Qualitative Effect of P, Pi, PID parameters

So far in this course, we have done the following:

- Developed dynamic models for chemical processes
- Studied the effect of changing inputs on the outputs and states
- Developed a procedure for interconnection of systems

In this lecture, we will address the issue of what happens when we connect a controller to a process. In particular, we will study how the parameters of the controller (which we can tune), affect the outputs.

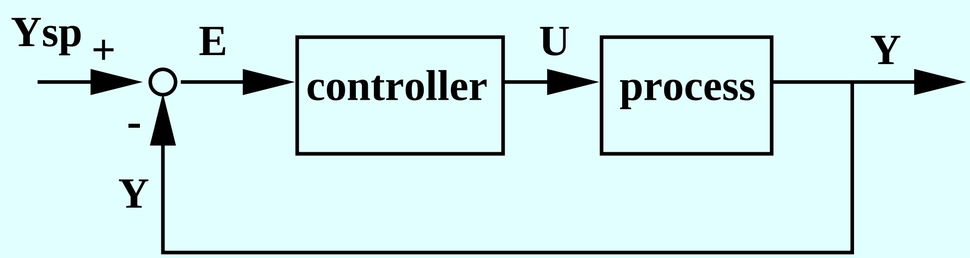
P Controller

Consider the following process:

$$
\begin{aligned}
\frac{d}{dt}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} &= \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 10 & -10 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} U \\
Y &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}
\end{aligned} \tag{1}
$$

