EEL 3003, INTRODUCTION TO ELECTRICAL ENGINEERING – SUMMER 2013

Lecture Notes – Lecture #2, Circuit Fundamentals

Lecture Outline:

Today's lecture had 3 major parts:

1) Review practice exercises from lec. #1. (Posted in assignments area.)
2) Quick overview of chapter 2.
3) Begin detailed coverage of Chapter 2.
   a) Definitions of basic circuit (network) terminology.
   b) Types of sources.
   c) Analysis with time-varying currents.
   d) Kirchoff's Current Law (review) and Kirchoff's Voltage Law.

Parts 2 & 3a-3b are covered in the PowerPoint slides posted on Blackboard, although I supplemented the slides with various ad-hoc examples sketched quickly on the whiteboard.

Then we had an in-class assignment to analyze the circuit in fig. 2.8 of the textbook to determine the number of terminals, junctions (not counting terminals), nodes, meshes, and loops. (Next semester, it would probably be a good idea to count the branches as well.)

Part 3c: Analysis with time-varying currents.

We did an example on the whiteboard in class of analyzing total charge transfer in the case of a time-varying current. The problem was: If a wire is carrying a time-varying current $i(t) = (3\text{A/s})t$, how much charge is transferred between times $t = 0$ and $t = 2$ sec.?

Set up as an integral and solve; here I'm writing out all the steps in gory detail, including all units:

$$Q(t) = \int_{t=0}^{2\text{ s}} i(t)\,\mathrm{d}t = \int_0^{2\text{ s}} \frac{3\text{ A}}{\text{s}} t\,\mathrm{d}t = \frac{3\text{ A}}{\text{s}} \int_0^{2\text{ s}} t\,\mathrm{d}t = \frac{3\text{ A}}{\text{s}} \left(\tfrac{1}{2} t^2 \Big|_0^{2\text{ s}} \right) = \frac{3\text{ A}}{\text{s}} \left(\frac{1}{2}(2\text{ s})^2 - \frac{1}{2}0^2 \right) =$$
$$= \frac{3\text{ A}}{\text{s}} \left(\frac{1}{2}(2\text{ s})^2 \right) = \frac{3\text{ A}}{\text{s}} \left(\frac{1}{2}(4\text{ s}^2) \right) = \frac{3\text{ A}}{\text{s}} (2\text{ s}^2) = 6\text{ A}\cdot\text{s} = 6\text{ C}. \tag{1}$$

You could also do it more simply, dropping the units until the very end:

$$Q(t) = \int_0^2 3t\,\mathrm{d}t = \tfrac{3t^2}{2}\Big|_0^2 = \frac{3\cdot 4}{2} = 6\text{ C}. \tag{2}$$

We didn't get to this next part in lecture, but you can similarly analyze power and energy in the time-varying case. Just as $P=IV$ for the time-independent case, instantaneous power is of course $p(t) = i(t)\cdot v(t)$. Thus, total energy transferred between times $t = t_0$ and $t = t'$ is:

$$E(t_0, t') = \int_{t=t_0}^{t\prime} p(t)\,\mathrm{d}t = \int_{t=t_0}^{t\prime} i(t)\cdot v(t)\,\mathrm{d}t, \tag{3}$$

so, you can see right away how to solve problems where you are given $i(t)$ and $v(t)$ and asked to find the total energy $E(t_0, t')$ transferred between two times t_0, t'. I give a problem like this in the practice exercises from Lecture 2, which are posted in the Assignments area & with the lecture slides.

Part 3d: Kirchhoff's Current Law & Kirchhoff's Voltage Law.

§2.2 – Kirchhoff's Current Law.

We went over the review problem at the start of lecture, and there is another, larger problem in the practice exercises for lecture 2.

§2.3 – Kirchhoff's Voltage Law.

General definition of voltage. I forgot to mention this is lecture, but in the case of time-varying voltages, there is a more general, differential definition of voltage, which is important in the case of source terminals that have a voltage that varies with the amount of charge transferred. Many real power sources, such as batteries and capacitors, indeed have a voltage that varies as they are discharged, so this case is important.

