Qualitative Effect of P, PI, PID parameters

So far in this course, we have done the following:

- Developed dynamic models for chemical processes
- Studied the effect of changing inputs on the outputs and states
- Developed a procedure for interconnection of systems

In this lecture, we will address the issue of what happens when we connect a controller to a process. In particular, we will study how the parameters of the controller (which we can tune), affect the outputs.

$\boxed{\text{P Controller}}$

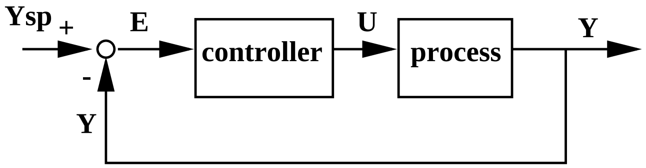
Consider the following process:

$$\frac{d}{dt}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 10 & -10 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \tag{1}$$

and the P controller

$$\begin{aligned} U &= k_c E \\ &= k_c(Y_{sp} - Y) \end{aligned} \tag{2}$$

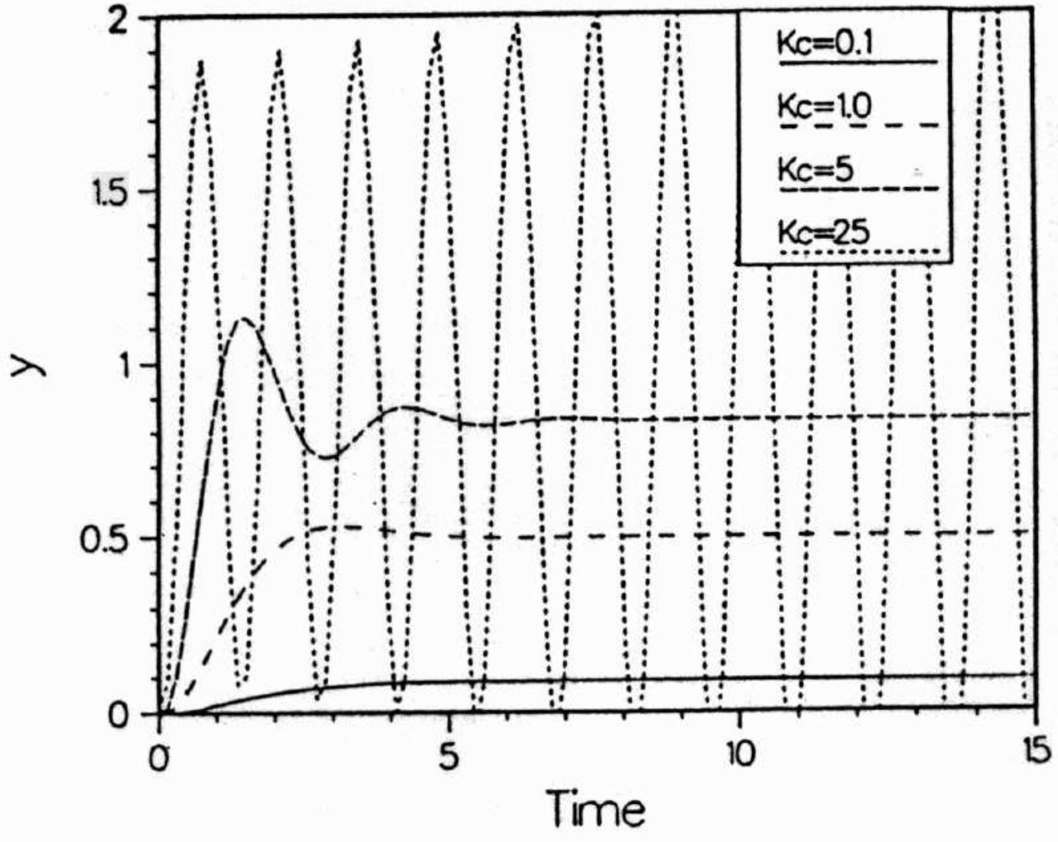


The closed-loop system is given by

$$\frac{d}{dt}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -k_c \\ 1 & -1 & 0 \\ 0 & 10 & -10 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} k_c \\ 0 \\ 0 \end{bmatrix} Y_{sp}$$

$$Y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \tag{3}$$

Suppose Y_{sp} undergoes a step change from 0 to 1. Clearly, we would like Y to reach the new value of the setpoint as soon as possible. What is the effect of the value of k_c on the output Y?



