Dynamics of Commercial Controllers

A controller is an electronic device that helps to:

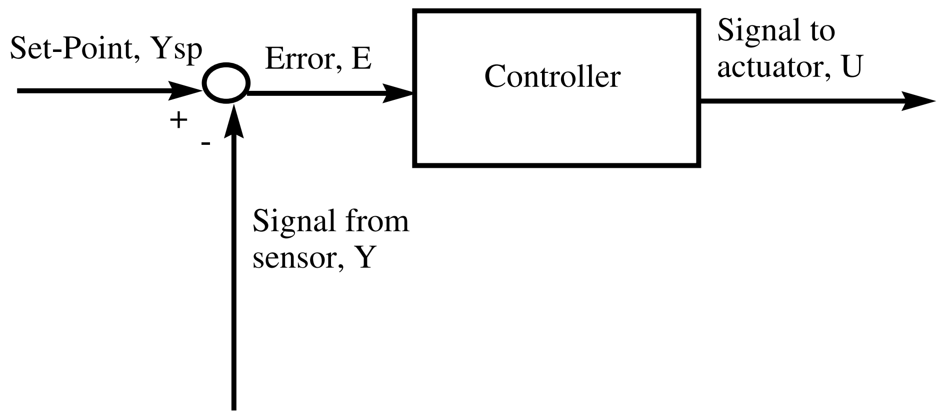
1. **regulate a process at its desired set-point despite disturbances to the process.**
2. **take the system to a new set-point safely, with desired dynamic characteristics.**

The vast majority of controllers used in the process industry (over 80%) are P, PI or PID controllers. In this lecture, we develop the dynamic models for these commercial controllers.

P = Proportional

PI = Proportional-Integral

PID = Proportional-Integral-Derivative



Y_{sp}: **Desired set-point**

Y: **Signal from sensor**

E: **Error(input to controller)** $E = Y_{sp} - Y$

U: **Signal to actuator (output of controller)**

1. P Controller

$$U = k_c E \tag{1}$$

The variable k_c is called the *controller gain* and is a *tunable* parameter, which is set by the user.

2. PI Controller

$$
\begin{aligned}
\frac{de_I}{dt} &= E \\
U &= k_c E + \frac{k_c}{\tau_I} e_I
\end{aligned}
\qquad (2)
$$

The variable τ_I is called the *integral time constant*. The PI controller has two tunable parameters: k_c and τ_I.

3. PID Controller

$$
\begin{aligned}
\frac{de_I}{dt} &= E \\
\frac{de_F}{dt} &= -\frac{1}{\lambda}e_F + \frac{1}{\lambda}E \\
U &= k_c E + \frac{k_c}{\tau_I}e_I + k_c\tau_D\left(-\frac{1}{\lambda}e_F + \frac{1}{\lambda}E\right)
\end{aligned} \tag{3}
$$

The variable τ_D is called the *derivative time constant.*

The variable e_F is the filtered derivative of the error with filter constant λ. The value of λ is not arbitrary but is set as the following

function of τ_D.

$$\lambda = \alpha \tau_D \quad (4)$$

In most commercial controllers, α is set at a value of 0.1. Thus, the PID controller has three tunable parameters: k_c, τ_I and τ_D.

Commercial Controllers in Standard Form

1. **P Controller**

$$U = 0.X + k_c.E \tag{5}$$

Note that there is no differential equation involved and thus the 'A' and 'B' matrices are both zero.

2. **PI Controller**

$$\begin{aligned} \frac{de_I}{dt} &= 0.e_I + 1.E \\ U &= \frac{k_c}{\tau_I}.e_I + k_c.E \end{aligned} \tag{6}$$

3. PID Controller

$$\begin{aligned}
\frac{d}{dt}\begin{bmatrix} e_I \\ e_F \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{\lambda} \end{bmatrix}\begin{bmatrix} e_I \\ e_F \end{bmatrix} + \begin{bmatrix} 1 \\ \frac{1}{\lambda} \end{bmatrix} E \\
U &= \begin{bmatrix} \frac{k_c}{\tau_I} & -\frac{k_c \tau_D}{\lambda} \end{bmatrix}\begin{bmatrix} e_I \\ e_F \end{bmatrix} + \begin{bmatrix} k_c + \frac{k_c \tau_D}{\lambda} \end{bmatrix} E
\end{aligned} \tag{7}$$