# Home Assignment

Problem 1: Consider the following linearized model in deviation form:

$$\frac{dX}{dt} = AX + BU \tag{1}$$

where

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \tag{2}$$

$$B = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \tag{3}$$

and $U$ is a scalar. Suppose the input undergoes the following step change:

$$U = \begin{cases} 0 & t < 0 \\ 3 & t \geq 0 \end{cases} \tag{4}$$

Compute how the $X$ vector changes with time for the following $A$ matrices:

1. $$A = \begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix}$$

2. $$A = \begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix}$$

3. $$A = \begin{bmatrix} k_1 & 0 \\ k_1 & k_2 \end{bmatrix}$$

Problem 2: Consider the following linearized model in deviation form:

$$\frac{dX}{dt} = AX + BU \tag{5}$$

where

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \tag{6}$$

$$B = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \tag{7}$$

and $U$ is a scalar. Suppose the input undergoes the following pulse change:

$$U = \begin{cases} 0 & t < 0 \\ 3 & 0 \leq t < 1 \\ 0 & t \geq 1 \end{cases} \tag{8}$$

Compute how the $X$ vector changes with time for the following $A$ matrices:

1.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -5 & 7 \\ -6 & -3 & -2 \end{bmatrix}$$

2.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 5 & 2 & 0 \\ 8 & 4 & 2 \end{bmatrix}$$

<u>Problem 3</u>: Consider the following linearized model in deviation form:

$$\frac{dX}{dt} = AX + BU \tag{9}$$

where

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \tag{10}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \tag{11}$$

$$U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \tag{12}$$

Suppose the input undergoes the following change:

$$U_1 = \begin{cases} 0 & t < 0 \\ 3 & t \geq 0 \end{cases} \tag{13}$$

$$U_2 = \begin{cases} 0 & t < 0 \\ 3.t & t \geq 0 \end{cases} \tag{14}$$

Compute how the $X$ vector changes with time for the following $A$ matrix:

1.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -5 & 7 \\ -6 & -3 & -2 \end{bmatrix}$$