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Lecture #6:
AC Network Analysis

EEL 3003
Introduction to Electrical Engineering
Summer Semseter, 2013
Instructor: Dr. Michael Frank

Administrative Announcements

- Outline of Today's Class Session:
  1. Quiz #2 on Chapter 3, Resistive Network Analysis
  2. Start Chapter 4, AC Network Analysis
     - §4.1 – Energy Storage (Dynamic) Circuit Elements
     - §4.2 – Time-dependent Signal Sources
     - §4.3 – Solution of Circuits Containing Energy Storage Elements (Dynamic Circuits)
     - §4.4 – Phasor Solution of Circuits with Sinusoidal Excitation
     - §4.5 – AC Circuit Analysis Methods
- Announcing Homework Assignment #3:
  - Read Ch. 4 of Textbook (Rizzoni 5th ed.)
  - Practice by doing at least these 8 textbook exercises:
    - 4.1*, 4.12*, 4.31, 4.59*, 4.66, 4.68, 4.71, 4.77*
  - Quiz on this material next Tuesday (June 11th).

Part I: Quiz #2 on Ch. 3 Resistive Network Analysis

- ½ hour for quiz
  - Please remain seated until time is up and I have collected all papers.
- Usual rules:
  - Calculator only

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Part II. Lecture: Start Ch. 4, AC Network Analysis

- §4.1 (pp. 150-167)
  - Energy Storage (Dynamic) Circuit Elements
- §4.2 (pp. 167-173)
  - Time-dependent Signal Sources
- §4.3 (pp. 173-175)
  - Solution of Circuits Containing Energy Storage Elements (Dynamic Circuits)
- §4.4 (pp. 175-191)
  - Phasor Solution of Circuits with Sinusoidal Excitation
- §4.5 (pp. 191-214)
  - AC Circuit Analysis Methods

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§4.1 - Energy Storage (Dynamic) Circuit Elements

- We'll study the following (ideal, linear, dynamic) elements:
  - Capacitors
  - Inductors

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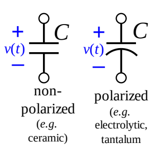
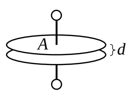
Ideal Capacitors

- A capacitor stores charge,
  - and resists a *change in voltage*.
- For charge Q stored on a capacitor with capacitance C at voltage V: $Q = CV$.
  - Differential form: $C = dq/dv$.
- A capacitor is equivalent to an open circuit for a DC current.
  - Voltage can't build up indefinitely.
    - If I into a capacitor is *constant*, $I = 0$.
- Unit of capacitance: the *farad* (F)
  - 1 F = 1 C/V



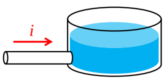


$C = \varepsilon A/d$
(ignoring fringe)

1 Farad →

Water Tank Analogy

- A rough analogue for a capacitor in a fluidic system is a storage tank for liquid
  - Fed by pipes at bottom of tank
  - Capacitance corresponds to area of tank
    - Constant capacitance = vertical tank walls
  - Gravity → height of water in tank is proportional to pressure at bottom of tank
    - Corresponds to voltage



Capacitance ~ area of tank
Current ~ flow into tank
Voltage ~ height of water
~ pressure at bottom
Charge ~ volume stored

Relationship between current and voltage in a capacitor

- Starting from differential form of capacitor equation: $C = \frac{\mathrm{d}q}{\mathrm{d}v}$
- Derive:

$$C\,\mathrm{d}v = \mathrm{d}q$$

$$C\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}q}{\mathrm{d}t}$$

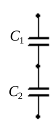
$$i(t) = C\frac{\mathrm{d}v}{\mathrm{d}t} = C\dot{v}$$

Integral form:

$$v(t) = v(0) + \frac{1}{C}\int_{t'=0}^{t} i(t')\,\mathrm{d}t'$$

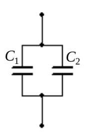
Equivalent networks of capacitors

- Series capacitors combine like parallel resistors,
  - And vice-versa.
- Because capacitance is current over (rate of change of) voltage
  - Whereas resistance is voltage over current
- The reciprocal of capacitance is called *elastance* (unit: "daraf")



