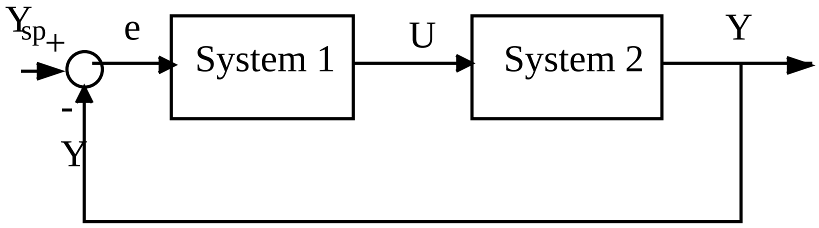
Interconnected Systems

Problem 1: Consider the following interconnected system:



Suppose System 1 is modeled as:

$$\begin{aligned}\frac{d}{dt}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} &= A_1\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + B_1 e \\ U &= C_1\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + D_1 e\end{aligned} \tag{1}$$

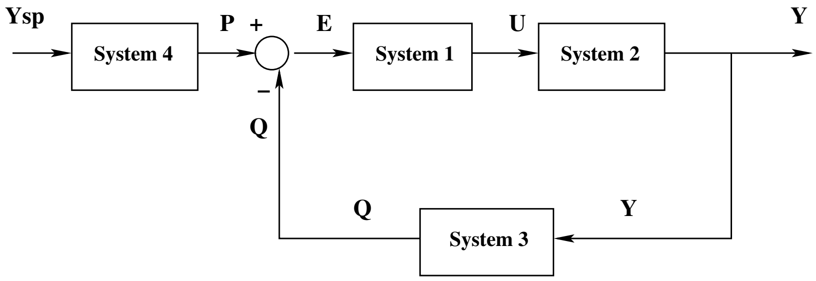
and System 2 is modeled as:

$$\begin{aligned}\frac{d}{dt}\begin{bmatrix} X_3 \\ X_4 \end{bmatrix} &= A_2\begin{bmatrix} X_3 \\ X_4 \end{bmatrix} + B_2 U \\ Y &= C_2\begin{bmatrix} X_3 \\ X_4 \end{bmatrix} + D_2 U\end{aligned} \tag{2}$$

Compute the overall system with input Y_{sp} and output Y for the following values of the matrices:

$$\begin{aligned}A_1 &= \begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix} \\ B_1 &= \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\ C_1 &= \begin{bmatrix} 1 & 1 \end{bmatrix} \\ D_1 &= [1] \\ A_2 &= \begin{bmatrix} -1 & 2 \\ 4 & -5 \end{bmatrix} \\ B_2 &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ C_2 &= \begin{bmatrix} -1 & 2 \end{bmatrix} \\ D_2 &= [0]\end{aligned} \tag{3}$$

Problem 2: Consider the following interconnected system:



Suppose System 1 is modeled as:

$$
\begin{aligned}
\frac{d}{dt}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} &= A_1 \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + B_1 E \\
U &= C_1 \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + D_1 E
\end{aligned} \tag{4}
$$

System 2 is modeled as:

$$
\begin{aligned}
\frac{d}{dt}\begin{bmatrix} X_3 \\ X_4 \\ X_5 \end{bmatrix} &= A_2 \begin{bmatrix} X_3 \\ X_4 \\ X_5 \end{bmatrix} + B_2 U \\
Y &= C_2 \begin{bmatrix} X_3 \\ X_4 \\ X_5 \end{bmatrix} + D_2 U
\end{aligned} \tag{5}
$$

System 3 is modeled as:

$$
\begin{aligned}
\frac{d}{dt}\begin{bmatrix} X_6 \\ X_7 \end{bmatrix} &= A_3 \begin{bmatrix} X_6 \\ X_7 \end{bmatrix} + B_3 Y \\
Q &= C_3 \begin{bmatrix} X_6 \\ X_7 \end{bmatrix} + D_3 Y
\end{aligned} \tag{6}
$$

System 4 is modeled as:

$$
\begin{aligned}
\frac{d}{dt}\begin{bmatrix} X_8 \end{bmatrix} &= A_4 \begin{bmatrix} X_8 \end{bmatrix} + B_4 Y_{sp} \\
P &= C_4 \begin{bmatrix} X_8 \end{bmatrix} + D_4 Y_{sp}
\end{aligned} \tag{7}
$$

Compute the overall system with input Y_{sp} and output Y for the following values of the matrices:

$$
\begin{aligned}
A_1 &= \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} \\
B_1 &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
C_1 &= \begin{bmatrix} 1 & 0 \end{bmatrix} \\
D_1 &= [1] \\
A_2 &= \begin{bmatrix} -1 & 2 & 0 \\ 4 & -5 & 1 \\ 0 & 3 & 8 \end{bmatrix} \\
B_2 &= \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \\
C_2 &= \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \\
D_2 &= [0] \\
A_3 &= \begin{bmatrix} -1 & 0 \\ 3 & -2 \end{bmatrix} \\
B_3 &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\
C_3 &= \begin{bmatrix} 1 & 0 \end{bmatrix} \\
D_3 &= [0] \\
A_4 &= [5] \\
B_4 &= [1] \\
C_4 &= [3] \\
D_4 &= [0]
\end{aligned} \tag{8}
$$