

T-F Lecture 5. Calculation of Work in the Stirling Cycle

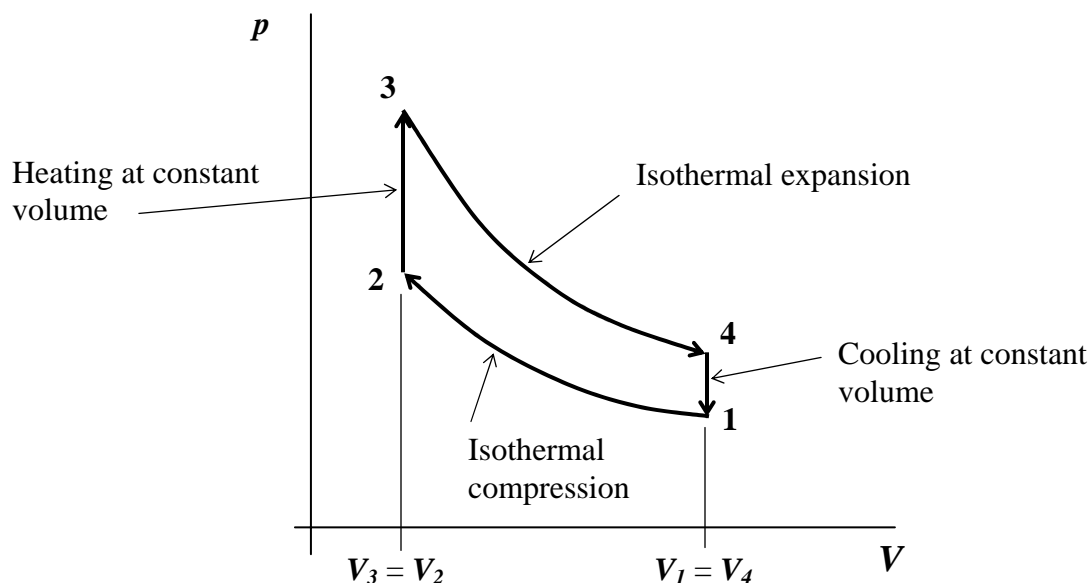
We have shown in the previous class that the Stirling cycle consists of four processes:
The four processes are:

- I. Isothermal expansion
- II. Cooling at constant volume
- III. Isothermal compression
- IV. Heating at constant volume

Let us number each of the states such that the initial state, state 1, corresponds to the ambient condition of pressure, temperature, and volume.

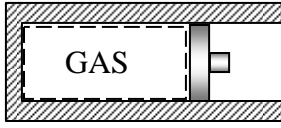
- Process 1 – 2. Isothermal compression
Process 2 – 3. Heating at constant volume
Process 3 – 4. Isothermal expansion
Process 4 – 1. Cooling at constant volume

The Stirling cycle is represented on a pressure-volume ($p - v$) diagram as:

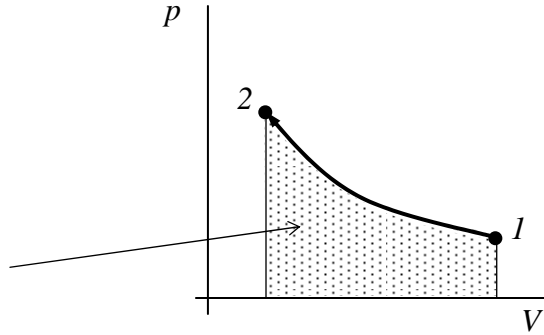


We shall now compute the work for each of the processes and for the cycle as a whole.

Recall the work expression for a piston-cylinder system.



$$W_{12} = \int_{V_1}^{V_2} p dV = \int_1^2 p dV$$



To evaluate this integral, we must have an expression for p as a function of V . A relationship among the p , V , and T variables is called an **equation of state**. We shall now obtain the expressions for work for each of the four processes of the Stirling engine.