and the P controller

$$\begin{aligned} U &= k_c E \\ &= k_c(Y_{sp} - Y) \end{aligned} \tag{2}$$

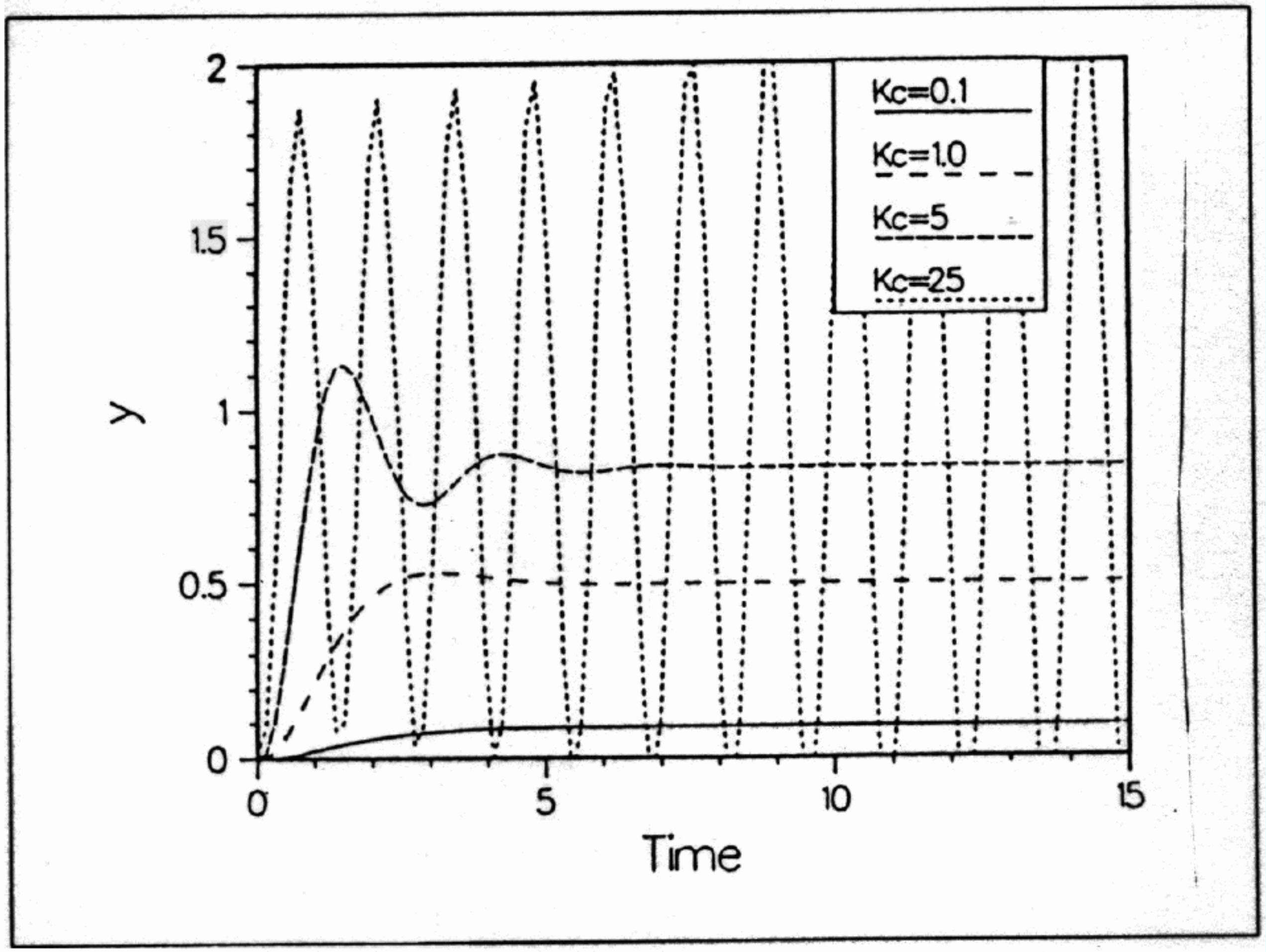


The closed-loop system is given by

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} &= \begin{bmatrix} -1 & 0 & -k_c \\ 1 & -1 & 0 \\ 0 & 10 & -10 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} k_c \\ 0 \\ 0 \end{bmatrix} Y_{sp} \\ Y &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \end{aligned} \tag{3}$$

Suppose Y_{sp} undergoes a step change from 0 to 1. Clearly, we would like Y to reach the new value of the setpoint as soon as possible.

What is the effect of the value of k_c on the output Y?



y
2
1.5
1
0.5
0
0
5
10
15
Time
Kc=0.1
Kc=1.0
Kc=5
Kc=25

- For small k_c, response is smooth and stable, but there is a large off-set.
- For medium k_c, response is oscillatory and stable, but the off-set is small.
- For large k_c, the response is oscillatory and unstable.

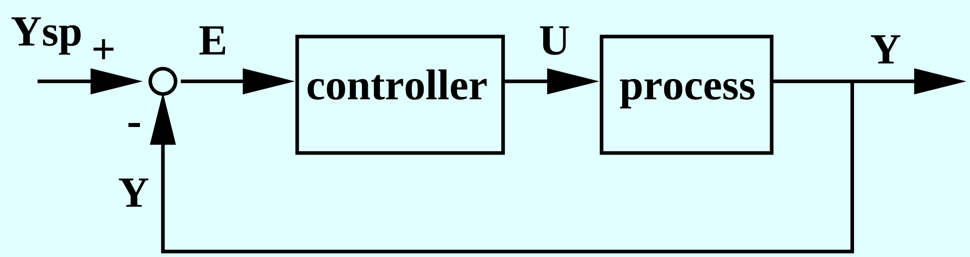
PI Controller

Consider the following process:

$$
\begin{aligned}
\frac{d}{dt}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} &= \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 10 & -10 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} U \\
Y &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}
\end{aligned} \tag{4}
$$

and the PI controller

$$
\begin{aligned}
\frac{dX_4}{dt} &= Y_{sp} - Y \\
U &= k_c E + \frac{k_c}{\tau_I} X_4
\end{aligned}
\tag{5}
$$

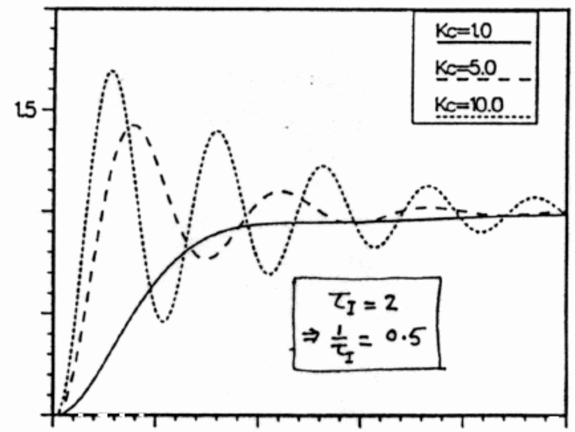


The closed-loop system is given by

$$
\begin{aligned}
\frac{d}{dt}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} &= \begin{bmatrix} -1 & 0 & -k_c & \frac{k_c}{\tau_I} \\ 1 & -1 & 0 & 0 \\ 0 & 10 & -10 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} k_c \\ 0 \\ 0 \\ 1 \end{bmatrix} Y_s \\
Y &= \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}
\end{aligned}
\tag{6}
$$

