Appendix II – I. Plate Efficiency

Many calculations that are done on distillation columns are done assuming ideal plates. Ideal plates are defined as plates on which the vapor and liquid streams exiting the tray are in both thermodynamic and mass transfer equilibrium, when there is net rate of ethanol transfer is zero. When equilibrium is achieved on a plate, the compositions of the liquid and vapor streams may be determined from the equilibrium relationship between the components. Ideal behavior, however, is not usually accomplished. Typical values for plate efficiency for sieve plates are between 60 and 80% (Perry 14-31).

1. Factors Affecting Plate Efficiency

To operate at a satisfactory efficiency, the most important factor is to have the plates operating properly. Another important factor is to have adequate and intimate contact between the vapor and liquid. The plate efficiency is lowered when the column is misoperated. This is done is by excessive foaming or entrainment, poor vapor distribution, short-circuiting, weeping, or dumping of liquid. Weeping is when the liquid starts dripping down the holes of the plate due to low vapor pressure.

Data for plate efficiency is plotted against a flow parameter, F, which is known as the *F factor* and defined as (McCabe 695):

$$F \equiv u\sqrt{\rho_V} \tag{1}$$

where u = the velocity of the vapor
ρ_V = the density of the vapor

Sample Calculation:

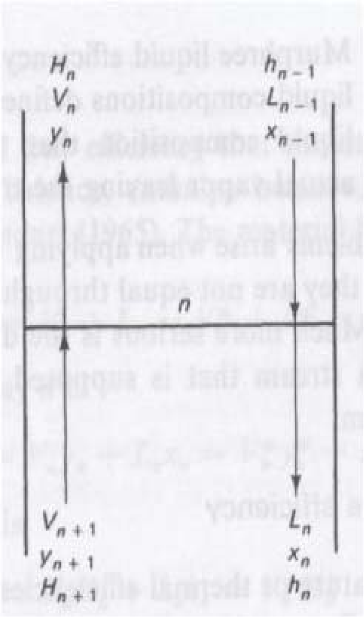
V=1.64 mol/min
U_V=0.808 m/s
F=0.808 m/s$\sqrt{(1.587 \text{ kg/m}^3)}$= 1.0 kg$^{1.5}$/m$^{0.5}$s

2. *Types of Plate Efficiency*

There are four different types of tray efficiencies that are used. These are the following: mass transfer efficiency, thermal efficiency, overall column efficiency, and point efficiency. The mass transfer and thermal efficiencies yield a different efficiency for each plate. The overall efficiency gives one efficiency for the entire column, and the point efficiency applies to only one point on one plate.

Figure 1 shows the liquid and vapor streams entering a plate *n* in a distillation column (Hines 445).

Figure 1: *n*th Plate Diagram



Note: The plates are numbered from top to bottom.

3. Murphree Efficiency

The mass transfer efficiency, also known as the Murphree efficiency, is one of the most widely used plate efficiencies. Developed by Murphree in 1925, it gives two efficiencies for each plate: one for the liquid stream and one for the vapor stream. The Murphree efficiency for the liquid stream is defined as (Hines 445):

$$E_{ML} = \frac{x_n - x_{n-1}}{x_n^* - x_{n-1}} \tag{2}$$

The more commonly used Murphree efficiency is for the vapor stream:

$$E_{MV} = \frac{y_n - y_{n+1}}{y_n^* - y_{n+1}} \tag{3}$$

where E_{ML} = Murphree liquid efficiency
E_{MV} = Murphree vapor efficiency
x_n, x_{n-1} = liquid phase composition leaving and entering the plate, respectively
y_n, y_{n+1} = vapor phase composition leaving and entering the plate, respectively
x_n^*, y_n^* = ideal liquid and vapor phase compositions that would be in equilibrium with the actual vapor and liquid leaving the tray

Table 1: Sample Calculations

	x	**v**	**x***	**v***
2	0.92			
3	0.89	0.92	0.84	0.94
4		0.88		

$$E_{MV} = \frac{0.92 - 0.88}{0.95 - 0.88} = 0.57$$

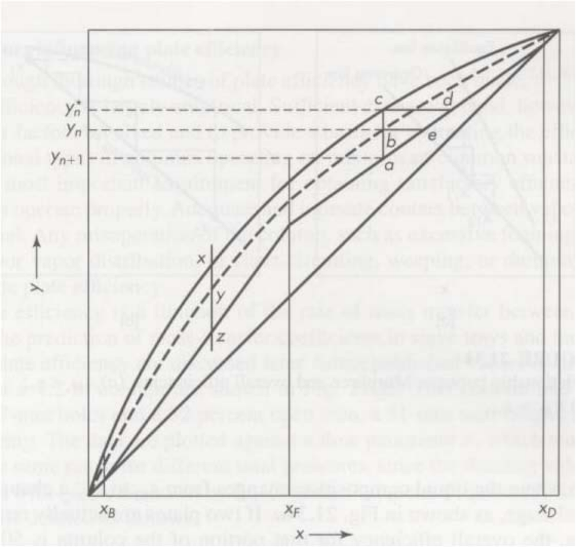
$$E_{ML} = \frac{0.89 - 0.92}{0.84 - 0.92} = 0.375$$

There are a few problems when using Murphree efficiencies. One is that there is a different Murphree efficiency for each plate. The main difficulty is the calculation of

x_n^* and y_n^*. These are the ideal compositions that would be in equilibrium with the non-ideal streams.

