

Eigenvalue Multiplicity	$\psi_i(t)$ functions	Exponential Matrix
$n=2$	simple: $\lambda_1 \neq \lambda_2$ $\psi_0(t) = -\frac{\lambda_2}{\lambda_1 - \lambda_2} e^{\lambda_1 t} - \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{\lambda_2 t}$ $\psi_1(t) = \frac{1}{\lambda_1 - \lambda_2} e^{\lambda_1 t} + \frac{1}{\lambda_2 - \lambda_1} e^{\lambda_2 t}$ double: $\lambda_1 = \lambda_2$ $\psi_0(t) = e^{\lambda_1 t} - \lambda_1 t e^{\lambda_1 t}$ $\psi_1(t) = t e^{\lambda_1 t}$	$e^{At} = \psi_0(t)I + \psi_1(t)A$
$n=3$	simple: $\lambda_1 \neq \lambda_2$ $\lambda_2 \neq \lambda_3$ $\lambda_3 \neq \lambda_1$ $\psi_0(t) = \frac{\lambda_2 \lambda_3}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)} e^{\lambda_1 t} + \frac{\lambda_1 \lambda_3}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)} e^{\lambda_2 t} + \frac{\lambda_1 \lambda_2}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)} e^{\lambda_3 t}$ $\psi_1(t) = \frac{-(\lambda_2 + \lambda_3)}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)} e^{\lambda_1 t} + \frac{-(\lambda_1 + \lambda_3)}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)} e^{\lambda_2 t} + \frac{-(\lambda_1 + \lambda_2)}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)} e^{\lambda_3 t}$ $\psi_2(t) = \frac{1}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)} e^{\lambda_1 t} + \frac{1}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)} e^{\lambda_2 t} + \frac{1}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)} e^{\lambda_3 t}$ one simple one double $\lambda_1 \neq \lambda_2 = \lambda_3$ $\psi_0(t) = \frac{\lambda_2^2}{(\lambda_1 - \lambda_2)^2} e^{\lambda_1 t} + \frac{\lambda_1^2 - 2\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)^2} e^{\lambda_2 t} - \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} t e^{\lambda_2 t}$ $\psi_1(t) = \frac{-2\lambda_2}{(\lambda_1 - \lambda_2)^2} e^{\lambda_1 t} + \frac{2\lambda_2}{(\lambda_1 - \lambda_2)^2} e^{\lambda_2 t} + \frac{\lambda_1 + \lambda_2}{\lambda_1 - \lambda_2} t e^{\lambda_2 t}$ $\psi_2(t) = \frac{1}{(\lambda_1 - \lambda_2)^2} e^{\lambda_1 t} - \frac{1}{(\lambda_1 - \lambda_2)^2} e^{\lambda_2 t} - \frac{1}{\lambda_1 - \lambda_2} t e^{\lambda_2 t}$ triple $\lambda_1 = \lambda_2 = \lambda_3$ $\psi_0(t) = e^{\lambda_1 t} - \lambda_1 t e^{\lambda_1 t} + \frac{\lambda_1^2}{2} t^2 e^{\lambda_1 t}$ $\psi_1(t) = t e^{\lambda_1 t} - \lambda_1 t^2 e^{\lambda_1 t}$ $\psi_2(t) = \frac{1}{2} t^2 e^{\lambda_1 t}$	$e^{At} = \psi_0(t)I + \psi_1(t)A + \psi_2(t)A^2$