EEL 3003 (INTRODUCTION TO ELECTRICAL ENGINEERING), SUMMER 2013

Lecture #5 – Resistive Network Analysis, Part II Supplemental Notes

Since today's lecture period got used up by my motivational presentation and the quiz review, I'm providing some of the other material I wanted to cover in the form of these notes. This covers the following sections of the textbook:

- §3.3 – The Mesh Current Method
- §3.4 – Node and Mesh Analysis with Controlled Sources

1. The Mesh Current Method (§3.3).

In lecture last Thursday (after the quiz), I analyzed this circuit from textbook problem #3.1 (fig. 1 below)—or actually, an already-simplified version of it—using node voltage analysis, and began the analysis of the same circuit using the mesh current method, but didn't finish. The problem is to find the two unknown voltages v_1 and v_2.

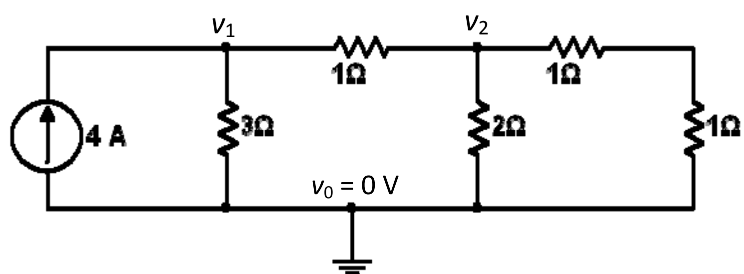

Figure 1. Circuit for Mesh Current Analysis Example.

Let's start over the mesh current analysis here. First, we can simplify the circuit a little (fig. 2) by replacing the two 1-Ω resistors in series with (equivalently) one 2-Ω resistor:

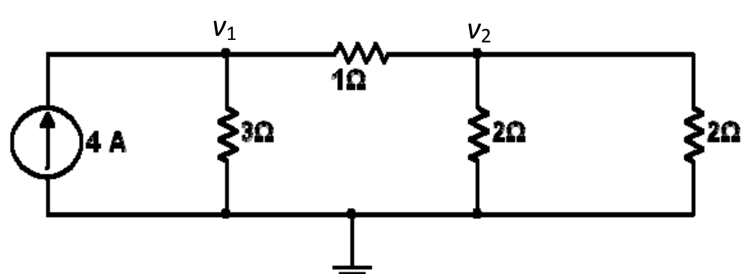

Figure 2. Circuit after combining series resistors.

And then we can replace the two parallel 2 Ω resistors with one equivalent resistor of resistance

$$R_{\text{eq}} = 2\,\Omega \parallel 2\,\Omega = \frac{1}{\frac{1}{2\,\Omega}+\frac{1}{2\,\Omega}} = \frac{1}{\left(\frac{2}{2\,\Omega}\right)} = \frac{2\,\Omega}{2} = 1\,\Omega \tag{1}$$

(see fig. 3 below). We also label the remaining unlabeled (ground) node v_0, for convenience. By convention, we say it is at 0 volts.

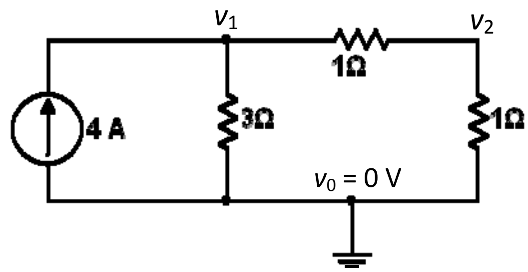

Figure 3. Circuit after combining parallel resistors.

Next, we label the mesh currents. By convention, we proceed around each mesh clockwise, starting in its lower-left corner (this is not strictly necessary, but it just helps keep things uniform).

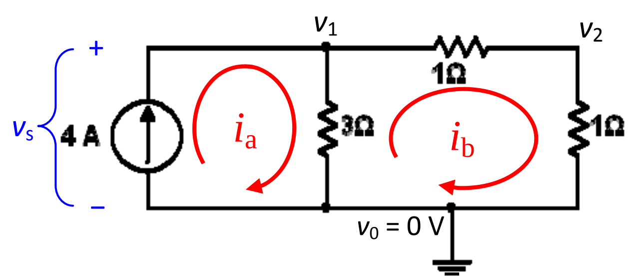

Figure 4. Circuit after labeling meshes & source voltage.

