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Lecture #11:
Transient Analysis (Part II)

EEL 3003
Introduction to Electrical Engineering
Summer Semseter, 2013

Instructor: Dr. Michael Frank

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Administrative Announcements

- Handing back Quiz #1&2 papers…
  - Plan to hand back Quiz #3 on Thurs.
- Announcing **MIDTERM EXAM**…
  - In-class next Tuesday, June 25th
  - Covering chapters 2-4
- Outline of today's class session:
  1. Review Quiz #3 solutions
  2. Continue Chapter 5, Transient Analysis
     - Cont. §5.2 – Writing Differential Equations for Circuits Containing Inductors and Capacitors
     - §5.3 – DC Steady-State Solution of Circuits Containing Inductors and Capacitors – Initial and Final Conditions
- Announcing Homework Assignment #4:
  - Read Ch. 5 & practice w. these exercises:
    - 5.7, 5.24, 5.35*, 5.39*, 5.74, 5.76, 5.79, 5.80
  - Quiz date no sooner than Thu., June 27th

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1. Review Quiz #3 solution

- Grade histogram so far (17/68 papers graded):
  - A – XX
  - B – XXXX
  - C – X
  - D – XX
  - F – XXXXXXXXX
- The plan is:
  - Hand back all quiz papers by this Thurs. so you can study what you missed for the exam.

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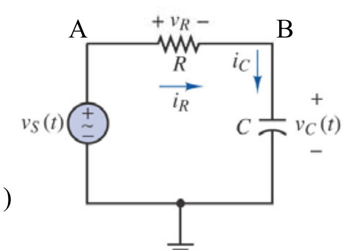
§5.2 – Writing Differential Equations for Circuits Containing Inductors and Capacitors

- On board last time, we worked out this ODE for this circuit using KVL:

$$\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{1}{RC}i - \frac{1}{R}\frac{\mathrm{d}v_\mathrm{S}}{\mathrm{d}t} = 0 \qquad (1)$$



- In-class assignment:
  - Start with KCL at node B, and derive a different ODE for this circuit that fits this general form:

$$(a)\frac{\mathrm{d}v_\mathrm{C}}{\mathrm{d}t} + (b)v_\mathrm{C} + (c)v_\mathrm{S} = 0 \qquad (a, b, c \text{ constants}) \qquad (2)$$

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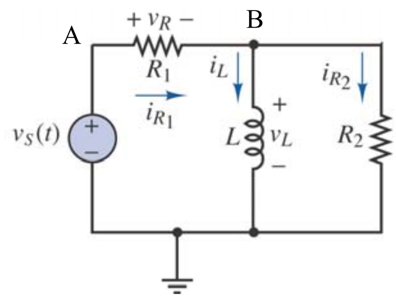
Another Example (Ex. 5.1, pp. 218-219)

- Derive diff. eq. for $i_L(t)$, given these values:
  - $R_1 = 10\ \Omega$
  - $R_2 = 5\ \Omega$
  - $L = 0.4$ H



- Start w. KCL @ node B:

$$i_{R_1} - i_L - i_{R_2} = 0 \qquad (3)$$

- Expand terms: $\frac{v_S - v_L}{R_1} - i_L - \frac{v_L}{R_2} = 0$ (4)

(cont.)

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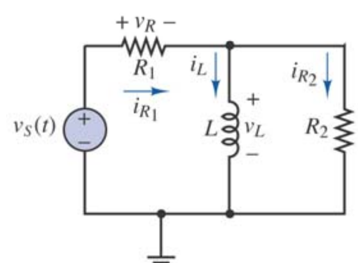
Example 5.1, *cont.*

- Use diff. eq. of inductor voltage to elim. v_L fr. eq.:

$$\frac{v_S}{R_1} - \frac{L}{R_1}\frac{\mathrm{d}i_L}{\mathrm{d}t} - i_L - \frac{L}{R_2}\frac{\mathrm{d}i_L}{\mathrm{d}t} = 0 \qquad (5)$$



- Then grouping:

$$\left(\frac{L}{R_1} + \frac{L}{R_2}\right)\frac{\mathrm{d}i_L}{\mathrm{d}t} + i_L = \frac{1}{R_1} v_S \qquad (6)$$

plug in values

$$(0.12\,\mathrm{s})\frac{\mathrm{d}i_L}{\mathrm{d}t} + i_L = (0.1\mathrm{S})v_S \qquad (7)$$

General Form for ODEs for First-Order Circuits

- As illustrated by the last couple of examples,
  - Whenever we have a circuit with only one reactive element, it will have an ODE of this general form:

$$a_1 \cdot \frac{\mathrm{d}x(t)}{\mathrm{d}t} + a_0 \cdot x(t) = b_0 \cdot f(t) \tag{8}$$

  - where:
    - $x(t)$ – A time-varying current or voltage in the circuit.
    - $f(t)$ – Some other time-varying quantity in the circuit.
    - a_1, a_2, b_0 – Constants.
- In-class exercise:
  - Find x, f, a_1, a_2, b_0 for both of the last two examples.

