (20) we know that $p_i^* = \frac{\sigma^2 q'^{-1}(\alpha)}{h_i(g-(M-1)q'^{-1}(\alpha))} = \frac{\sigma^2 \Gamma_1}{h_i(g-(M-1)\Gamma_1)} = \frac{\overline{\varphi}_1}{h_i}$, and thus $p_i^* = \frac{\overline{\varphi}_1}{h_i}$

$$p_i^* = \begin{cases} \overline{p}_i \text{ for all } i \in \mathcal{M}, & \text{if } 0 \leq \alpha < q'(\Gamma_M) \\ \begin{cases} \overline{p}_i & \text{for } i = 1, \ldots, l-1, \\ \frac{\left(\sum\limits_{j=1}^{l-1} \overline{\varphi}_j + \sigma^2\right) q'^{-1}(\alpha)}{h_i(g-(M-l)q'^{-1}(\alpha))} & \text{for } i = l, \ldots, M, \end{cases} & \text{if } q'(\Gamma_l) \leq \alpha' q'(\Gamma_{l-1}), \text{ where } l = 2, 3, \ldots, M \\ \frac{\sigma^2 q'^{-1}(\alpha)}{h_i\left(g-(M-1)q'^{-1}(\alpha)\right)} \text{ for all } i \in \mathcal{M}, & \text{if } q'(\Gamma_1) \leq \alpha \leqq' (0) \\ 0 \text{ for all } i \in \mathcal{M}, & \text{if } \alpha > q'(0). \end{cases} \tag{20}$$

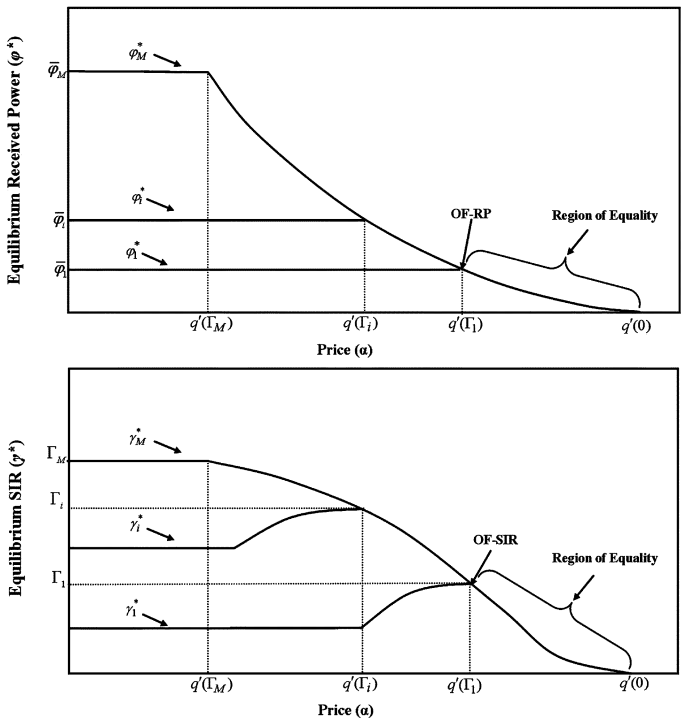

Fig. 2. The received power and the SIR achieved for individual users vs. the price at the NE of SP-NPCG. Note that $\varphi_i^* \leq \varphi_j^*$ and $\gamma_i^* \leq \gamma_j^*$ for all $i < j$ for any given α, where φ_i^* and γ_i^* are the received power and the SIR of user i at the NE, respectively. For $\alpha \geq q'(\Gamma_1)$, the received power as well as the SIR achieved by all users at the NE are the same (region of equality). If $\alpha = q'(\Gamma_1)$, then the NE satisfies max-min QoS (OF-RP and OF-SIR). For $\alpha > q'(0)$, all users are switched off, and for $0 \leq \alpha < q'(\Gamma_M)$, every user transmits at its maximum power at the NE. The QoS achieved by any user at the NE also has a similar shape as equilibrium SIR.

for all $i \in \mathcal{M}$. Thus, from Theorems 1 and 3, we conclude statement b) above. ∎

Fig. 2 shows the received power and the SIR for each user at the NE with respect to price, in which we note that the SIR, and thus the QoS, for a given user i at the NE of SP-NPCG G_s is maximized when $\alpha = q'(\Gamma_i)$. This can be proved by using (20). Corollary 1-b obtains the dynamic prices for satisfying Pareto efficiency and fairness simultaneously. Note that Γ_1 is time varying, whose values should be dynamically obtained from (19) for $i = 1$.

C. Binary-Pricing Noncooperative Power Control Game (BP-NPCG)

Although the proposed pricing scheme with single-price $\alpha = q'(\Gamma_1)$ provides each user with equal QoS in an optimum and Pareto-efficient manner at the NE, it cannot be used for satisfying γ^{th}-max-min SIR, or for aggregate throughput optimization. In what follows, we extend the proposed pricing scheme to the binary-pricing scheme to attain any one of the above goals in a Pareto-efficient manner.

Let the set $\mathcal{F}$ denote the group of users to be dropped (not to transmit), and $\mathcal{F}'$ denote the remaining users. Power control and switch-off mechanisms can be jointly applied by using the proposed pricing scheme with binary-pricing for members of $\mathcal{F}$ and $\mathcal{F}'$ in a complementary manner, as will be shown by the following theorem.

Theorem 9: Define the BP-NPCG $G_b = \langle \mathcal{F} \cup \mathcal{F}', (P_i), (U_i) \rangle$, where $U_i(p_i, \mathbf{p}_{-i}) = q(\gamma_i) - \alpha_i \gamma_i$ in which α_i is equal to either of the two different values. In BP-NPCG, if $\alpha_i \geq q'(0)$ for all $i \in \mathcal{F}$ and $\alpha_i = \alpha$ where $\beta < \alpha \leq q'(0)$ for all $i \in \mathcal{F}'$, then the game G_b has a unique NE at which the transmit power for all $i \in \mathcal{F}$ is zero ($p_i^* = 0$) and, for all $i \in \mathcal{F}'$, the transmit power is the one calculated in Theorem 8 by replacing $\mathcal{M}$ with $\mathcal{F}'$ (and thus M with $|\mathcal{F}'|$, where $|\mathcal{F}'|$ is the number of members in $\mathcal{F}'$).