$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

(voltages add, currents equal)



$$C = C_1 + C_2$$

(currents add, voltages equal)

$$\frac{1}{C} = \frac{\mathrm{d}v/\mathrm{d}t}{i(t)} \qquad C = \frac{i(t)}{\mathrm{d}v/\mathrm{d}t}$$

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Energy Storage in Capacitors

- Energy stored when charging up a constant capacitor *C* from 0 V to voltage *V*:

$$E = \int_{t=0}^{T} p(t)\,\mathrm{d}t = \int_{t=0}^{T} i(t)\cdot v(t)\,\mathrm{d}t$$

$$= \int_{t=0}^{T} C\frac{\mathrm{d}v}{\mathrm{d}t}\cdot v(t)\,\mathrm{d}t = C\int_{v=0}^{V} v\,dv$$

$$= C\left(\frac{1}{2}v^2\right)\Bigg|_{v=0}^{V} = C\left(\frac{1}{2}V^2\right) = \frac{1}{2}CV^2.$$

Note: Time is irrelevant!

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Capacitive Reactance

- This will be important later…
- Under an AC excitation with frequency *f*, a capacitor has a *reactance* of:

$$X_C = \frac{-1}{2\pi\, f\, C}\Omega$$

- The −1 in numerator → a capacitor's voltage *lags* its current by a phase angle of π/2.
  - We'll discuss what this means later on…

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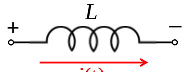
Ideal Inductors



- An inductor stores magnetic flux,
  - And resists a change in current
  - Equiv. to short-circuit for DC current
    - *v* = const. → *v*=0.
- Unit: 1 Henry (H)
  - 1 H = 1 Ω·s

$$v(t) = L\frac{\mathrm{d}i(t)}{\mathrm{d}t}$$

$$i(t) = i(0) + \frac{1}{L}\int_{t=0}^{t'} v(t')\,\mathrm{d}t'$$

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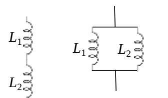
Hydraulic Analogy

- Inductance is analogous to *inertance* in fluidic systems
  - *E.g.* a low-viscosity liquid moving in a long coiled tube carries a lot of inertia (momentum)
    - Takes pressure applied over time to gradually accelerate the mass of liquid to higher flow rates
- Similarly, current moving in a coiled wire has an electrical analogue to momentum
  - Caused by the magnetic fields that form in the coil
    - It takes a voltage (electromotive force) applied over time to gradually "accelerate" the current to higher intensity

Equivalent networks of inductors

- Series inductors combine like series resistors,
  - And parallel inductors like parallel resistors.
- Because inductance is voltage over (rate of change of) current
  - Resistance is voltage over current
- The reciprocal of inductance is called *reluctance* (unit: S/s)



$L = L_1 + L_2$ (voltages add, currents equal)

$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$ (currents add, voltages equal)

$$L = \frac{v(t)}{\mathrm{d}i/\mathrm{d}t} \qquad \frac{1}{L} = \frac{\mathrm{d}i/\mathrm{d}t}{v(t)}$$

Energy Stored in an Inductor

- Energy stored when "spinning up" a constant inductor L from $i(0) = 0$ A to current $i(T) = I$:

$$E = \int_{t=0}^{T} p(t)\,\mathrm{d}t = \int_{t=0}^{T} i(t)\cdot v(t)\,\mathrm{d}t$$
$$= \int_{t=0}^{T} i(t)\cdot L\frac{\mathrm{d}i}{\mathrm{d}t}\,\mathrm{d}t = L\int_{i=0}^{I} i\,di$$
$$= L\left(\frac{1}{2}i^2\right)\Bigg|_{i=0}^{I} = L\left(\frac{1}{2}I^2\right) = \frac{1}{2}LI^2.$$

Inductive Reactance

- This will be important later…
- Under an AC excitation with frequency *f*, an inductor has a reactance of:

$$X_L = 2\pi f\ L\ \Omega$$

- The + sign means → an inductor's voltage *leads* its current by a phase angle of π/2.
  - We'll discuss what this means later on…