FAMU-FSU
College of Engineering

Lecture #6:
Resistive Network Analysis (Part III)

EEL 3003
Introduction to Electrical Engineering
Summer Semseter, 2013

Instructor: Dr. Michael Frank

Administrative Announcements

- Outline of Today's Lecture:
  1. Continue Chapter 3, Resistive Network Analysis
     - §3.4 – Node & mesh analysis w. Controlled sources
     - §3.5 – Principle of Superposition
     - §3.6 – Norton/Thevenin Equivalent Circuits
     - §3.7 – Maximum Power Transfer Theorem
     - §3.8 – Nonlinear Circuit Elements ← light coverage
- Reminder: Current Homework Assignment:
  - Read Ch. 3 of Textbook (Rizzoni 5th ed.)
  - Practice exercises:
    - 3.6, 3.10, 3.17, 3.43, 3.60, 3.72, 3.74*, 3.75, ~~3.76, 3.81~~ ← last 2 optional
  - Quiz Tuesday (June 4th).

Continue Chapter 3, Resistive Network Analysis

- ~~§3.1 – Network Analysis~~
- ~~§3.2 – The Node Voltage Method~~
- §3.3 – The Mesh Current Method
  - See supplemental notes I posted after last lecture
- §3.4 – Node & mesh analysis w. Controlled sources
- §3.5 – Principle of Superposition
- §3.6 – Norton/Thévenin Equivalent Circuits
- §3.7 – Maximum Power Transfer Theorem
- §3.8 – Nonlinear Circuit Elements – covering lightly

FAMU-FSU College of Engineering

§3.4 – Node & Mesh Analysis with Controlled Sources

- Each controlled source gives us a *constraint equation* which we plug into the analysis.
  - Gives controlled value of voltage or current in terms of controlling voltage or current.
    - Does not increase number of unknowns, since controlled value is completely determined by controlling value.
- Let's quickly go through the example in the textbook…

Example of Node/Mesh Analysis with Controlled Sources

- Consider this circuit (from fig. 3.24, p. 101):

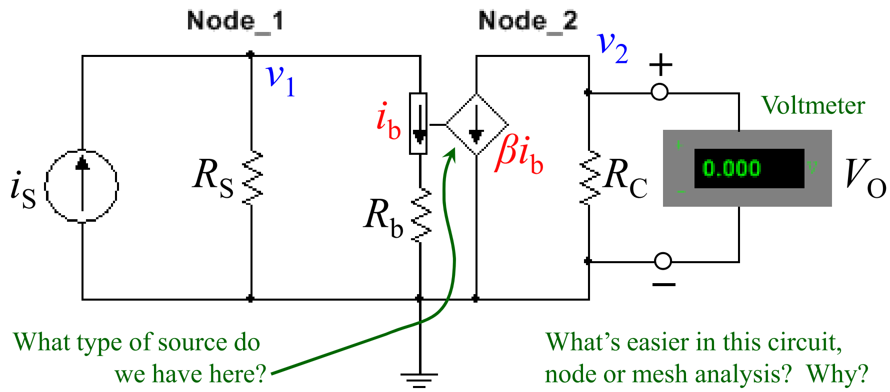


Fig. 3.24 Example, continued

- KCL equation for Node 1:

$$i_S = v_1\left(\frac{1}{R_S} + \frac{1}{R_b}\right) \qquad \text{(eq. 1)}$$

- KCL equation for Node 2:

$$\beta i_b + \frac{v_2}{R_C} = 0 \qquad \text{(eq. 2)}$$

Fig. 3.24 Example, continued

- Next, use current-divider rule to express the controlling current i_b in terms of i_S:

$$i_b = i_S \frac{1/R_b}{1/R_b + 1/R_S} = i_S \frac{R_S}{R_b + R_S} \qquad \text{(eq. 3)}$$

- So, the controlled current is:

$$\beta i_b = \beta i_S \frac{R_S}{R_b + R_S} \qquad \text{(eq. 4)}$$

Constraint equation

FAMU-FSU College of Engineering

Fig. 3.24 example, cont.