Recall that for a constant voltage V, we had define it as $V = E/Q$, electrostatic potential energy transferred per unit of charge transferred. This is fine if the voltage either does not change at all, or changes by only a negligible amount as the quantity Q of charge is transferred. But, given a real, finite energy source, generally the electrostatic potential a.k.a. voltage $v(q)$ of a source's output terminal will eventually fall as more and more charge is delivered. In such a case, the equation $V = E/Q$ can still be fine for expressing the average voltage at which the charge was delivered, but it no longer represents the instantaneous voltage of the terminal at all times. In such a case, an accurate definition of the voltage $v(q)$ of a wire after a given quantity q of charge has been transferred through it is:

$$v(q) = \frac{\mathrm{d}E(q)}{\mathrm{d}q}, \tag{4}$$

where $E(q)$ is the cumulative energy transferred through the wire after a cumulative quantity q of charge has been transferred. If it is desired to express the voltage as a function of time, instead of a function of charge, then one can write

$$v(t) = v\big(q(t)\big) = \frac{\mathrm{d}E(q(t))}{\mathrm{d}q(t)} = \frac{\mathrm{d}E(q(t))}{i(t)\,\mathrm{d}t} = \frac{1}{i(t)}\frac{\mathrm{d}}{\mathrm{d}t}E\left(\int_{\tau=0}^{t} i(t)\,\mathrm{d}t\right), \tag{5}$$

which is then phrased just in terms of the time-varying current $i(t)$ and the energy function $E(\cdot)$.

Note, now, that in the case of a true constant-voltage supply, where the voltage is independent of the amount of charge transferred, that is if $v(q)$ = const. = V, then by substituting into (4) and rearranging, we have that $\mathrm{d}E = V\,\mathrm{d}q$, thus (integrating both sides starting from 0) $E(q) = Vq$, and so we have that $(\mathrm{d}/\mathrm{d}t)E(q(t)) = V\,\mathrm{d}q/\mathrm{d}t = V\cdot i(t)$, and so eq. (5) reduces to $v(t) = V$ as expected, a nice check. This just confirms that the easy equation $V = E/Q$ is consistent with the more general definition $v(q) = \mathrm{d}E(q)/\mathrm{d}q$ in the case where $v(q)$ is constant.

Labeling branch voltages. Anyway, now that we understand a voltage a little better, let's get to the real topic of this section, KVL (Kirchoff's Voltage Law). As a starting point for KVL, you have to label the branch voltages around a loop. It's important to realize that a *branch* voltage v always just represents a voltage *difference* between the nodes at the two ends of the branch, *i.e.*,

$$v = v_+ - v_- \tag{6}$$

where v_+ denotes the voltage of the node at the "+" terminal of the branch, and v_- denotes the voltage of the "−" terminal of the branch. It is important to note that the "+" and "−" *are only labels* to help us remember the order of the terms on the right side of eq. (6). In general, they have nothing to do with which terminal of the branch actually has the higher voltage. It's possible that the "−" terminal has a higher voltage than the "+" terminal, in which case the value v of the branch voltage comes out negative.

Therefore, really we are free to label the terminals of each branch in whatever order we like. In particular, we can always label the terminals of all the branches making up any loop so that, as we go clockwise (say) around the loop, we enter each branch through its "−" terminal and exit through its "+" terminal. Or, we could label them all in the opposite order. For example, consider the following loop:

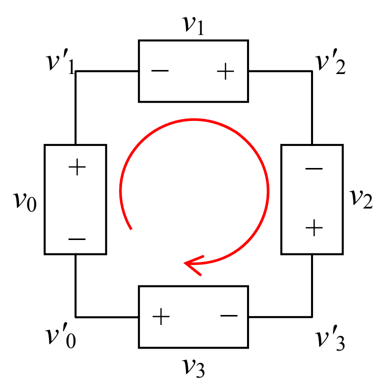


Figure 1. A loop with all branches labeled −/+ clockwise.

Note that the terminals of all the branches have been labeled +/− in such a way that when circumnavigating the loop in clockwise order, we always enter each branch through its "−" terminal and leave it through its "+" terminal. Note also that we have labeled the branch voltages v_k (k = 0 to 3) and node voltages v'_ℓ (ℓ = 0 to 3) in such a way that, for each branch k (with k = 0, 1, 2, 3) we have:

$$v_k = v_k^+ - v_k^- = v'_{(k+1)\bmod N} - v'_k \tag{7}$$

where N = 4, the number of branches in the loop. If you're not familiar with the mod function, all you have to know, for the present purpose, is that $N \bmod N = 0$, and for all ℓ in the range 0 to N−1, the value of ℓ mod $N = \ell$. (In abstract algebra terms, our node indices form a cyclic group—basically, they wrap around.)