Proof: Choosing $\alpha_i \geq q'(0)$ for all $i \in \mathcal{F}$ imposes that $\widehat{\gamma} = 0$ be the unique maximizer of U_i, i.e., $b_i(\mathbf{p}_{-i}) = 0$. Similarly, for all $i \in \mathcal{F}'$, the unique maximizer of U_i is $\widehat{\gamma} = q'^{-1}(\alpha_i)$, i.e., $b_i(\mathbf{p}_{-i}) = \min\left\{\overline{p}_i, \frac{q'^{-1}(\alpha)}{gh_i}\left(\sum_{j \neq i} p_j h_j + \sigma^2\right)\right\}$ for all $i \in \mathcal{F}'$. Now, $b_i(\mathbf{p}_{-i}) = 0$, for all $i \in \mathcal{F}$ implies that $p_i^* = 0$ for all $i \in \mathcal{F}$, meaning that users in $\mathcal{F}$ cause no interference to users in $\mathcal{F}'$. Thus, the problem is reduced to finding the fixed point for $b_i(\mathbf{p}_{-i})$ for all $i \in \mathcal{F}'$, which is equivalent to the fixed point of a SP-NPCG $\langle \mathcal{F}', (P_i), (U_i) \rangle$ with $U_i(p_i, \mathbf{p}_{-i}) = q(\gamma_i) - \alpha\gamma_i$, which can be calculated by using Theorem 8 and replacing $\mathcal{M}$ with $\mathcal{F}'$ (thus replacing M with $|\mathcal{F}'|$) and assuming (without loss of generality) that members of $\mathcal{F}'$ are indexed from 1 to $|\mathcal{F}'|$ in an increasing order of their maximum received power. In other words, the fixed point of the best response function set for all $i \in \mathcal{F}'$ together with $p_i^* = 0$ for all $i \in \mathcal{F}$ constitute the fixed point of the best response function set (i.e., $p_i^* = b_i\left(\mathbf{p}_{-i}^*\right)$ for all $i \in \mathcal{F} \cup \mathcal{F}'$). ∎

Theorem 9 states that the binary-price enables us to divide the users into two groups so that at the NE, one group is dropped and the other is controlled by tuning their prices. In the following two corollaries, two specific cases that may be more interesting are stated (their proofs are immediate from Theorems 2–4 and 9).

Corollary 2: In the BP-NPCG G_b defined in Theorem 9, assume $\alpha_i = q'\left(\gamma^{\text{th}}\right)$ for $i = \{k, k+1 \ldots, M\}$, and $\alpha_i \geq q'(0)$ for $i = \{1, 2, \ldots, k-1\}$, for a given γ^{th}. The index k is chosen so that if $\gamma^{\text{th}} \leq \widetilde{\Gamma}_1$, then $k = 1$, and if $\widetilde{\Gamma}_1 < \gamma^{\text{th}} \leq \widetilde{\Gamma}_M$, then $k \in \{2, \ldots, M\}$ for which $\widetilde{\Gamma}_{k-1} < \gamma^{\text{th}} \leq \widetilde{\Gamma}_k$. Now, at the NE, the γ^{th}-max-min SIR goal (and consequently the Pareto efficiency) is satisfied.

Corollary 3: In the BP-NPCG G_b defined in Theorem 9, if $\alpha_M \leq q'\left(g\frac{\overline{\varphi}_M}{\sigma^2}\right)$ and if $\alpha_i \geq q'(0)$ for all $i \neq M$, then at the NE, the user M transmits at its maximum power and other users are switched off. Therefore, at the NE, the O-AT goal (and consequently the Pareto efficiency) is satisfied.

Corollaries 2 and 3 dynamically obtain the prices for satisfying the γ^{th}-max-min SIR and for achieving the optimum aggregate throughput, respectively. Note that we can also use Corollary 2 to trade off between fairness and aggregate throughput optimization by setting the value of γ^{th} such that a given threshold for aggregate throughput is achieved.

VI. Distributed Goal-Driven Power Control Algorithms

In any distributed power control algorithm that converges to the NE of either the SP-NPCG or the BP-NPCG, or equivalently attains the max-min SIR or the γ^{th}-max-min SIR or optimizes the aggregate SIR, each user needs to get its optimal price. Optimal prices for each goal can be obtained at the base station if each user informs the base station of its path gain and its maximum transmit power. In this way, the base station should find the solutions to the OF-SIR, $\gamma^{\rm th}$-max-min SIR, or the O-AT; and then announce them back to each user (in terms of the pricing, or equivalently the target SIR). This centralized decision-making can be replaced by a distributed one if each user can set its price (or equivalently its target SIR) in a distributed and optimal manner. In what follows, we first assume that the optimal prices for a given goal are given to each user by the base station and subsequently propose a distributed scheme for obtaining the prices by users.

A. Centralized (Base Station) Price Setting

The work presented in [27] provides a framework for understanding the convergence of the existing power control algorithms, where it is shown that for a standard function $\mathbf{f}(\mathbf{p})$, if there is a unique fixed point $\mathbf{p}^*$ so that $\mathbf{p}^* = \mathbf{f}(\mathbf{p}^*) = [f_1(\mathbf{p}^*), f_2(\mathbf{p}^*), \ldots, f_M(\mathbf{p}^*)]^{\rm T}$, then for any initial power vector, the power control algorithm $\mathbf{p}(t) = \mathbf{f}(\mathbf{p}(t-1))$ converges to $\mathbf{p}^*$, where t is the time step number. As we proved in Theorems 7 and 9 for the SP-NPCG and the BP-NPCG, respectively, there is a unique fixed point for their corresponding best response function set (i.e., the NE). Thus, if every user updates its transmit power along with its own best response function, the NE is attained. The following theorem can be proved taking similar steps as in [27].

Theorem 10: The distributed goal-driven power control (DGD-PC) algorithm is defined by $\mathbf{p}(t) = \mathbf{b}(\mathbf{p}(t-1))$, where $\mathbf{b}(\mathbf{p})$ is the best response function set, or equivalently

$$p_i(t) = \min\left\{\overline{p}_i, \frac{q'^{-1}(\alpha_i)}{gh_i}\left(\phi(t-1) - h_i p_i(t-1)\right)\right\} \quad (21)$$

for all $i \in \mathcal{M}$, where ϕ is the total received power plus noise at the base station, i.e., $\phi = \sum_{i=1}^{M} \varphi_i + \sigma^2$, and $\beta < \alpha_i \leq q'(0)$ is the price introduced in the SP-NPCG or in the BP-NPCG, which is given to user i by the base station. The algorithm (21) converges to the NE for the corresponding game.