- Plug constraint equation back into the KCL equation that involved the controlled current source (eq. 2):

$$\beta i_S \frac{R_S}{R_b + R_S} + \frac{v_2}{R_C} = 0 \qquad \text{(eq. 5)}$$

- Thus, v_2 can be solved in terms of i_S:

$$v_2 = -\beta i_S \frac{R_S R_C}{R_b + R_S} \qquad \text{(eq. 6)}$$

Fig. 3.24 example, cont.

- Here both v_1 & v_2 are solved in terms of i_S:

$$v_1 = \frac{i_S}{1/R_S + 1/R_b} \qquad \text{(eq. 7)}$$

$$v_2 = -\beta i_S \frac{R_S R_C}{R_b + R_S} \qquad \text{(eq. 6 again)}$$

- So, if the independent source current i_S, the coefficient β, & the R's are given, we're done.

§3.5 – The Principle of Superposition

- In a linear circuit,
  - which means, one containing only linear devices,
    - which are: resistors, capacitors, & inductors,
- the effects of multiple sources are additive,
  - which means, the solution with multiple sources is just the sum of solutions found with individual sources…
    - you just have to simplify the multi-source circuits to single-source circuits in the right way.

FAMU-FSU College of Engineering

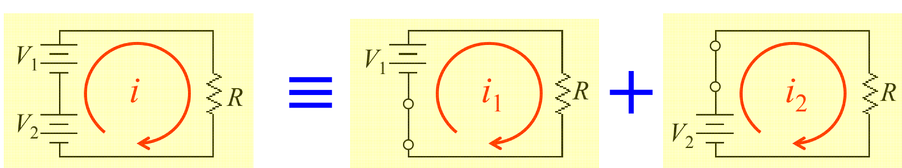
Rules for "Zeroing Out" Sources

- When simplifying a multiple-source circuit to a single-source circuit,
  - for purposes of later applying the Superposition Principle to combine your solutions,
- Set all of the *other* sources "equal to 0" in the following way:
  - Zero voltage source = closed (short) circuit.
  - Zero current source = open (disconnected) circuit.

FAMU-FSU College of Engineering

Simple Example of the Superposition Principle (from fig. 3.27, p. 106)

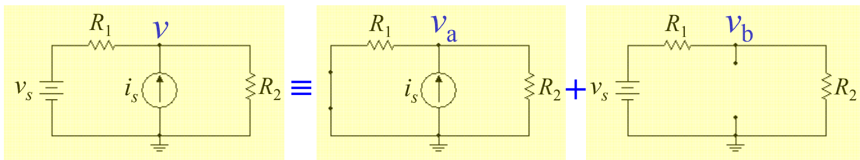
- Voltage sources in series, with a single-resistor load.



$$i = \frac{V_1 + V_2}{R} = \frac{V_1}{R} + \frac{V_2}{R} = i_1 + i_2$$

Slightly More Complicated Superposition Example (Fig. 3.28)

- Note current source gets replaced by an open circuit.

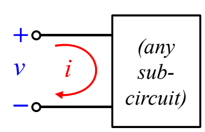


Currents on each branch of the circuit add (like in previous slide), therefore, because of Ohm's law, voltages on each node add as well, so:

$$v = v_{\mathrm{a}} + v_{\mathrm{b}}$$

§3.6 – One-Port Networks & Equivalent Circuits

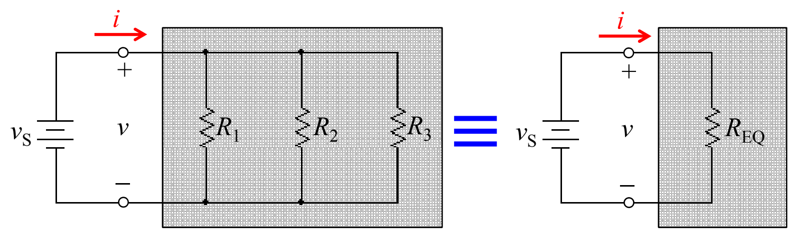
- Recall that "one port" means "two terminals."
  - Thus, any subcircuit with exactly 2 terminals comprises a one-port network.