Kc=0.1
Kc=1.0
Kc=5
Kc=25
y
2
1.5
1
0.5
0
0
5
10
15
Time

- **For small k_c, response is smooth and stable, but there is a large off-set.**
- **For medium k_c, response is oscillatory and stable, but the off-set is small.**
- **For large k_c, the response is oscillatory and unstable.**

$\boxed{\text{PI Controller}}$

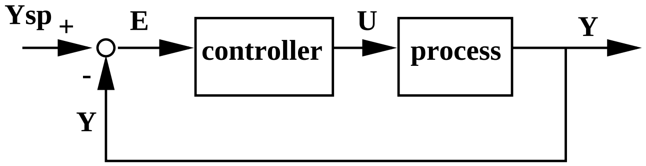
Consider the following process:

$$\frac{d}{dt}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 10 & -10 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \tag{4}$$

and the PI controller

$$\begin{aligned} \frac{dX_4}{dt} &= Y_{sp} - Y \\ U &= k_c E + \frac{k_c}{\tau_I} X_4 \end{aligned} \tag{5}$$



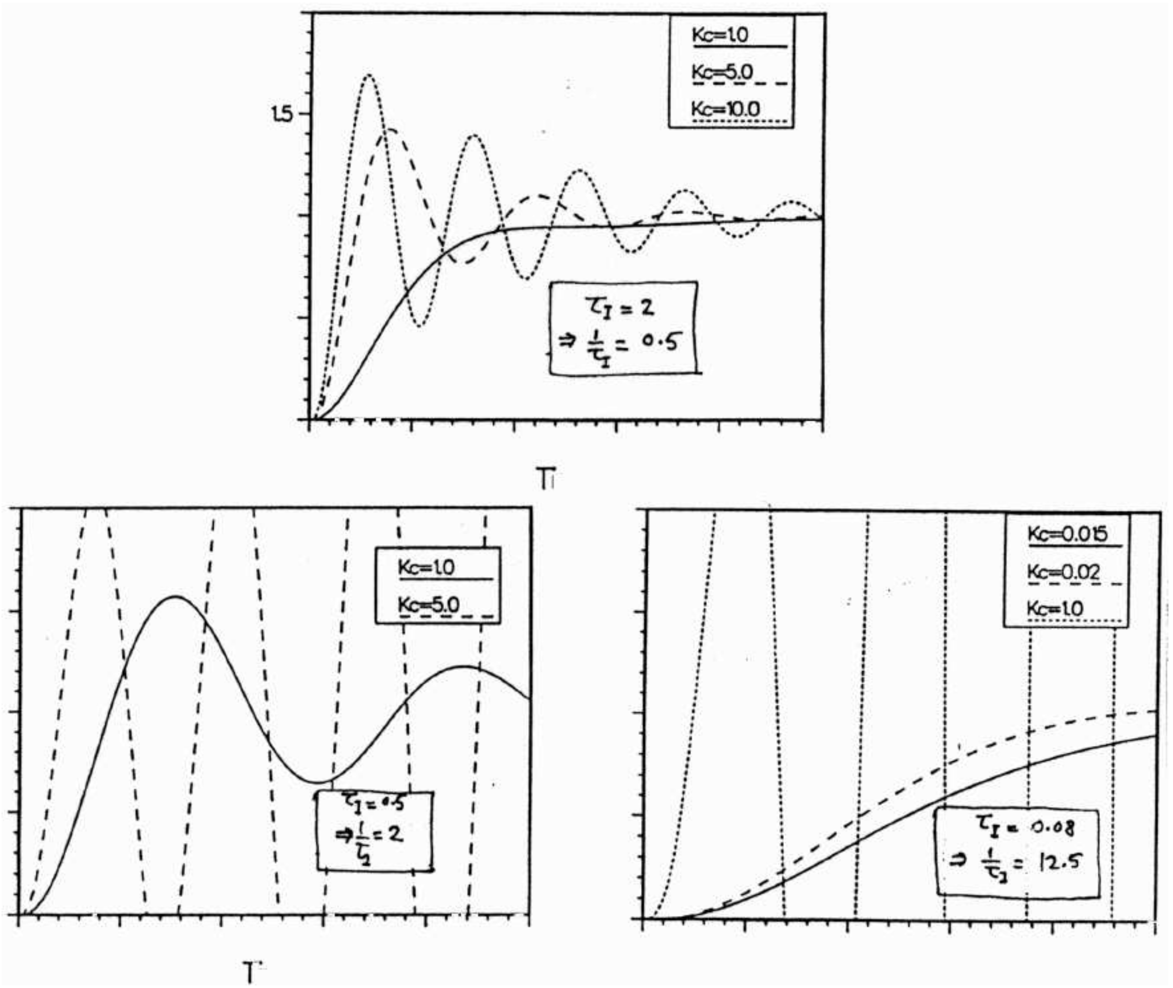
The closed-loop system is given by

$$\frac{d}{dt}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -k_c & \frac{k_c}{\tau_I} \\ 1 & -1 & 0 & 0 \\ 0 & 10 & -10 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} k_c \\ 0 \\ 0 \\ 1 \end{bmatrix} Y_s$$

$$Y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} \tag{6}$$

Suppose Y_{sp} undergoes a step change from 0 to 1. Clearly, we would like Y to reach the new value of the setpoint as soon as possible.

What is the effect of the value of k_c and τ_I on the output Y?



- **Integral action eliminates off-set.**

- **As τ_I is decreased ($=>$ $\dfrac{1}{\tau_I}$ is increased), the range of values for k_c which lead to stable output response, decreases.**

PID Controller

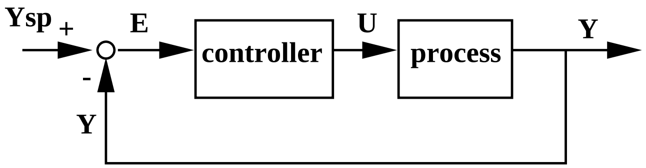
Consider the following process:

$$\frac{d}{dt}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 10 & -10 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \tag{7}$$