Now, the statement of Kirchhoff's voltage law, with this kind of labeling, is simply:

$$\sum_{k=0}^{N-1} v_k = 0. \tag{8}$$

That this is true can be proven easily by just expanding out the definitions of v_k and noting that all of the terms necessarily cancel:

$$\begin{aligned}\sum_{k=0}^{N-1} v_k &= \sum_{k=0}^{N-1} (v'_{(k+1) \bmod N} - v'_k) \\ &= (v'_1 - v'_0) + (v'_2 - v'_1) + (v'_3 - v'_2) + (v'_0 - v'_3) \\ &= 0.\end{aligned}$$

Of course, this must always be true with our labeling, since the term for each node voltage appears once in the sum with a "+" sign, from the preceding branch's "+" terminal, and once with a "–" sign, from the following branch's "–" terminal.

If you label some of the branches in the opposite order, you can still use KVL, you just have to flip the sign of those terms in the sum.

There is a practice exercise on applying KVL in the list of practice exercises I posted for lecture 2.

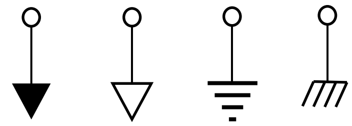
Ground terminals. One more concept, which I didn't get to in lecture: By convention, we always choose one node in any given circuit and label it "ground," and declare that its voltage is always 0, as a matter of definition, and use it as a reference voltage that all of the other node voltages in the circuit are defined relative to. We can always do this because voltages are always relative anyway, which is true because electrostatic potential energy (which voltage is based on) is relative. As I mentioned in lecture #1, there can be no absolute electrostatic potential, because (as a thought experiment) if you imagine a hollow sphere of positive charge surrounding you at a vast distance, its presence will uniformly raise the electrostatic potential of every point inside it, but without affecting the physics of any experiment that you can perform inside it. Absolute electrostatic potential therefore cannot be measured, and therefore, it is physically meaningless. Like other forms of potential energy, electrostatic potential energies (and thus voltages) are only defined up to an arbitrary additive constant.[1]

There's also a nice analogy between electrical "ground" and the real (planetary surface) ground. Just as you can stand on top of a hill of any altitude, and declare the ground there to be a region of 0 gravitational potential energy, for purposes of a local physics experiment, similarly you can choose any node of a circuit and declare it to be your "ground" node for measuring electric potentials. It is just a name; it does not affect the physics of the circuit. If you like, you *could* tie the nominal "ground" node

[1] Well, the absolute energy of a system (including its potential energy) can, perhaps, be measured, at least in principle, in the context of general relativity, since it affects gravity, but that is beyond the scope of this discussion. Anyway, even in GR, the total energy of a system, including its internal self-gravitational potential energy, only becomes clearly well-defined in the limiting case of observing a system from an infinite distance away, in an otherwise-flat space – which is an idealization that may not be achievable in this universe.

of a circuit to the real (Earth) ground, but you do not have to. Ultimately, Earth's own voltage level is itself really only relative and free-floating as well, it is not guaranteed to always be exactly constant relative to (say) other objects in the solar system, or the galaxy, or the intergalactic medium.

Some popular symbols for the ground terminal include the following:



This terminal can be attached anywhere in a circuit, or to multiple places in the circuit. If multiple ground symbols are attached at different places in a given circuit diagram, it means that all of those places are part of the same node (the ground node). If a ground symbol is attached in only a single place in the circuit, then it stands an exception to the general rule that any useful subcircuit must have more than one terminal (since its "use" is purely conventional, anyway, to designate a voltage reference; again, it does not really affect the physics of the circuit).

One thing that *is* important, though, is that, if you physically connect multiple real subcircuits together to form a larger circuit, you should make sure the various subcircuits' designated ground nodes are, in fact, *actually* physically tied together directly to each other (forming a "common ground"), since otherwise you are liable to become very confused regarding the relative voltage levels between different parts of the combined circuit, since the "ground" nodes in the various subcircuits are not likely to actually be (or stay) at the same voltage level if they are not physically tied together. This can easily result in all kinds of errors in the circuit's operation.

That's all for now…

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