EEL 3705, Digital Logic Design, Fall 2006

Midterm Exam #1 - SOLUTIONS

About the exam: This exam tests the following Course Instructional Objectives (CIOs), from the Syllabus:

1. **[Numbers]** Convert numbers between different number systems (including binary, octal, and hexadecimal).
2. **[2s-comp]** Solve arithmetic equations in two's complement and interpret results regarding overflow conditions
3. **[SynAnal]** Derive digital circuits from optimized Boolean equations and compute the Boolean equations of a digital circuit.

There will be one question worth 35 points, covering each CIO. All 3 problems together are worth 105 points. 100 points is required for full credit on the exam. The exam is open book, open notes. You may use a calculator to check your answers, but for full credit, you must show your work, including all steps involved in hand-calculations.

Problem #1. (CIO #1, Numbers) Perform the following number conversions. Show your hand calculations in the space to the right. (Note: When writing binary, octal, and hexadecimal numbers, please place a comma after every 4th digit for readability.)

a) 971_{10} = $11,1100,1011_2$

971 – 512 = 459 → 1111001011_2
459 – 256 = 203
203 – 128 = 75
75 – 64 = 11
(no 32s or 16s)
11 – 8 = 3
(no 4s) 3 = 11_2

$011_2 \rightarrow 3_8$
$001_2 \rightarrow 1_8$
$111_2 \rightarrow 7_8$
$1_2 \rightarrow 1_8$

= 1713_8

= $3CB_{16}$

$1011_2 = B_{16}$
$1100_2 = C_{16}$
$11_2 = 3_{16}$

b) $3DF_{16}$ = $0011,1101,1111_2$

$3_{16} \rightarrow 0011_2$
$D_{16} \rightarrow 1101_2$
$F_{16} \rightarrow 1111_2$

$111_2 \rightarrow 7_8$
$011_2 \rightarrow 3_8$
$111_2 \rightarrow 7_8$
$001_2 \rightarrow 1_8$

= 1737_8

= 991_{10}

$3\times16^2 + 13\times16 + 15$
$= 768+208+15 = 991$

Problem #2. (CIO #2, 2s-comp) Consider the following two sequences of 8 bits:

$x =$ 10101011
$y =$ 01001000

a) If a two's complement signed number representation is being used, what number is represented by each of the above bit sequences? Give your answer in ordinary decimal with a ± sign. Show the hand-calculations used to determine the answer.

$x = -128 + 32 + 8 + 2 + 1 = -85$

$y = 64 + 8 = 72$

b) Compute the sum $x + y$ by hand in binary. Both inputs and the result should be interpreted in 8-bit, two's complement signed number representation. Check your result in decimal. Is there an overflow? Explain how you know.

```
    1
  10101011     = -85
+ 01001000     = +72
  11110011       -13
```

There is no overflow because the result is correctly represented as an 8-bit two's complement signed number. Also the carries into and out of the high bit are the same (0).

c) Compute the the difference $x - y$ by hand in binary. Both inputs and the result should be interpreted in 8-bit, two's complement signed number representation. Check your result in decimal. Is there an overflow? Explain how you know.

```
   0 1
  10101011      =   -85
- 01001000      = - +72
  01100011 = 99 ≠  -157   (01100011₂ = +99₁₀)
```

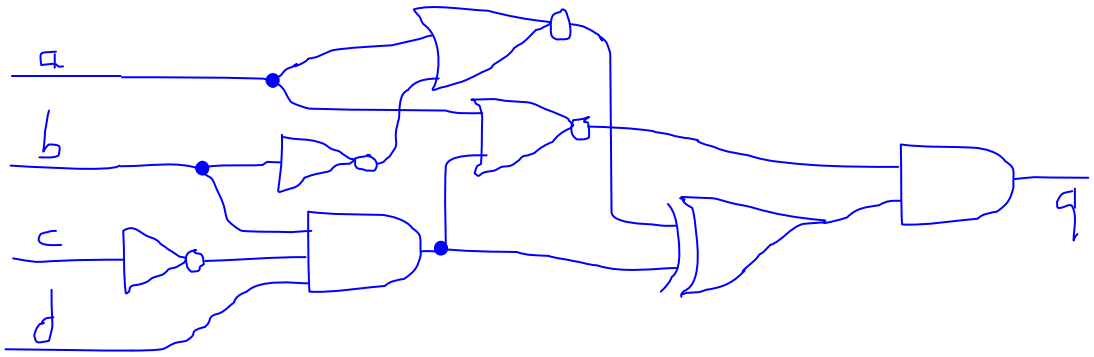
There is an overflow because the desired result, –157, is not representable as an 8-bit two's complement signed number (since it is less than –128). Also, the borrows in and out of position 7 are different.

Problem #3. (CIO #3, SynAnal)

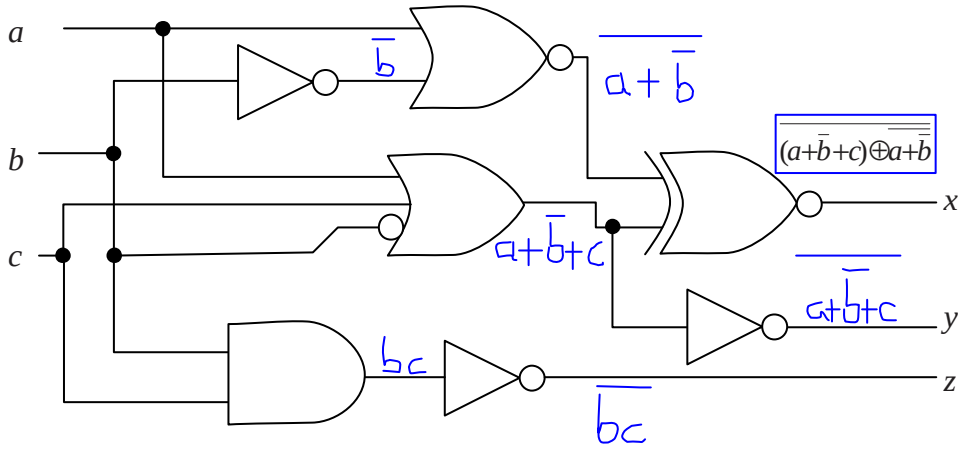
a) Using standard logic gate icons, draw a digital circuit that implements the following Boolean algebra equation:

$$q = \overline{(a + b\bar{c}d)}(b\bar{c}d \oplus \overline{(a + \bar{\bar{b}})})$$

Be sure that all inputs and outputs are properly labeled, and that all gates are drawn correctly. You do not need to simplify the expression or create an optimal implementation, but for full credit, please try not to redundantly recompute a given subexpression.



b) Write down Boolean algebra equations that describe the Boolean functions (what the book calls "switching functions") that are computed by the following circuit. You do not need to minimize or simplify your expressions. Instead, write down expressions that correspond directly to the structure of the circuit below.



$$x = \overline{(a + \bar{b} + c) \oplus \overline{a + \bar{b}}}$$
$$y = \overline{a + \bar{b} + c}$$
$$z = \overline{bc}$$