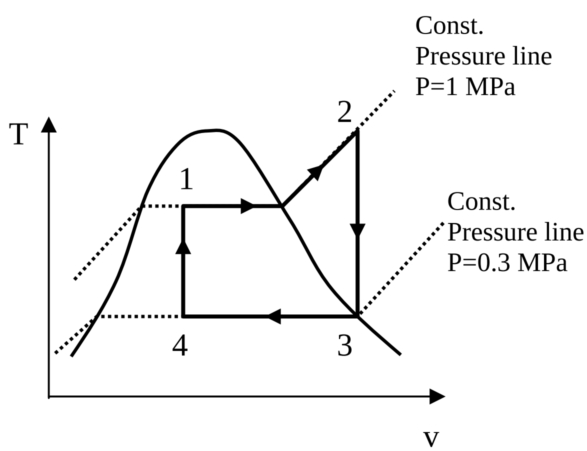
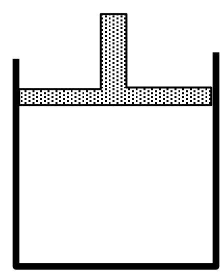
EML 3002 ME Tools   Example   2006

1. A piston/cylinder assembly (with an initial volume of V_1=0.1 m^3, P_1=1 MPa) contains 1 kg of a mixture of saturated liquid water and saturated steam as shown. The system is undergoing through four processes 1-2-3-4-1 as indicated on the T-v diagram. 2-3 and 4-1 are constant volume process. (40%)

   (a) Determine the initial quality (x_1) and temperature (T_1);
   (b) determine the temperature (T_2) at the state 2;
   (c) determine the quality (x_4) and temperature (T_4) at the state 4;
   (d) determine the amount of the heat transfer Q_{12} added and work done W_{12} during the process 1-2;
   (e) determine the amount of the heat transfer Q_{23} lost and work done W_{23} during the process 2-3.





(a) $\text{v}_1 = \frac{0.1}{1} = 0.1 m^3/kg,\ P_1 = 1 MPa,\ T_1 = 179.9°C$

From saturated steam table : $v_{f,1} = 0.001127, v_{g,1} = 0.1944$

$$v_1 = v_{f,1} + x_1(v_{g,1} - v_{f,1}),\quad x_1 = \frac{0.1 - 0.001127}{0.1944 - 0.001127} = 0.51$$

(b) $P_3 = 0.3MPa$, from saturation steam table : $v_{g,3} = 0.6058 = v_2 > v_{g,1}$

Use superheated table and interpolate between 1,000°C & 1,100°C : $P_2 = 1MPa,\ v_2 = 0.6058(m^3/kg)$

$$T_2 = 1000 + \frac{1100 - 1000}{0.6335 - 0.5871}(0.6058 - 0.5871) = 1040.3(°C)$$

(c) $T_4 = 133.55°C$ from saturation steam table with $P_4 = 0.3MPa$

$v_{f,4} = 0.001073,\ v_{g,4} = 0.6058, v_4 = v_1 = 0.1m^3/kg$ (constant volume process)

$$x_4 = \frac{0.1 - 0.001073}{0.6058 - 0.001073} = 0.165$$

$$(d)\ \ W_{12} = \int_1^2 PdV = P_1(V_2 - V_1) = P_1 m(v_2 - v_1) = (1000)(1)(0.6058 - 0.1) = 505.8(kJ)$$

$$\Delta U_{12} = U_2 - U_1 = m(u_2 - u_1) = Q_{12} - W_{12}$$

$u_1 = u_{f,1} + x_1 u_{fg,1} = 761.68 + (0.51)(1822) = 1690.9,$ from saturated steam table

u_2 needs to be interpolated from the superheated table between 1,000°C and 1,100°C

$$\mathrm{u}_2 = 4050.5 + \frac{4255.1 - 4050.5}{1100 - 1000}(1040 - 1000) = 4132.3(kJ/kg)$$

$$Q_{12} = (1)(4132.3 - 1690.9) + 505.8 = 2947.2(kJ)$$

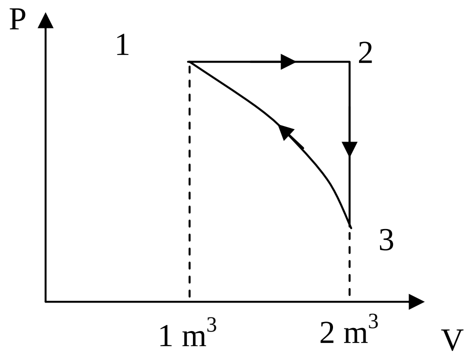
$$(e)\ W_{23} = \int_2^3 PdV = 0$$

$$\Delta U_{23} = m(u_3 - u_2) = Q_{23}$$

$$Q_{23} = m(u_{g,3} - u_2) = (1)(2543.6 - 4132.3) = -1588.7(kJ);$$

$u_{g,3}$ from saturated steam table with P = 0.3 MPa

2. The cylinder/piston assembly is filled with 1 kg of air (an ideal gas with R=0.287 kJ/kg K and C_v=0.72 kJ/kg, k=1.4.) The initial pressure inside the cylinder is P_1=100 kPa. The system is going through three processes 1-2-3-1 to complete a cycle as shown. (1-2:constant pressure and its volume expands from 1 m^3 to 2 m^3; 2-3: constant volume; 3-1: adiabatic.) (30%)
   (a) Determine the temperatures at all states (T_1, T_2 & T_3);
   (b) determine the work (W_{12}) and the heat transfer (Q_{12}) between 1 and 2;
   (c) determine the work (W_{23}) and the heat transfer (Q_{23}) between 2 and 3;
   (d) determine the work (W_{31}) between 3 and 1.



From1-2:constantprerssure

$$(b)W_{12}=\int_1^2 PdV=P_1(V_2-V_1)=(100)(2-1)=100(kJ)$$

$$(a)T_1=\frac{P_1V_1}{mR}=\frac{(100)(1)}{(1)(0.287)}=348(K)$$

$$(a)T_2=\frac{P_2V_2}{mR}=\frac{(100)(2)}{(1)(0.287)}=697(K)$$

$$(b)\Delta U_{12}=U_2-U_1=mC_V(T_2-T_1)=(1)(0.72)(697-348)=Q_{12}-W_{12}=Q_{12}-100$$

$$Q_{12}=351.2(kJ)$$

$$(c)W_{23}=\int_2^3 PdV=0\ \text{No external work}$$

From3 to1, adiabaticprocess: $\left(\frac{\mathrm{T}_3}{\mathrm{T}_1}\right)=\left(\frac{V_1}{V_3}\right)^{k-1},\frac{T_3}{348}=\left(\frac{1}{2}\right)^{1.4-1}=0.758$

$$(a)T_3=263.7(K)$$

$$(c)\Delta U_{23}=mC_V(T_3-T_2)=Q_{23}-W_{23}$$

$$Q_{23}=(1)(0.72)(263.7-697)+0=-311.9(kJ)$$

$$(d)\ \text{Adiabatic process } Q_{31}=0$$

$$\Delta U_{31}=Q_{31}-W_{31}$$

$$W_{31}=-\Delta U_{31}=mC_V(T_3-T_1)=(1)(0.72)(263.7-348)=-60.7(kJ)$$