The cylinder/piston assembly is filled with 1 kg of air (an ideal gas with R=0.287 kJ/kg K and C_v=0.72 kJ/kg, k=1.4.) The system is going through four processes 1-2-3-1 to complete a cycle. (1-2: constant temperature and its volume expands from 1 m^3 to 2 m^3; 2-3: constant volume; 3-1: adiabatic). It is known that heat transfer from 2 and 3 is equal to -100 kJ (losing heat).

(a) Sketch the three processes on the T-V diagram shown below;

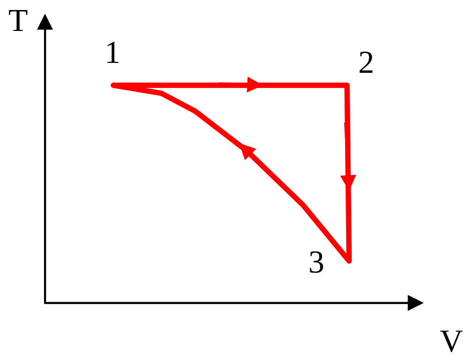
(b) determine all three temperature T_1, T_2 and T_3;

Note: Regardless of what temperature you found from (b), for parts (c), (d) & (e), use T_1=500K, Q_{23} is unknown now, V_3=V_2=2 m^3 and V_1=1 m^3 for your calculation.

(c) determine all the works (W_{12}, W_{23}, and W_{31});

(d) determine all the heat transfers (Q_{12}, Q_{23}, and Q_{31});

(e) determine all the internal energy changes (ΔU_{12}, ΔU_{23}, and ΔU_{31}).



From 1-2: constant temperature: $T_1=T_2$

From 2-3: constant volume: $W_{23} = \int_1^2 PdV = 0$

$$\Delta U_{23} = Q_{23} - W_{23} = Q_{23} = -100(kJ)$$
$$\Delta U_{23} = mC_V(T_3 - T_2) = (1)(0.72)(T_3 - T_2)$$
$$T_3 - T_2 = -138.9(K)....(A)$$

From 3-1: adiabatic process: $(\frac{V_3}{V_1})^{k-1} = (\frac{T_1}{T_3})$, $(\frac{2}{1})^{1.4-1} = 1.32 = \frac{T_1}{T_3} = \frac{T_2}{T_3}$

$$T_2 = 1.32T_3 \; ....(B)$$

(b) From (A) and (B): $T_1=T_2$=573(K) and T_3=434.1(K)

$(c)\quad \Delta U_{12} = 0 = Q_{12} - W_{12},\ \ Q_{12} = W_{12} = \int_1^2 PdV = \int_1^2 \frac{mRT}{V} dV = mRT \ln(\frac{V_2}{V_1})$

$W_{12} = (1)(0.287)(500)\ln(\frac{2}{1}) = 99.5(kJ)$

$W_{23} = \int_2^3 PdV = 0$ No external work

$\Delta U_{31} = Q_{31} - W_{31} = -W_{31}$

$W_{31} = -\Delta U_{31} = -mC_v(T_1 - T_3)$

From 3 to 1, adiabatic process: $\left(\frac{\mathrm{T}_3}{\mathrm{T}_1}\right) = \left(\frac{V_1}{V_3}\right)^{k-1}, \frac{T_3}{500} = \left(\frac{1}{2}\right)^{1.4-1} = 0.758$

$T_3 = 379(K)$

$W_{31} = -\Delta U_{31} = -(1)(0.72)C_v(500 - 379) = -87.1(kJ)$

$(d)\quad \Delta U_{12} = Q_{12} - W_{12},\ \ Q_{12} = W_{12} = 99.5(kJ)$ since $\Delta \mathrm{U}_{12}$=0

$\Delta U_{23} = Q_{23} - W_{23}, W_{23} = 0$

$Q_{23} = \Delta U_{23} = mC_v(T_3 - T_2) = (1)(0.72)(379 - 500) = -87.1(kJ)$

Adiabatic process $\mathrm{Q}_{31} = 0$

$(e)\ \Delta U_{12} = 0$ constant temperature process

$\Delta U_{23} = mC_V(T_3 - T_2) = (1)(0.72)(379 - 500) = -87.1(kJ)$

$\Delta U_{31} = mC_V(T_1 - T_3) = (1)(0.72)(500 - 379) = 87.1(kJ)$