- In a 1-port network w/o any reactance,
  - which means, no capacitors or inductors,
    - only components like resistors/diodes,
- the electrical properties of that network are completely described by its *i*/*v* characteristic,
  - and the network can be described using a simplified *equivalent circuit* model.

Equivalent Circuits Example

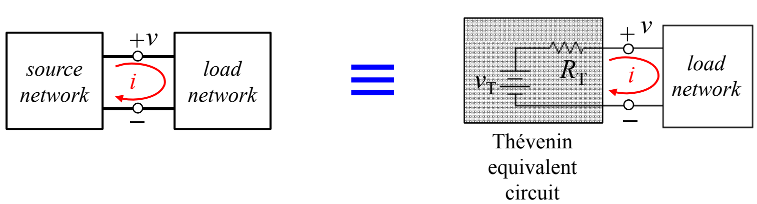
- Gray box on left is a 2-terminal subcircuit (one-port network)
  - It is equivalent to the simplified circuit on the right.



$$R_{EQ} = 1/(1/R_1 + 1/R_2 + 1/R_3)$$

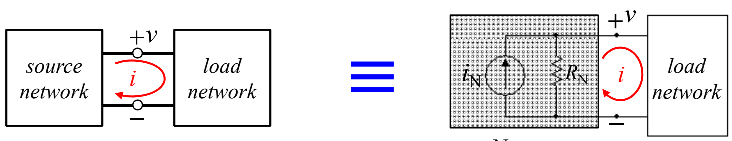
Thévinin's Theorem

- Any 1-port network of ideal voltage & current sources & resistors is equivalent to a *single* ideal voltage source v_T in series with a *single* resistor R_T.
  - v_T is called the *Thévinin equivalent voltage* of the circuit.
  - Similarly, R_T is called the *Thévinin equivalent resistance*.



Norton's Theorem

- Any 1-port network of ideal voltage & current sources & resistors is equivalent to a single ideal *current* source i_N in *parallel* with a single resistor R_N.
  - i_N is called the *Norton equivalent current* of the circuit.
  - Similarly, R_N is called the *Norton equivalent resistance*.



Norton equivalent circuit

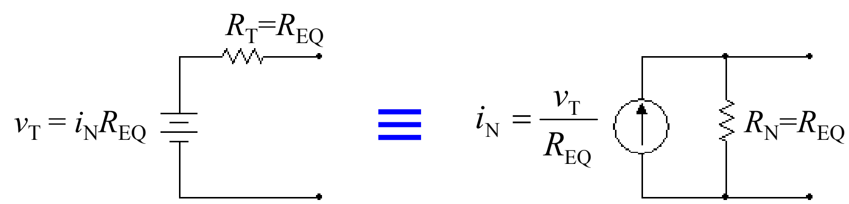
Finding Thévinin/Norton Equivalent Circuits

This is rather easy!!!

- **Step 1:**
  - Set all sources equal to 0 and simplify to find the Thév./Norton equivalent resistance $R_T = R_N = R_{EQ}$.
- **Step 2:**
  - Thévenin voltage v_T = open-circuit voltage at output port (with load removed).
- **Step 3:**
  - Norton current i_N = short-circuit current at output port (shorting over the load)

Equivalence of Thévenin & Norton Equivalent Circuits

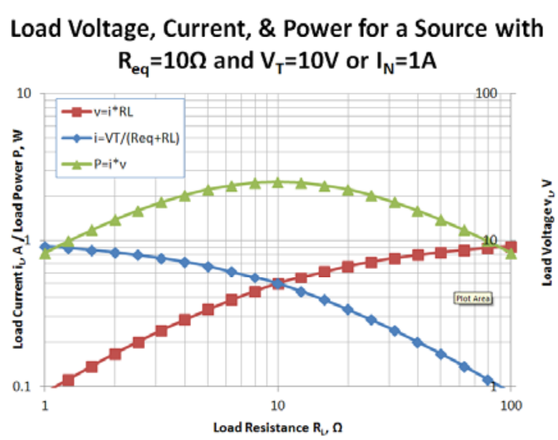
- Since any one-port network has a Thévenin circuit & a Norton circuit that are both ≡ to the original network,
  - Obviously, therefore, the Thévenin circuit ≡ the Norton circuit.