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Properties of the General Form

- Dividing last eq. (8) through by a_0 gives:

$$\frac{a_1}{a_0}\frac{\mathrm{d}x(t)}{\mathrm{d}t} + x(t) = \frac{b_0}{a_0} f(t) \tag{9}$$

- Let us define quantities:
  - $\tau = a_1 / a_0$ – The "time constant"
  - $K_S = b_0 / a_0$ – The "DC gain"
- Then eq. (9) becomes:

$$\tau \frac{\mathrm{d}x(t)}{\mathrm{d}t} + x(t) = K_S f(t) \tag{10}$$

- Exercise: What are τ and K_S of prev. examples?

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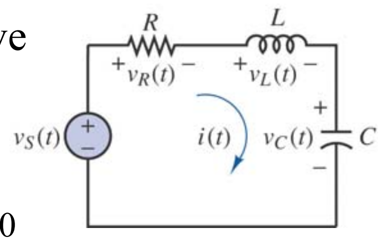
Second-Order Circuit Example (Example on pp. 220-221)

- Note this circ. has 2 reactive elements (ind. & cap.)
  - KVL equation:



$$v_S(t) - Ri(t) - L\frac{\mathrm{d}i(t)}{\mathrm{d}t} - \frac{1}{C}\int_{\tau=-\infty}^{t} i(\tau)\,\mathrm{d}t = 0 \quad (11)$$

  - Differentiate to get 2nd-order ODE:

$$R\frac{\mathrm{d}i(t)}{\mathrm{d}t} + L\frac{\mathrm{d}^2 i(t)}{\mathrm{d}t^2} + \frac{1}{C}i(t) = \frac{\mathrm{d}v_S(t)}{\mathrm{d}t} \quad (12)$$

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Another Form

- Subst. $i(t) = C\,\mathrm{d}v_C/\mathrm{d}t$ into orig. eq. (11) to get:

$$RC\frac{\mathrm{d}v_C(t)}{\mathrm{d}t} + LC\frac{\mathrm{d}^2 v_C(t)}{\mathrm{d}t^2} + v_C(t) = v_S(t) \quad (13)$$

General form of ODEs for Second-Order Circuits

- Eqs. (12) & (13) can each be expressed in the general form:

$$a_2 \frac{\mathrm{d}^2 x(t)}{\mathrm{d}t^2} + a_1 \frac{\mathrm{d}x(t)}{\mathrm{d}t} + a_0 x(t) = b_0 f(t)$$

- Again we can divide through by a_0, to get:

$$\frac{a_2}{a_0} \frac{\mathrm{d}^2 x(t)}{\mathrm{d}t^2} + \frac{a_1}{a_0} \frac{\mathrm{d}x(t)}{\mathrm{d}t} + x(t) = \frac{b_0}{a_0} f(t)$$

Properties of Second-Order Circuits

- Now we define:
  - ω_n – ("omega sub n") The *natural frequency*
  - ζ – ("zeta") The *damping ratio*
  - K_S – ("K sub S") The *DC gain*

$$\omega_n = \sqrt{\frac{a_0}{a_2}}, \quad \zeta = \frac{a_1}{2} \sqrt{\frac{1}{a_0 a_2}}, \quad K_S = \frac{b_0}{a_0}$$

- And get:

$$\frac{1}{\omega_n^2} \frac{\mathrm{d}^2 x(t)}{\mathrm{d}t^2} + \frac{2\zeta}{\omega_n} \frac{\mathrm{d}x(t)}{\mathrm{d}t} + x(t) = K_S f(t)$$

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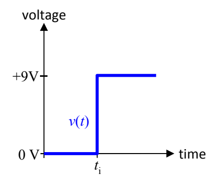
§5.3 – DC Steady-State Solution of Circuits Containing Inductors & Capacitors – Initial & Final Conditions

- Just before a step impulse at the source,
  - And as $t \to \infty$ after the step,
- We can assume eventually all voltages and currents in the circuit settle down to stable equil. values.
  - Set all time derivatives equal to 0.



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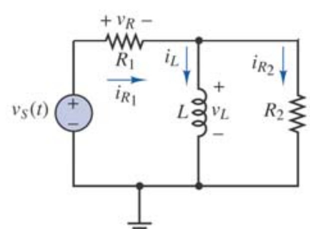
Steady-State for Example 5.1

- Differential equation was:

$$\underbrace{\left(\frac{L}{R_1}+\frac{L}{R_2}\right)}_{\tau}\frac{\mathrm{d}i_L}{\mathrm{d}t}+i_L=\underbrace{\frac{1}{R_1}}_{K_S}v_S$$