Note that for $\beta < \alpha_i \leq q'(0)$, the best response function of each game (the SP-NPCG or the BP-NPCG) is (21). Hence, the NE of the game with the proposed pricing scheme can be reached in a distributed manner by each user knowing only its own uplink gain, the total received power at the base station, and its own price given by the base station. No user is required to know the path gains, the (peak) transmit powers, and the prices of others. However, as stated earlier, all users should inform the base station of their path gains and maximum transmit powers. In the sequel, three distributed power control algorithms are presented, requiring neither the knowledge of users' path gains and maximum transmit powers by the base station nor the provision of the optimum values of prices by the base station to users. Instead, they only require the base station to broadcast the total number of active users.

B. Distributed Price Setting

In what follows, we propose three distributed algorithms, each converging to the optimal solution of max-min SIR, O-AT, or $\gamma^{\rm th}$-max-min SIR, respectively, in which each user sets its transmit power (or equivalently its price or its target SIR) in a distributed manner requiring only to know its own uplink gain, the total received power plus noise at the base station, the number of active (transmitting) users, and the additive white Gaussian noise (AWGN) power at the receiver. The number of active (transmitting) users can be broadcast by the base station to users, which we assume would be the case. Furthermore, there are well-known algorithms by which each user can obtain the number of active users by only knowing the total received power at the base station (for details, see [28]–[31]).

Theorem 11: Define the distributed optimum-fairness goal-driven power control (DOFGD-PC) algorithm as

$$p_i(t+1) = \min\left\{\overline{p}_i, \frac{\phi(t) - \sigma^2}{Mh_i}\right\} \text{ for all } i \in \mathcal{M} \quad (22)$$

where ϕ is the total received power plus noise at the base station. The algorithm (22) converges to the optimal solution of the max-min SIR if users start transmitting at their maximum power, i.e., $p_i(0) = \overline{p}_i$ for all $i \in \mathcal{M}$.

Proof: The fixed point of (22) is $p_i^* = \min\left\{\overline{p}_i, \frac{\phi^* - \sigma^2}{Mh_i}\right\}$, where $\phi^* = \sum_{j\in\mathcal{M}} \varphi_j^* + \sigma^2$, in which $\varphi_j^* = h_j p_j^*$. Equivalently

$$\min\left\{\overline{\varphi}_i, \frac{\phi^* - \sigma^2}{M}\right\} = \varphi_i^* \text{ for all } i \in \mathcal{M}. \quad (23)$$

Thus we have

$$\sum_{j=1}^{M} \varphi_j^* = M\varphi_i^* \text{ for all } i \in \mathcal{M}, \quad (24)$$

or equivalently $\varphi_j^* = \varphi_i^*$ for all $i, j \in \mathcal{M}$. This means that at the fixed point, the received power at the base station for each user is the same as that of other users. One can easily show that if $\varphi_i(0) = \overline{\varphi}_i$ for all $i \in \mathcal{M}$, we have $\frac{\sum_{j\in\mathcal{M}} \varphi_j(t)}{M} \geq \overline{\varphi}_1$ and consequently $\min\left\{\overline{\varphi}_i, \frac{\sum_{j\in\mathcal{M}} \varphi_j(t)}{M}\right\} \geq \overline{\varphi}_1$ for all $t \geq 0$. Thus, in this case we have

$$\varphi_j(t) \geq \overline{\varphi}_1, \text{ for all } i \in \mathcal{M} \text{ for all } t \geq 0. \quad (25)$$

It is evident that for all $t \geq 0$ and for all $i \in \mathcal{M}$, $\frac{\sum_{j\in\mathcal{M}} \varphi_j(t+1)}{M} \leq \frac{\sum_{j\in\mathcal{M}} \varphi_j(t)}{M}$, and thus $\min\left\{\overline{\varphi}_i, \frac{\sum_{j\in\mathcal{M}} \varphi_j(t+1)}{M}\right\} \leq \min\left\{\overline{\varphi}_i, \frac{\sum_{j\in\mathcal{M}} \varphi_j(t)}{M}\right\}$, or equivalently $\varphi_i(t+1) \leq \varphi_i(t)$, which implies that $\varphi_i(t)$ is decreasing in time. Since from (25), $\varphi_i(t)$ is lower-bounded by $\overline{\varphi}_1$ for all $i \in \mathcal{M}$, and any fixed point of the algorithm has the property that $\varphi_k^* = \varphi_i^* \leq \overline{\varphi}_1$ for all $i, k \in \mathcal{M}$, we have $\lim_{t\to\infty} \varphi_i(t) = \overline{\varphi}_1$ for all $i \in \mathcal{M}$. Thus, under the DOFGD-PC algorithm, if users start transmitting at their maximum powers, the max-min SIR would be satisfied at the steady state. ■

Theorem 12: Define the distributed O-AT goal-driven power control (DOATGD-PC) algorithm as

$$p_i(t+1) = \left\{ \begin{matrix} \overline{p}_i, \text{ if } |\mathcal{A}(t)|\,\overline{\varphi}_i + \sigma^2 \geq \phi(t) \\ 0, \text{ O.}W. \end{matrix} \right\} \text{ for all } i \in \mathcal{M}, \tag{26}$$

where $\mathcal{A}(t)$ is the set of active users at time t, i.e., $\mathcal{A}(t) = \{i\,|p_i(t) \neq 0 \forall i \in \mathcal{M}\}$ and $|\mathcal{A}(t)|$ is the number of its members. The distributed power-update function (26) has a unique fixed point, which is the unique solution of O-AT as well. Furthermore, for any initial power vector, the algorithm converges to its fixed point.

Proof: One can easily show that the unique solution of O-AT is a fixed point of (26) (i.e., $p_M^* = \overline{p}_M$ and $p_i^* = 0$ for all $i \in \mathcal{M}$). Thus, the algorithm has a fixed point. Now, we show that this fixed point is unique. It is evident that at the fixed point we have $|\mathcal{A}| \neq 0$ because if $|\mathcal{A}| = 0$, then we must have $\sigma^2 < \sigma^2$, which is a contradiction. It can also be observed that at the fixed point we must have $|\mathcal{A}| = 1$ because if $|\mathcal{A}| > 1$, then $\phi \leq |\mathcal{A}|\,\overline{\varphi}_i + \sigma^2$ must hold for all users in the set $\mathcal{A}$, which contradicts the fact that we have $\phi > |\mathcal{A}|\,\overline{\varphi}_i + \sigma^2$ at least for $i = \arg\min_{j\in\mathcal{A}}\overline{\varphi}_j$. Now, we show that the set $\mathcal{A}$ at the fixed point only includes user M. This is observed by noting that when $|\mathcal{A}| = 1$, the inequality $\phi \leq \overline{\varphi}_i + \sigma^2$ always holds for user M, and when user M is the only member of $\mathcal{A}$, we have $\phi > \overline{\varphi}_i + \sigma^2$ for each user $i \neq M$. In what follows, we show that starting from any initial power vector, the algorithm (26) converges to its unique fixed point.