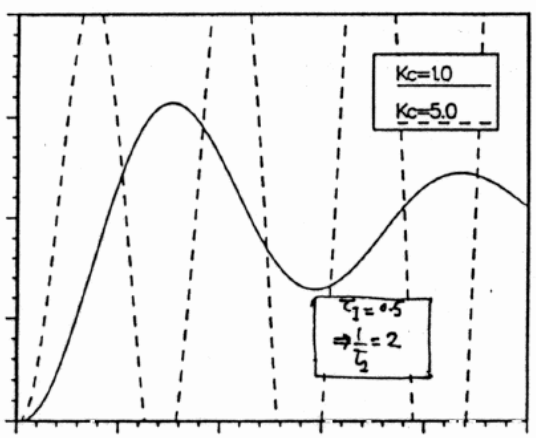
Suppose Y_{sp} undergoes a step change from 0 to 1. Clearly, we would like Y to reach the new value of the setpoint as soon as possible.

What is the effect of the value of k_c and τ_I on the output Y?



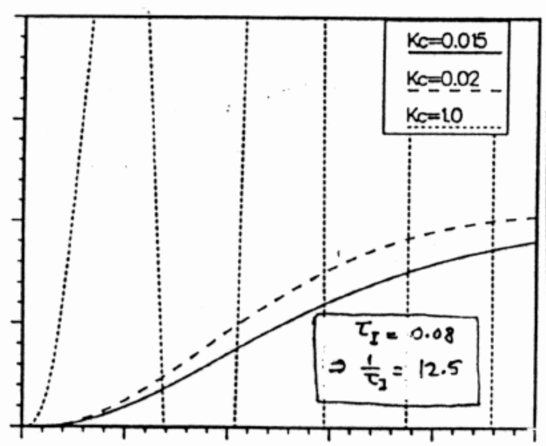
1.5
Kc=1.0
Kc=5.0
Kc=10.0
τI = 2
⇒ 1/τI = 0.5

T



Kc=1.0
Kc=5.0
τI = 0.5
⇒ 1/τI = 2

T



Kc=0.015
Kc=0.02
Kc=1.0
τI = 0.08
⇒ 1/τI = 12.5

- Integral action eliminates off-set.
- As τ_I is decreased (=> $\frac{1}{\tau_I}$ is increased), the range of values for k_c which lead to stable output response, decreases.

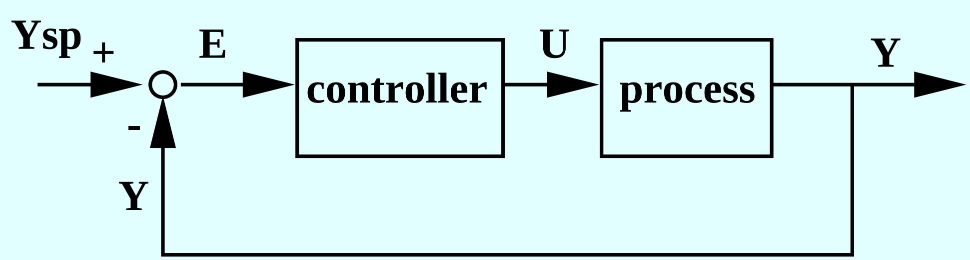
PID Controller

Consider the following process:

$$
\begin{aligned}
\frac{d}{dt}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} &= \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 10 & -10 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} U \\
Y &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}
\end{aligned} \tag{7}
$$