- In steady-state, $\mathrm{d}i_L/\mathrm{d}t \to 0$ (const. current), so in the limit as $t \to \infty$,

$$i_L = K_S v_S = \frac{v_S}{R_1} \qquad \text{so} \qquad v_S = R_1 i_L$$

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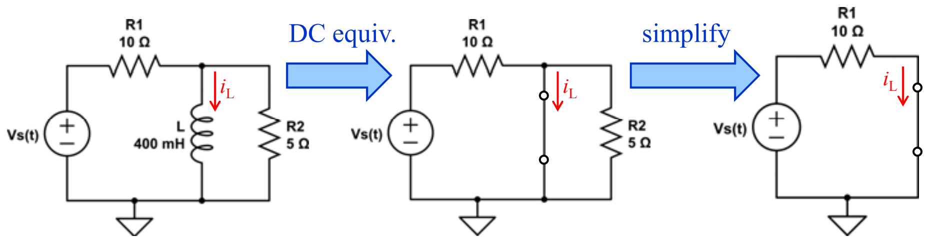
Another Method

- Under DC excitation, replace reactances:
  - $L \longrightarrow$ closed circuit
  - $C \longrightarrow$ open circuit



Thus, clearly, in steady state, $i_L = v_s/R_1$

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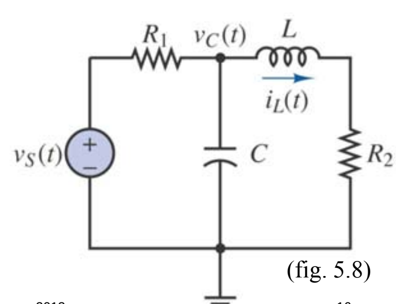
Example: Finding the Steady-State Equations of a 2nd-Order Circuit

- Method 1: ODE comes out as:

$$\underbrace{\left(\frac{R_1 CL}{R_1+R_2}\right)}_{1/\omega_n^2}\frac{d^2 i_L(t)}{dt^2}+\underbrace{\left(\frac{R_1R_2C+L}{R_1+R_2}\right)}_{2\zeta/\omega_n}\frac{di_L(t)}{dt}+i_L(t)=\underbrace{\frac{1}{R_1+R_2}}_{K_S}v_S(t)$$

- Set all derivs. = 0 to get:

$$i_L(t)=\left(\frac{1}{R_1+R_2}\right)v_S(t)$$



(fig. 5.8)

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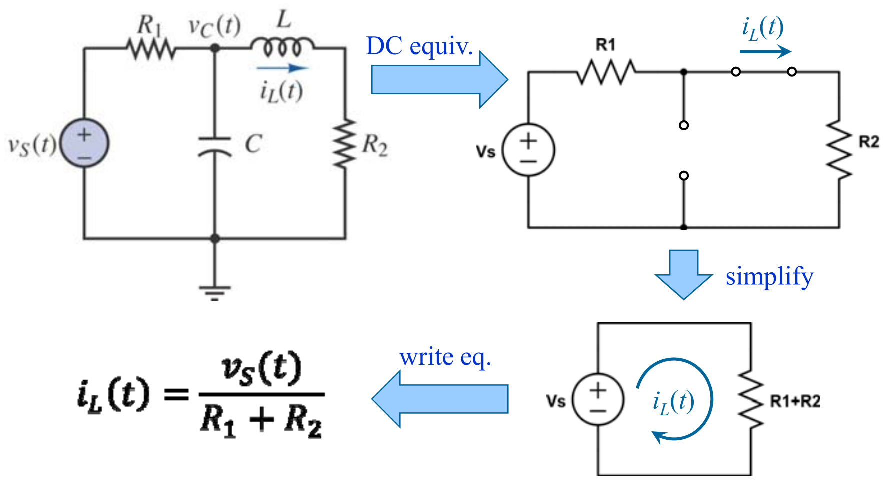
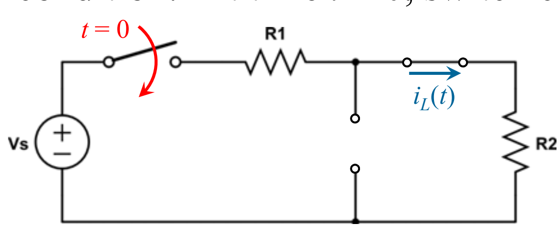
Or, by simplifying circuit…



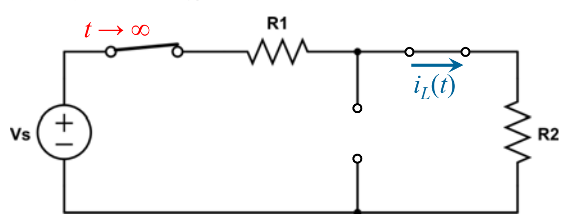
$$i_L(t) = \frac{v_S(t)}{R_1 + R_2}$$

Initial/Final Conditions

- Initial condition: At time $t = 0$, switch closes:



- Final condition: State as time $t \to \infty$:



Continuity Rules

- The main variables for 1st- and 2nd-order transient analysis are usually:
  - v_C – The voltage across a capacitor
  - i_L – The current through an inductor
- *These variables can't change instantaneously!*
- Therefore,
  - $v_C(0^+) = v_C(0^-)$ $\lim_{\delta t \to 0}: v_C(0 - \delta t) = v_C(0 + \delta t)$
  - $i_L(0^+) = i_L(0^-)$