1 For a given $t \geq 1$, we have $p_i(t) = \overline{p}_i$ for all $i \in \mathcal{A}(t)$ and $p_i(t) = 0$ for all $i \notin \mathcal{A}(t)$, implying that at each time step, a given user either transmits at its maximum power or is switched off.

2 For user M, (i.e., $i = M$), we have $i \in \mathcal{A}(t)$ for all $t \geq 1$, since $\phi(t) \leq |\mathcal{A}(t)|\,\overline{\varphi}_i + \sigma^2$ always holds for user M.

3 For any $|\mathcal{A}(t)| \geq 2$ (i.e., in addition to user M, at least one more user also belongs to $\mathcal{A}(t)$), there is at least one user $i = \arg\min_{j\in\mathcal{A}(t)}\overline{\varphi}_j$ that $i \in \mathcal{A}(t)$ and $i \notin \mathcal{A}(t+1)$ because we have $\phi(t) > |\mathcal{A}(t)|\,\overline{\varphi}_i + \sigma^2$ at least for user $i = \arg\min_{j\in\mathcal{A}(t)}\overline{\varphi}_j$.

4 If $i \notin \mathcal{A}(t)$, then we have $i \notin \mathcal{A}(t+1)$. To validate this, we first show that $\max_{j\notin\mathcal{A}(t)}\overline{\varphi}_j < \min_{k\in\mathcal{A}(t)}\overline{\varphi}_k$. Since for each user $j \notin \mathcal{A}(t)$, we have $|\mathcal{A}(t-1)|\,\overline{\varphi}_j + \sigma^2 < \phi(t-1)$, and since for each user $k \in \mathcal{A}(t)$, we have $|\mathcal{A}(t-1)|\,\overline{\varphi}_k + \sigma^2 \geq \phi(t-1)$, then if $\max_{j\notin\mathcal{A}(t)}\overline{\varphi}_j \geq \min_{k\in\mathcal{A}(t)}\overline{\varphi}_k$, they contradict each other at least for users $\arg\max_{j\notin\mathcal{A}(t)}\overline{\varphi}_j$ and $\arg\min_{k\in\mathcal{A}(t)}\overline{\varphi}_k$. If $i \notin \mathcal{A}(t)$, then from statement 1 above and $\max_{j\notin\mathcal{A}(t)}\overline{\varphi}_j < \min_{k\in\mathcal{A}(t)}\overline{\varphi}_k$, we conclude that $|\mathcal{A}(t)|\,\overline{\varphi}_i + \sigma^2 < \phi(t)$, which implies that $i \notin \mathcal{A}(t+1)$.

The above statements 3 and 4 say that $|\mathcal{A}(t)|$ is decreasing in time. Thus, using the above statements 1 and 2, we conclude that in at most $M-1$ time steps, the algorithm converges to its unique fixed point, i.e., $\mathcal{A}(t) = \{M\}$ for all $t \geq M-1$. ■

We now propose a distributed power control algorithm under which if the OF-SIR for all users is lower than a threshold, those users with the lowest maximum received power switch off one by one until the OF-SIR for the remaining users becomes equal to or higher than the threshold. It can also be used for trading off between fairness and aggregate SIR optimization. The distributed optimum limited-fairness goal-driven power control algorithm (DOLFGD-PC) is defined below.

DOLFGD-PC Algorithm: Assume ε is a small positive constant, γ^{th} is the threshold value for OF-SIR (assuming that it is feasible, i.e., $\gamma^{\text{th}} \leq \overline{\varphi}_M/\sigma^2$), and the number of active users is broadcast by the base station.

1—Let $p_i(t) = \overline{p}_i$ and increment t.

2—Let $p_i(t) = \min\left\{\overline{p}_i, \frac{\phi(t-1)-\sigma^2}{|\mathcal{A}(t)|h_i}\right\}$.

If $|\phi(t) - \phi(t-1)| < \varepsilon$ and $\gamma_i(t) < \gamma^{\text{th}}$ then

if $p_i(t) = \overline{p}_i$, increment t and go to step 3,

else increment t and go to step 1,

else increment t and repeat step 2.

3—Let $p_i(t) = 0$, increment t, and if $\frac{g\overline{\varphi}_i}{|\mathcal{A}(t)|\overline{\varphi}_i+\sigma^2} \geq \gamma^{\text{th}}$, go to step 1, else repeat step 3.

Theorem 13: Under the DOLFGD-PC algorithm, the least number of users are dropped one by one in decreasing order of their max-received power, until the remaining users optimally attain the same SIR that is equal to or higher than γ^{th}, i.e., it converges to the solution of γ^{th}-max-min SIR.

Proof: One can easily see that if $\frac{g\overline{\varphi}_1}{M\overline{\varphi}_1+\sigma^2} \geq \gamma^{\text{th}}$, then all users operate in Step 2 for $t \geq 1$, and from Theorem 11 we conclude that all users attain the same SIR equal to the max-min SIR (i.e., $\frac{g\overline{\varphi}_1}{M\overline{\varphi}_1+\sigma^2}$), which is higher than γ^{th}. If $\frac{g\overline{\varphi}_1}{M\overline{\varphi}_1+\sigma^2} < \gamma^{\text{th}}$ and $|\phi(t) - \phi(t-1)| < \varepsilon$ (i.e., when the distributed algorithm momentarily converges), then only user 1 is transmitting at its maximum power (as Theorem 11 implies). Thus, user 1 goes to Step 3 and remains there while others go to Step 1 and then proceed to Step 2. If $\frac{g\overline{\varphi}_2}{(M-2)\overline{\varphi}_2+\sigma^2} \geq \gamma^{\text{th}}$, then similarly we conclude that all active users operate in Step 2, attaining the same SIR equal to or higher than γ^{th}. Otherwise, user 2 is dropped and the same process is repeated. In fact, the algorithm DOLFGD-PC drops users one by one in decreasing order of their max-received power until the remaining users attain the same SIR that is equal to or higher than γ^{th}. ■

Remark: The proposed DOLFGD-PC algorithm can also be used to trade off between fairness and the aggregate throughput. Suppose that the tradeoff is defined by the maximum possible number of users attaining the same SIR in an optimal manner that is higher than a threshold denoted by $\gamma_{\text{of}}^{\text{th}}$, while the aggregate SIR is also higher than a threshold denoted by Sth. This tradeoff is achieved by replacing the fixed threshold γ^{th} in the DOLFGD-PC algorithm by a dynamic threshold $\gamma^{\text{th}}(t) = \max\left\{\gamma_{\text{of}}^{\text{th}}, \frac{S^{\text{th}}}{|\mathcal{A}(t)|}\right\}$.