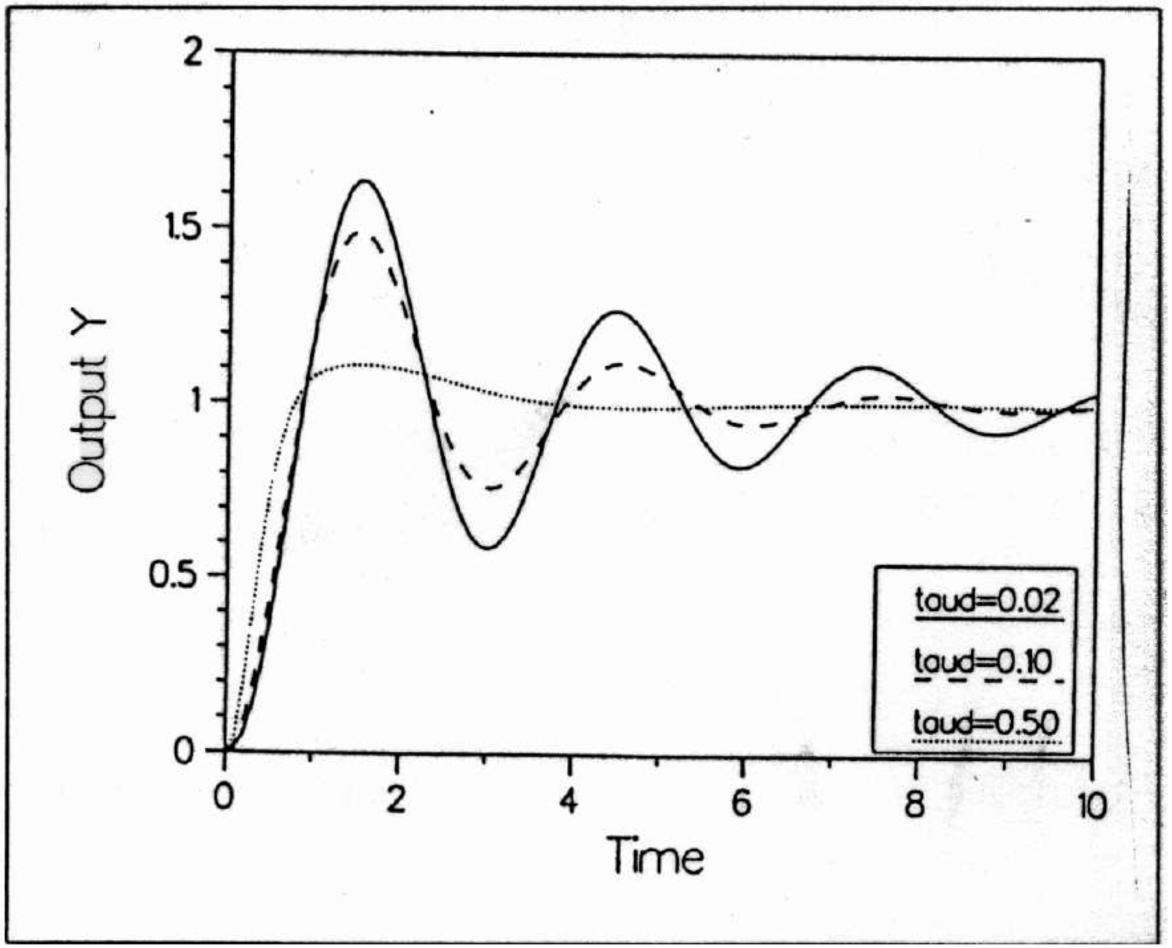
and the PID controller

$$\begin{aligned}
\frac{dX_4}{dt} &= E \\
\frac{dX_5}{dt} &= -\frac{1}{\alpha\tau_D}X_5 + \frac{1}{\alpha\tau_D}E \qquad (8) \\
U &= k_c(1+\frac{1}{\alpha})E + \frac{k_c}{\tau_I}X_4 - \frac{k_c}{\alpha\tau_D}X_5
\end{aligned}$$



Suppose Y_{sp} undergoes a step change from 0 to 1. Clearly, we would like Y to reach the new value of the setpoint as soon as possible.
What is the effect of the value of k_c, τ_I, and τ_D on the output Y?

The value of α is set to 0.1 for most commercial controllers. Thus there are only three parameters to vary (k_c, τ_I, and τ_D)



- **Integral action eliminates off-set.**

- **Derivative action has a stabilizing effect on the process. Thus, the presence of the τ_D term allows the use of higher values of k_c as compared to the PI controller.**

Summary

- **Proportional action** alone is not sufficient to tightly control the output to the desired set-point. Increasing k_c reduces offset but destabilizes the process.

- **Integral action** eliminates offset. However as the integral term becomes larger (τ_I becomes small), the response is destabilized.

- **Derivative action** has a stabilizing effect on the closed-loop system which allows for higher values of proportional and integral terms to be used for faster response.