After obtaining the efficiency, it is useful to know how to use the efficiency. The efficiency can be graphed onto the McCabe-Thiele diagram for the process, as done in Figure 2 below.

Figure 2: McCabe-Theile with Efficiency



In Figure 2, the dashed line is the effective equilibrium curve when the Murphree efficiency equals 0.60, and the darker curve above it is the ideal plate theoretical curve. As shown in the figure, the ideal plate enriches the vapor more than the actual plate, shown by comparing lines *ac* to *ab*. Also, to achieve the desired product, more actual plates are needed than ideal plates, as seen by comparing the size of the equilibrium steps *acd* to *abe.* The bigger the steps, the fewer steps are needed to accomplish the same product.

4. Point Efficiency

For large diameter columns, the efficiency at each different point along each plate is different. This is because as the liquid flows from the inlet side to the outlet side, its composition of *A* decreases for the case of stripping. Therefore, the vapor that contacts the entering liquid will be higher in concentration of *A* when it leaves the tray (see Figure 3). The efficiency at each point is defined as (Geankoplis 668):

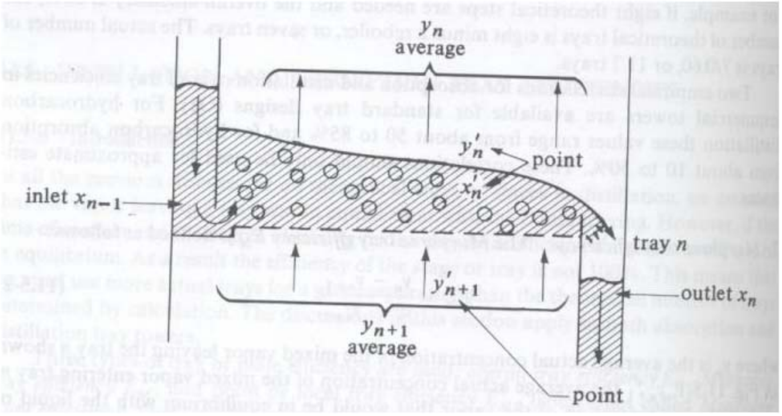
$$E_{MVP} = \frac{y'_n - y'_{n+1}}{y'^{*}_n - y'_{n+1}} \qquad (4)$$

where E_{MVP} = Murphree vapor phase point efficiency
y' = vapor phase composition at a specific point

The efficiency of a plate can be related to the point efficiency by integrating the point efficiency over the entire plate.

For small diameter columns, it can be assumed that the vapor agitates the liquid enough so the liquid on the plate can be assumed to be well-mixed. For this case, $y'_n = y_n$ and $y'_{n+1} = y_{n+1}$ and $y'^{*}_n = y^{*}_n$; therefore, $E_{MVP} = E_{MV}$.

Figure 3: Plate Vapor Liquid Flow Diagram



5. *Overall Column Efficiency*

The overall column efficiency is very widely used. The overall column efficiency does not take into account the fact that each tray has a different efficiency. Overall efficiency is defined as (Geankoplis 667):

$$E_o = \frac{N_t}{N_a} \qquad (5)$$

where E_o = overall column efficiency
N_t = number of theoretical trays, not including reboiler
N_a = number of actual trays

Sample:

$$E_o = \frac{2.9}{9.0} = 0.32$$

6. *Predicting Efficiency*

To predict the efficiency of the overall column, use the following equation (Perry 14-35):

$$E_O = \frac{1}{1 + 3.7(10^4)\dfrac{KM}{h'\rho_l T}} \qquad (6)$$

where E_O = overall column efficiency, fractional
K = vapor liquid equilibrium ratio, $y°/x$
$y°$ = gas-phase concentration at equilibrium, mole fraction
x = liquid-phase concentration, mole fraction
M = molecular weight
h' = effective liquid depth, mm
ρ_l = liquid density, kg/m^3
T = Temperature, K

Sample Calculation:

$$E_O = \frac{1}{1 + 3.7(10^4)\dfrac{2 * 46 g/mol}{10 mm * 766 kg/m^3 * 321 K}} = 0.42$$

The above calculation was performed for both ethanol and n-propanol at the top and bottom of the column. The sample above was for ethanol at the top of the column. The results of the calculations are shown in the table below.

Table 2: Results

Component	Position	Density (kg/m^3)	Temperature (K)	Efficiency
Ethanol	Top	766	321	0.42
Ethanol	Bottom	715	363	0.43
N-Propanol	Top	782	321	0.36
N-Propanol	Bottom	733	363	0.37

Using the above results, an average overall column efficiency can be assumed to be 40%.

7. Experimentally Determining Efficiency

In order to experimentally determine the efficiency of a plate one must take samples of the liquid stream both entering and leaving the plate and a vapor sample leaving the plate. Then these samples must be analyzed by gas chromatography to obtain their compositions. These compositions must then be used in conjunction with equation (2) to calculate individual plate efficiency. In order to utilize equation (2), x^* must be evaluated, which is done by using the equilibrium relationship with the composition of the vapor stream leaving the plate.

To experimentally determine the overall column efficiency, see equation 7. In order to use this equation one must know the number of plates that would be theoretically needed to perform the separation. To find this, one must perform a McCabe-Thiele analysis using the operating lines, equilibrium curve, and outlet compositions. Equation 7 yields an overall efficiency using the number of plates determined using the McCabe-Thiele and the actual number of plates used.