The distributed target-SIR assignment algorithms corresponding to the DOFGD-PC, or the DOFGD-PC, or the DOATGD-PC are

$$\widehat{\gamma}_i(t+1) = g\frac{p_i(t+1)h_i}{\phi(t) - h_i p_i(t)} \text{ for all } i \in \mathcal{M} \tag{27}$$

where $p_i(t+1)$ is the transmit power for user i obtained in a distributed manner by using the corresponding algorithm. These

TABLE I
LIST OF PARAMETERS/FUNCTIONS AND THEIR RESPECTIVE VALUE/ASSIGNMENTS IN CASE STUDIES

Parameter / Function	Value / Assignment
M, the total number of users	6
R_c, chip rate	10^6 bits/second
R, bit rate	10^4 bits/second
σ^2, AWGN power at the receiver	5×10^{-15} Watts
$\overline{p}$	2 Watts
λ,	0.09
$q(\gamma)$	$\frac{R}{2}\log_2(1+\gamma)$
$q'(\gamma)$	$\frac{R}{2\ln 2(1+\gamma)}$
$q'^{-1}(\gamma)$	$\frac{\frac{R}{2\ln 2}-\alpha}{\alpha}$

three distributed target SIR assignment algorithms converge to the solution of the max-min SIR, the O-AT, or the γ^{th}-max-min SIR, respectively, and enable users in the NPCG to set their prices iteratively in a distributed manner by

$$\alpha_i(t+1) = q'\left(\widehat{\gamma}_i(t+1)\right) \tag{28}$$

so that the resulting NE (at the steady state) satisfies the corresponding goal.

VII. Numerical Results

Now, we provide numerical results of applying our proposed schemes. Assume that six users are located in the area covered by a given base station and the upper bound on the transmit power for all users are the same and equal to 2 W. We adopt a simple and well-known model [32] for the path gain as $h_i = \lambda d_i^{-4}$, where d_i is the distance between the user i and the base station and λ is the attenuation factor that represents the power variation due to the shadowing effect. We take $\lambda = 0.09$. The system parameters are listed in Table I. In our case studies, we use the logarithmic function of the SIR given by (4) with $k = R/2$ as the QoS function. This QoS function as well as its first derivative and its inverse function used in this section are also shown in Table I. The distance vector is $\mathbf{d} = [d_1, d_2, \ldots, d_M]^{\mathrm{T}}$. Now, we consider two different distance vectors $\mathbf{d}^{(1)} = [1100,\ 880,\ 680,\ 470,\ 360,\ 290]^{\mathrm{T}}$ m, and $\mathbf{d}^{(2)} = [3900,\ 880,\ 680,\ 470,\ 360,\ 290]^{\mathrm{T}}$ m, (their only difference being the locations of user 1). The path gains due to $\mathbf{d}^{(1)}$ and $\mathbf{d}^{(2)}$ are $\mathbf{h}^{(1)} = 10^{-10}\times[0.0006,\ 0.0015,\ 0.0042,\ 0.0184,\ 0.0536,\ 0.1272]^{\mathrm{T}}$ and $\mathbf{h}^{(2)} = 10^{-10}\times[0.000004,\ 0.0015,\ 0.0042,\ 0.\ 0184,\ 0.0536,\ 0.1272]^{\mathrm{T}}$, respectively. For each distance vector $\mathbf{d}^{(k)}$ $(k = 1, 2)$, the maximum transmit power for each user i denoted by $\overline{\varphi}_i^{(k)}$ and $\Gamma_i^{(k)}$ [defined in (19)] for $i = 1, 2, \ldots, M$, where k is the index for the respective distance vector, are easily calculated by using $\mathbf{h}^{(k)}$ and the parameters' values in Table I.

If no pricing is applied, at the NE, selfish users would transmit at their maximum power to maximize their utility function, and none of the goals mentioned in Section III-E is attained. We apply the proposed schemes in three scenarios (S1–S3) in Table II, each with a given distance vector. In each scenario, to attain a specific goal, we calculate the optimal values of prices as described in Section V. The goals, the distance vectors, the optimal prices, the corresponding NE transmit powers, and the NE SIRs for each scenario are given in Table II. We also simulate each scenario where all users update their transmit powers using our proposed distributed power control algorithm corresponding to the goals stated in Table II (i.e., for S1, DOFGD-PC; for S2, DOLFGD-PC; and for S3, DOATGD-PC).

Scenario 1: We begin by considering six users at $\mathbf{d}^{(1)}$ from the base station whose goal is to provide fairness in an optimum and Pareto-efficient manner (S1). The OF-RP (OF-SIR) is achieved from (8) when the received power at the base station for each user is equal to $\overline{\varphi}_1^{(1)} = 0.0012\times 10^{-10}$ W. The OF-SIR (9) and the OF-QoS (10) are equal to 19.83864 and 21906 bps, respectively. If we apply our proposed pricing scheme by centrally setting the price to $\alpha = q'\left(\Gamma_1^{(1)}\right) = 346.1587$ (as shown in Corollary 1-b), at the NE, each user (with complete information) transmits at a power level required for attaining the OF-SIR (as well as the OF-QoS) point [see Table II (S1)]. We simulate the case where each user updates its transmit power using our proposed distributed DOFGD-PC algorithm (21). The initial transmit power vector is set at its maximum value. The transmit power, and the SIR versus each iteration are shown in Fig. 3. Note that the algorithm rapidly converges to the NE, where at its steady state, each user transmits at a power level required for attaining the OF-SIR.