Next, we write the KVL equations for each mesh. Since we have a current source, the voltage across it ($v_s = v_1 - v_0$) is initially unknown; we'll solve for it in our solution. The KVL equations are then:

$$\begin{aligned} &\text{Mesh } a\text{:} && v_s - (i_a - i_b)3\Omega = 0 \\ &\text{Mesh } b\text{:} && -(i_b - i_a)3\Omega - i_b 1\Omega - i_b 1\Omega = 0. \end{aligned} \tag{2}$$

The terms here need some explanation. In the first equation, we are proceeding around mesh *a*, the left-hand loop, in a clockwise direction starting from lower-left. We first go up by an unknown voltage of $+v_s$. Then we cross a resistor, which means we are going down (−) in voltage if the current is in the direction of the mesh *a* arrow. However, the total current in that direction is not just i_a, but is $(i_a - i_b)$, since there is another mesh current component i_b passing through that resistor in the opposite direction. The voltage drop across a resistor is $\Delta V = iR$ where $i = (i_a - i_b)$ is the current in the direction of the drop and R is the resistance, so the voltage drop in this case is $\Delta V = iR = (i_a - i_b)\cdot 3\Omega$. Since it is a *drop* in voltage (*i.e.*, since we're crossing a resistor in the direction of a nominal current i), the change in voltage (later node minus earlier node) when going in that direction is then $-\Delta V = -(i_a - i_b)\cdot 3\Omega$, so that's the second term on the first line.

On the second line, first term: Again, we put a minus sign in front, because we are crossing a resistor with reference to a nominal current direction (the mesh current i_b, starting from the lower-left corner of mesh *b*), but now, the total current in that direction is $i = i_b - i_a$, since i_b is the direction we are going, and i_a is the opposite direction, so it gets a minus sign. The resistance is still R=3Ω. So, that term overall is $-(i_b - i_a)\cdot 3\Omega$. Note that this is the opposite sign of the corresponding term for Mesh *a*, which is appropriate since we are crossing that resistor now in the opposite direction, so the change in voltage is opposite.

The second term in the Mesh b KVL equation is $-i_b\cdot 1\Omega$ since we have a voltage drop across a resistor, as we proceed in the direction of the current i_b across a resistance of 1Ω. Likewise for the third term.

Now, simplifying the equations (2) to standard linear form, we have:

$$\begin{aligned} v_s + (-3\Omega)i_a + (3\Omega)i_b &= 0 \\ (3\Omega)i_a + (-5\Omega)i_b &= 0. \end{aligned} \tag{3}$$

It might look at first as if we have three unknowns but only two equations, but wait! One of our variables (i_a) is actually not an unknown – it is equal to the source current i_s = 4A. In general, each of your known sources will turn one unknown variable into a known one, so before you start solving your equations, make sure you have plugged in all the source values. Doing this, we have:

$$\begin{aligned} v_s + (-3\Omega)(4\text{A}) + (3\Omega)i_b &= 0 \\ (3\Omega)(4\text{A}) + (-5\Omega)i_b &= 0. \end{aligned} \tag{4}$$

By convention, simplify the constants and put them on the right-hand side:

$$\begin{aligned} v_s + (3\Omega)i_b &= 12\text{ V} \\ (-5\Omega)i_b &= -12\text{ V}. \end{aligned} \tag{5}$$

At this point, we have two equations and only two unknown, so we know we can solve. In fact, the second equation can be solved for $\boldsymbol{i_b}$ directly:

$$i_b = \frac{-12\text{ V}}{-5\Omega} = \frac{12}{5}\text{A} = 2.4\text{ A}, \tag{5}$$

which we can then plug back into the first of equations (5) to find v_s:

$$v_s + (3\Omega)(2.4\text{ A}) = 12\text{ V}$$
$$v_s + 7.2\text{ V} = 12\text{ V}$$
$$v_s = 4.8\text{ V}. \tag{6}$$

In a more complicated circuit, the direct substitution method might be too cumbersome, and we might have to use determinants or resort to computer-based methods to solve the system of equations, but substitution is adequate for this simple case.

Anyway, now that the source voltage v_s is known, we can easily proceed to find v_1 and v_2. The node voltage $v_1 = v_0 + v_s = 0\text{V} + v_s = v_s = 4.8$ V. And once v_1 is known, v_2 can be found by subtracting the voltage drop from v_1 to v_2 across the top 1Ω resistor:

$$v_2 = v_1 - (1\ \Omega)i_b$$
$$= (4.8\text{ V}) - (1\ \Omega)(2.4\text{ A})$$
$$= (4.8\text{ V}) - (2.4\text{ V})$$
$$= 2.4\text{ V}. \tag{7}$$

2. Node and Mesh Analysis with Controlled Sources (§3.4)

The discussion of this in the textbook seems fine, and so far, I don't have anything to add to it…

[add some more examples here later]