and the PID controller

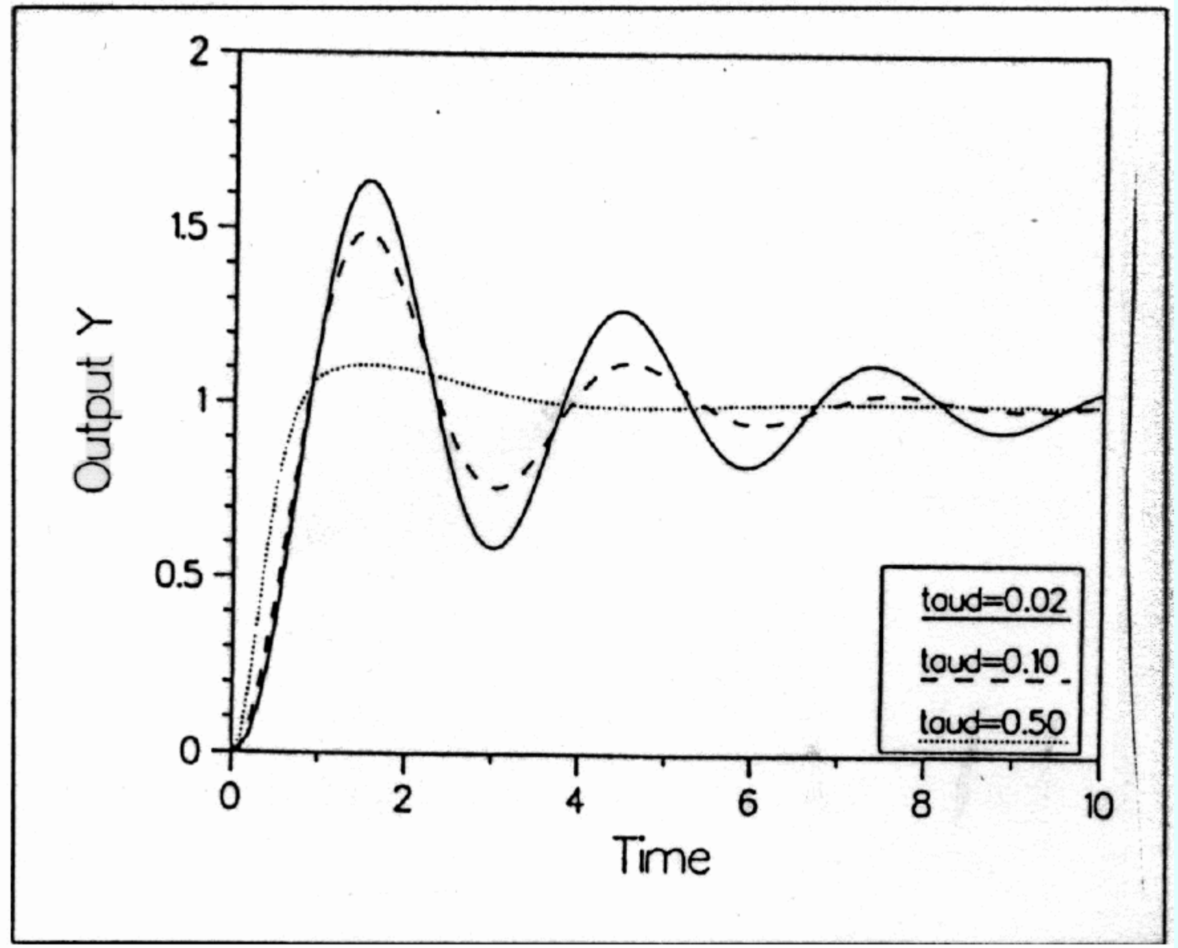
$$\begin{aligned}
\frac{dX_4}{dt} &= E \\
\frac{dX_5}{dt} &= -\frac{1}{\alpha\tau_D}X_5 + \frac{1}{\alpha\tau_D}E \qquad (8) \\
U &= k_c(1+\frac{1}{\alpha})E + \frac{k_c}{\tau_I}X_4 - \frac{k_c}{\alpha}X_5
\end{aligned}$$



Suppose Y_{sp} undergoes a step change from 0 to 1. Clearly, we would like Y to reach the new value of the setpoint as soon as possible.

What is the effect of the value of k_c, τ_I, and τ_D on the output Y?

The value of α is set to 0.1 for most commercial controllers. Thus there are only three parameters to vary (k_c, τ_I, and τ_D)


Output Y
Time
taud=0.02
taud=0.10
taud=0.50

- Integral action eliminates off-set.
- Derivative action has a stabilizing effect on the process. Thus, the presence of the τ_D term allows the use of higher values of k_c as compared to the PI controller.

Summary

- Proportional action alone is not sufficient to tightly control the output to the desired set-point. Increasing k_c reduces offset but destabilizes the process.
- Integral action eliminates offset. However as the integral term becomes larger (τ_I becomes small), the response is destabilized.
- Derivative action has a stabilizing effect on the closed-loop system. This allows for higher values of proportional and integral terms to be used, thereby making the response faster.