Scenario 2: Now assume that the goal is to optimize the aggregate throughput (S2). From Theorem 2, we know that the aggregate throughput is optimized if users 1–5 do not transmit, and user 6 transmits at its maximum power. We use the centralized scheme to obtain $\alpha \geq q'(0) = 7213.5$ for users 1–5 and $\alpha \leq q'\left(g\overline{\varphi}_6^{(2)}/\sigma^2\right) = 0.0142$ for user 6 [see Table II (S2)]. Simulation results for updating the transmit power by each user according to our proposed DOATGD-PC algorithm (26) are shown in Fig. 4. Note that by using our proposed distributed algorithm, at the steady state, user 6 transmits at its maximum power while users 1–5 are dropped (no transmission), and thus the aggregate throughput is optimized.

Scenario 3: Now assume that user 1 moves to a farther point $d_1 = 3900$ m from the base station, and hence $\mathbf{d} = \mathbf{d}^{(2)}$. The system could be made optimum-fair again if the centralized scheme is applied and the base station announces $\alpha = q'\left(\Gamma_1^{(2)}\right) = 512.7260$ to all users or if the distributed DOFGD-PC scheme is employed by all users. Due to the very low OF-RP (i.e., $\overline{\varphi}_1^{(2)} = 0.7781 \times 10^{-15}$ W), all users experience an unsatisfactory OF-SIR (9), namely $\gamma_{\mathrm{of}} = g\overline{\varphi}_1^{(2)}/\left((M-1)\overline{\varphi}_1^{(2)} + \sigma^2\right) = 8.7518$, meaning that all users are punished. Now, it is better to drop user 1 from the network and let the remaining five users get an equal QoS in an optimum manner. By dropping user 1, the OF-RP and the OF-SIR for users 2–6 is $\overline{\varphi}_2^{(2)} = 0.0030 \times 10^{-10}$ W and $\gamma_{\mathrm{of}} = g\overline{\varphi}_2^{(2)}/\left((M-2)\overline{\varphi}_2^{(2)} + \sigma^2\right) = 24.8963$. Note that dropping user 1 triples the OF-SIR for the remaining users. In other words, for Scenario 3 the goal is γ^{th}-max-min SIR for a given $8.7518 \leq \gamma^{\mathrm{th}} < 24.8963$. This can be

TABLE II
OPTIMAL PRICES FOR THREE DIFFERENT SCENARIOS/GOALS AND THE NE (THE TRANSMIT POWER AND THE SIR) FOR THE CORRESPONDING NPCG

Senario/Goal	Dist. Vector	Optimal Prices (α) for the Specified Goal	Parameters at NE for Each User	User 1	User 2	User 3	User 4	User 5	User 6
S1: max-min SIR	$\mathbf{d}^{(1)}$	$\alpha = 346.1587$ for all users (SP-NPCG)	T. Power	2	0.8192	0.2921	0.0667	0.0229	0.0097
			SIR	19.8386	19.8386	19.8386	19.8386	19.8386	19.8386
S2: Aggregate Throughput Optimization	$\mathbf{d}^{(1)}$	$\alpha = 0.0142$ for user 6, and $\alpha = 7213.5$ for user 1-5 (BP-NPCG)	T. Power	0	0	0	0	0	2
			SIR	0	0	0	0	0	508990
S3: γ^{th}-max-min SIR for a given $8.7518 \leq \gamma^{\text{th}} < 24.8963$	$\mathbf{d}^{(2)}$	$\alpha = 278.5522$ for user 2-6, and $\alpha = 7213.5$ for user 1 (BP-NPCG)	T. Power	0	2	0.7131	0.1627	0.0560	0.0236
			SIR	0	24.8963	24.8963	24.8963	24.8963	24.8963

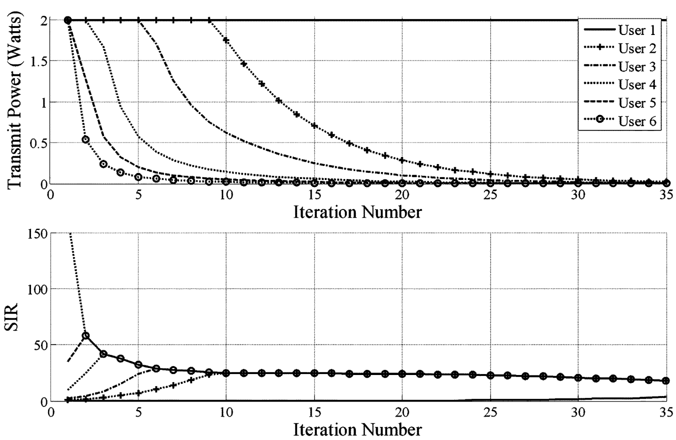

Fig. 3. The transmit power and the SIR of each user vs. the iteration number for the DOFGD-PC algorithm (S1). Note that at the steady state, all users get their OF-SIR (equal to 19.83864 bps).

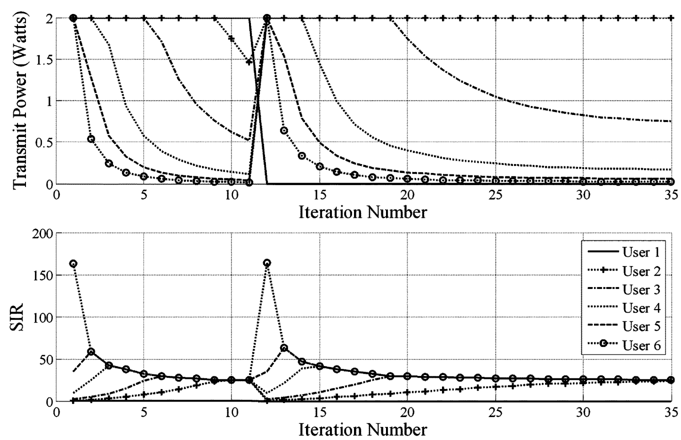

Fig. 5. The transmit power and the SIR of each user vs. the iteration number for the DOLFGD-PC algorithm (S3). Note that at the steady state, user 1 is turned off and other users get their OF-SIR (equal to 24.8963 bps).