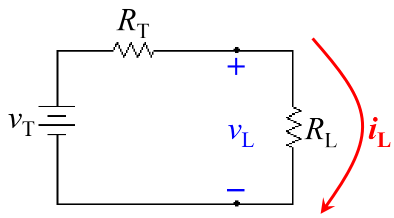
- Thus`, you can always transform a subcircuit that has only a voltage source & series resistor into one that has only a current source & parallel resistor, and vice-versa.

Response of a resistive load to a linear source

- When $R_L >> R_{EQ}$,
  $v_L \rightarrow v_T$;
- When $R_L << R_{EQ}$,
  $i_L \rightarrow i_N$;
- When $R_L = R_{EQ}$,
  $v_L = v_T/2$ &
  $i_L = i_N/2$.
  - This is the maximum power point, $p = p_{max} = v_T i_N/4$.
- See also supplemental note posted.



§3.7 – Maximum Power Transfer Theorem



$$p_L = i_L^2 R_L \qquad i_L = \frac{v_T}{R_T + R_L}$$

$$p_L = \frac{v_T^2 R_L}{(R_T + R_L)^2}$$

Let $\frac{dp_L}{dR_L} = 0,$ (Steps on next slide) → $R_L = R_T.$

Full Derivation of MPTT Result

$$\frac{d}{dR_L}\frac{v_T^2 R_L}{(R_T + R_L)^2} = 0 \qquad \frac{d}{dR_L}\frac{R_L}{(R_T + R_L)^2} = 0$$

$$\frac{d}{dR_L} R_L (R_T + R_L)^{-2} = R_L \frac{d}{dR_L}(R_T + R_L)^{-2} + (R_T + R_L)^{-2}\frac{d}{dR_L} R_L$$

$$= R_L(-2)(R_T + R_L)^{-3} + (R_T + R_L)^{-2} = \frac{1 - 2R_L/(R_T + R_L)}{(R_T + R_L)^2} = 0$$

$$1 - 2R_L/(R_T + R_L) = 0 \qquad 2R_L = R_T + R_L \qquad R_L = R_T$$

§3.8 – Nonlinear Circuit Elements

1. Description of Nonlinear Elements
   - Exponential *i*-*v* curve example (not yet covered)
2. Graphical (Load-Line) Analysis of Nonlinear Circuits
   - Discussed graphical method in class
     - (next slides)
   - Did not yet cover analytical solution examples
     - Homework problems optional
     - No quiz question on this topic this semester

Equations to Calculate Source Loading

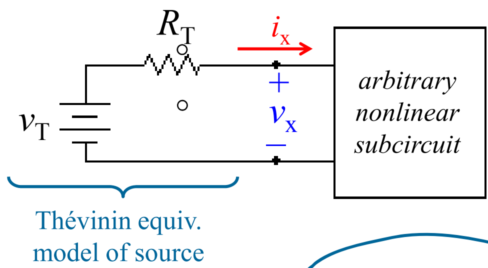
- Given any linear source (v_T, R_{EQ}) or (i_N, R_{EQ}),
  - To calculate the voltage v_L on the load given the current i_L through the load, or vice-versa:

$$v_L = v_T - i_L R_{EQ} = (i_N - i_L) R_{EQ}$$

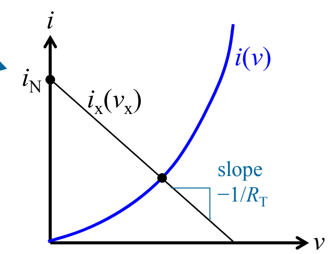
$$i_L = i_N - v_L / R_{EQ} = (v_T - v_L) / R_{EQ}$$

  - This can be helpful for taking measurements of an unknown load using a known source.

Load-Line Equation



Useful to solve a circuit with an arbitrary nonlinear element (*e.g.*, diode) when you only know its i-v curve.



$$i_x = \frac{v_T - v_x}{R_T} = \left(\frac{v_T}{R_T}\right) - \left(\frac{1}{R_T}\right) v_x$$