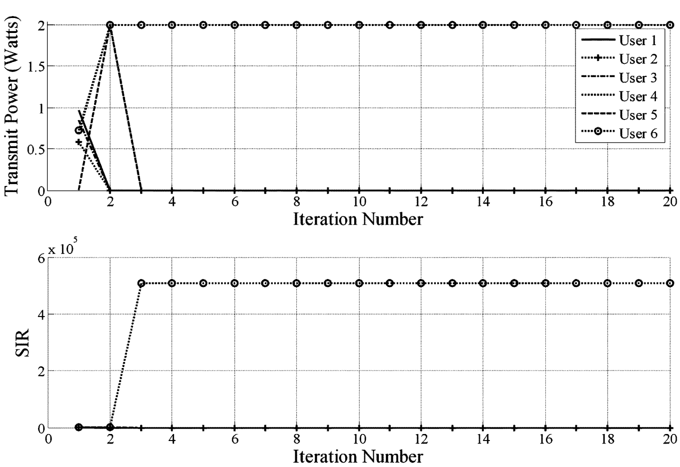

Fig. 4. The transmit power and the SIR of each user vs. the iteration number for the DOATGD-PC algorithm (S2). Note that the at steady state, user 6 transmits at its maximum power while users 1–5 are dropped (no transmission), and thus the aggregate throughput is optimized.

achieved by using the centralized scheme for determining the optimal price (Corollary 2) and declaring a binary price $\alpha = q'(24.8963) = 278.5522$ for users 2–6 and $\alpha \geq q'(0) = 7213.5$ for user 1 by the base station [see Table II (S3)]. Fig. 5 shows simulation results of employing our proposed distributed DOLFGD-PC algorithm for obtaining optimal prices. Note that as Fig. 5 illustrates, by using our proposed distributed algorithm, at the steady state, user 1 is turned off and other users get their OF-SIR $\gamma_{\text{of}} = 24.8963$.

VIII. CONCLUSION

We proposed a novel pricing scheme for a noncooperative power control game and showed that tuning the price (either single-pricing or binary-pricing) for each user in the proposed scheme enables us to satisfy different goals (such as Pareto efficiency, max-min SIR (OF-SIR), γ^{th}-max-min SIR, O-AT, or trading off between fairness and aggregate throughput) in a controlled manner at the NE. We also showed that for the proposed pricing scheme, the NE is unique and Pareto efficient. Specifically we showed that the proposed pricing scheme with a single-price is adequate for a single service network, as it forces the NE to be fair in an optimal manner. Furthermore, the proposed pricing scheme with a binary-price enables us to apply limited fairness or to optimize the aggregate throughput.

We analytically obtained the optimal prices for each goal. In a centralized scheme, we require the base station to dynamically announce the optimal prices to users so that the preassigned goal will be satisfied at the NE. In this scheme, each user must inform the base station of its path gain and its maximum transmit power. To avoid such communication between the base station and users, for each goal we presented a distributed scheme for updating users' transmit power or equivalently for setting their optimal price (target SIR setting), which converges to the corresponding goal without requiring the base station to know users' path gains and their (peak) transmit powers or to provide the optimal prices to users. It instead requires each user to know the number of active users, which can be broadcast by the base station.

APPENDIX I
PROOF OF THEOREM 1

First, we show that $p_i^* = \overline{\varphi}_1/h_i$ for all $i \in \mathcal{M}$ is an optimal solution to the max-equal QoS problem (6). As the QoS objective is a strictly increasing function of SIR for all users, the *same QoS* constraint imposes that the SIR at the base station be the same for all users. Substituting the same SIR γ for all users in (2) gives

$$\gamma \leq g\frac{\overline{\varphi}_1}{(M-1)\overline{\varphi}_1+\sigma^2}. \tag{29}$$

This implies that the max-equal QoS is achieved when the received power of all users is equal to the min-max received power, i.e., $\overline{\varphi}_1 = \min_i \overline{\varphi}_i$. The optimal solution to (6) is $p_i^* = \overline{\varphi}_1/h_i$ for all $i \in \mathcal{M}$. It was shown in [21] that the optimal solution to the max-min SIR problem is unique and results in the equality in the SIR sense, i.e., $\gamma_i(\mathbf{p}) = \gamma_j(\mathbf{p})$ for all $i,j \in \mathcal{M}$; conversely, any equal SIR with at least one user transmitting at its maximum power is a solution to the max-min SIR problem. We know that the max-equal QoS is achieved when at least one user (the one with the smallest maximum received power) transmits at its maximum power. Thus, in power control, the max-equal QoS and the max-min QoS problems have the same solution.

APPENDIX II
PROOF OF THEOREM 8

It is easy to see that $\Gamma_1 \leq \Gamma_2 \leq \cdots \leq \Gamma_M$ and $q'(\Gamma_M) \leq \cdots \leq q'(\Gamma_2) \leq q'(\Gamma_1) \leq q'(0)$. The fixed point of the best response function is

$$p_i^* = b_i\left(\mathbf{p}_{-i}^*\right), \text{ for all } i \in \mathcal{M} \tag{30}$$

where $b_i\left(\mathbf{p}_{-i}^*\right)$ is given by (16).

From Theorem 7, we know that the fixed point of the best response function set (16) is unique. Thus, we only need to show that (20) satisfies (30) for any $\alpha \geq 0$. From (16), we know that for $0 \leq \alpha \leq \beta$, we have $b_i(\mathbf{p}_{-i}) = \overline{p}_i$ for all $i \in \mathcal{M}$ whose fixed point is $p_i^* = \overline{p}_i$ for all $i \in \mathcal{M}$. For $\beta < \alpha \leq q'(0)$, U_i is maximized at $\widehat{\gamma} = q'^{-1}(\alpha)$. If $\beta < \alpha < q'(\Gamma_M)$, then $\widehat{\gamma} > \Gamma_M$. In this case, $p_i^* = \overline{p}_i$ for all $i \in \mathcal{M}$ is the fixed point of the best response function set (30) because from $\widehat{\gamma} > \Gamma_M$, for all $i \in \mathcal{M}$ we have

$$\frac{\widehat{\gamma}}{gh_i}\left(\sum_{j=1,j\neq i}^{M}\overline{\varphi}_j+\sigma^2\right) > \frac{\overline{\varphi}_M}{h_i}\left(\frac{\sum_{j=1,j\neq i}^{M}\overline{\varphi}_j+\sigma^2}{\sum_{j=1}^{M-1}\overline{\varphi}_j+\sigma^2}\right) \geq \frac{\overline{\varphi}_i}{h_i}.$$

Thus

$$\min\left\{\overline{p}_i, \frac{\widehat{\gamma}}{gh_i}\left(\sum_{j=1,j\neq i}^{M}\overline{\varphi}_j+\sigma^2\right)\right\} = \overline{p}_i, \text{ for all } i \in \mathcal{M}.$$

Therefore, for $\beta < \alpha < q'(\Gamma_M)$ (and consequently for $0 \leq \alpha < q'(\Gamma_M)$), we have $p_i^* = \overline{p}_i$ for all $i \in \mathcal{M}$. For $q'(\Gamma_M) \leq \alpha < q'(\Gamma_1)$, there is a unique $l \in \{2,\ldots,M\}$ so that $q'(\Gamma_l) \leq \alpha < q'(\Gamma_{l-1})$ or equivalently $\Gamma_{l-1} < \widehat{\gamma} = q'^{-1}(\alpha) \leq \Gamma_l$, which implies

$$\overline{\varphi}_{l-1} < \frac{\widehat{\gamma}}{g}\left(\sum_{j=1}^{l-2}\overline{\varphi}_j+(M-l+1)\overline{\varphi}_{l-1}+\sigma^2\right)$$

and

$$\frac{\widehat{\gamma}}{g}\left(\sum_{j=1}^{l-1}\overline{\varphi}_j+(M-l)\overline{\varphi}_l+\sigma^2\right) \leq \overline{\varphi}_l.$$

The latter two inequalities can be stated by

$$\overline{\varphi}_{l-1}'\varphi^* = \frac{\left(\sum_{j=1}^{l-1}\overline{\varphi}_j+\sigma^2\right)\widehat{\gamma}}{g-(M-l)\widehat{\gamma}} \leq \overline{\varphi}_l. \tag{31}$$

This suggests that transmitting at the maximum power by users 1 to $l-1$, i.e., $p_i^* = \overline{p}_i$ for $i = 1,2,\ldots,l-1$, and setting the transmit power to $p_i^* = \frac{\varphi^*}{h_i}$ for $i = l, l+1, \ldots, M$, satisfies (30) for $q'(\Gamma_l) \leq \alpha < q'(\Gamma_{l-1})$, i.e., for $i = 1,2,\ldots,l-1$

$$\min\left\{\overline{p}_i, \frac{\widehat{\gamma}}{gh_i}\left(\sum_{j=1,j\neq i}^{l-1}\overline{\varphi}_j+(M-l+1)\varphi^*+\sigma^2\right)\right\} = \overline{p}_i \tag{32}$$

and for $i = l, l+1, \ldots, M$

$$\min\left\{\overline{p}_i, \frac{\widehat{\gamma}}{gh_i}\left(\sum_{j=1}^{l-1}\overline{\varphi}_j+(M-l)\varphi^*+\sigma^2\right)\right\} = \frac{\varphi^*}{h_i} \tag{33}$$

which we will show in the sequel. Since $\widehat{\gamma} > \Gamma_{l-1}$ for $q'(\Gamma_l) \leq \alpha < q'(\Gamma_{l-1})$ we have

$$\begin{aligned}
&\frac{\widehat{\gamma}}{gh_i}\left(\sum_{j=1,j\neq i}^{l-1}\overline{\varphi}_j+(M-l+1)\varphi^*+\sigma^2\right)\\
&= \frac{\overline{\varphi}_{l-1}}{h_i}\left(\frac{\sum_{j=1,j\neq i}^{l-1}\overline{\varphi}_j+(M-l+1)\varphi^*+\sigma^2}{\sum_{j=1}^{l-2}\overline{\varphi}_j+(M-l+1)\overline{\varphi}_{l-1}+\sigma^2}\right)\\
&> \frac{\overline{\varphi}_{l-1}}{h_i} \geq \frac{\overline{\varphi}_i}{h_i} = \overline{p}_i
\end{aligned}$$

because $\overline{\varphi}_{l-1} < \varphi^*$, and for $i = 1,2,\ldots,l-1$, $\overline{\varphi}_i \le \overline{\varphi}_{l-1}$, and $\sum_{j=1}^{l-2}\overline{\varphi}_j \le \sum_{j=1,j\neq i}^{l-1}\overline{\varphi}_j$. Therefore, (32) holds. Similarly, since $\widehat{\gamma} \le \Gamma_l$ for $q'(\Gamma_l) \le \alpha < q'(\Gamma_{l-1})$, we have

$$\begin{aligned}\frac{\widehat{\gamma}}{gh_i}&\left(\sum_{j=1}^{l-1}\overline{\varphi}_j + (M-l)\varphi^* + \sigma^2\right)\\ &= \frac{\overline{\varphi}_l}{h_i}\left(\frac{\sum_{j=1}^{l-1}\overline{\varphi}_j + (M-l)\varphi^* + \sigma^2}{\sum_{j=1}^{l-1}\overline{\varphi}_j + (M-l)\overline{\varphi}_l + \sigma^2}\right)\\ &\le \frac{\overline{\varphi}_i}{h_i}\end{aligned}$$

because $\varphi^* \le \overline{\varphi}_l$, and $\overline{\varphi}_l \le \overline{\varphi}_i$ for $i = l, l+1,\ldots,M$. Thus, for $i = l, l+1,\ldots,M$

$$\begin{aligned}\min&\left\{\overline{p}_i\frac{\widehat{\gamma}}{gh_i}\left(\sum_{j=1}^{l-1}\overline{\varphi}_j + (M-l)\varphi^* + \sigma^2\right)\right\}\\ &\qquad = \frac{\widehat{\gamma}}{gh_i}\left(\sum_{j=1}^{l-1}\overline{\varphi}_j + (M-l)\varphi^* + \sigma^2\right).\end{aligned}$$

It is easily observed from (31) that $\widehat{\gamma}\left(\sum_{j=1}^{l-1}\overline{\varphi}_j + (M-l)\varphi^* + \sigma^2\right)/g = \varphi^*$; therefore, (33) holds. If $q'(\Gamma_1) < \alpha \le q'(0)$, then $\widehat{\gamma} = q'^{-1}(\alpha) \le \Gamma_1 = g\overline{\varphi}_1/\left((M-1)\overline{\varphi}_1 + \sigma^2\right)$, and consequently $\widehat{\gamma} \le \Gamma_i$ for all i, then all users can achieve the same SIR $\widehat{\gamma}$ (which maximizes their utility) in such a way that the received power for all users are the same and is equal to or less than $\overline{\varphi}_1$. In this case, substituting the same SIR $\widehat{\gamma}$ for all users in (1) results in the feasible transmit power $p_i^* = \frac{\sigma^2\widehat{\gamma}}{h_i\left(g-(M-1)\widehat{\gamma}\right)}$ for all $i \in \mathcal{M}$, which satisfies (30). For $\alpha > q'(0)$, from (16) we know that $b_i(\mathbf{p}_{-i}) = 0$ for all $i \in \mathcal{M}$ whose fixed point is $p_i^* = 0$ for all $i \in \mathcal{M}$.

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