PROCESS 1 – 2. ISOTHERMAL COMPRESSION

The process is at constant temperature. Assume an ideal gas. The ideal gas equation of state is:

$$pV = mRT$$

Since the temperature is constant for this process, $T_1 = T_2$, we have the following relationship between the pressure and volume:

$$pV = mRT_1 = \text{constant}$$

In particular,

$$p = \frac{mRT_1}{V}$$

Substitute for p in the expression for work:

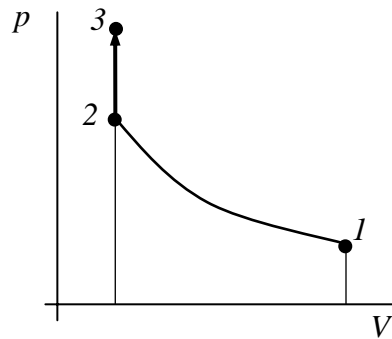
$$W_{12} = \int_{V_1}^{V_2} p dV = \int_1^2 p dV = mRT_1 \int_1^2 \frac{dV}{V} = mRT_1 \int_1^2 d \ln V = mRT_1 \ln \frac{V_2}{V_1}$$

Note that since $V_2 < V_1$ the term $\ln \frac{V_2}{V_1} < 0$ and $W_{12} < 0$. This is consistent with our sign convention that work done on the system (to compress the gas) is negative!

PROCESS 2 – 3. HEATING AT CONSTANT VOLUME

For this constant volume process $V_3 = V_2$, it follows that $dV = 0$. The expression for work becomes:

$$W_{23} = \int_2^3 p dV = 0$$



ISOTHERMAL EXPANSION, PROCESS 3 – 4.

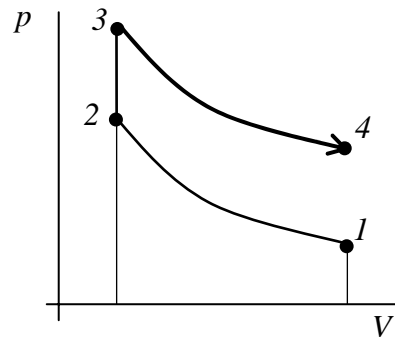
For this constant temperature process we have $T_3 = T_4 = \text{constant}$, thus:

$$pV = mRT_3 = \text{constant}$$

The expression for work becomes:

$$W_{34} = \int_3^4 p dV = mRT_3 \int_3^4 \frac{dV}{V} = mRT_3 \int_1^2 d \ln V$$

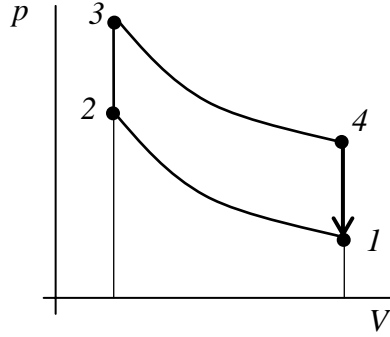
$$W_{34} = mRT_3 \ln \frac{V_4}{V_3}$$



COOLING AT CONSTANT VOLUME, PROCESS 4 – 1.

For this process $V_4 = V_1$, hence $dV = 0$, and the expression for work becomes:

$$W_{41} = \int_4^1 p dV = 0$$



NET WORK FOR THE CYCLE

The net work for the cycle is the sum of the work for each of the processes.

$$W_{net} = W_{12} + W_{23} + W_{34} + W_{41} = mRT_1 \ln \frac{V_2}{V_1} + 0 + mRT_3 \ln \frac{V_4}{V_3} + 0$$

Since $V_4 = V_1$ and $V_3 = V_2$, the above expression can be simplified to:

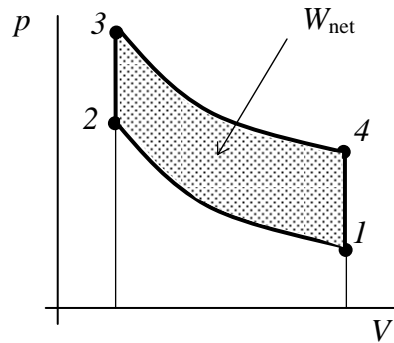
$$W_{net} = mRT_1 \ln \frac{V_2}{V_1} + mRT_3 \ln \frac{V_4}{V_3} = -mRT_1 \ln \frac{V_1}{V_2} + mRT_3 \ln \frac{V_1}{V_2}$$

Finally we obtain

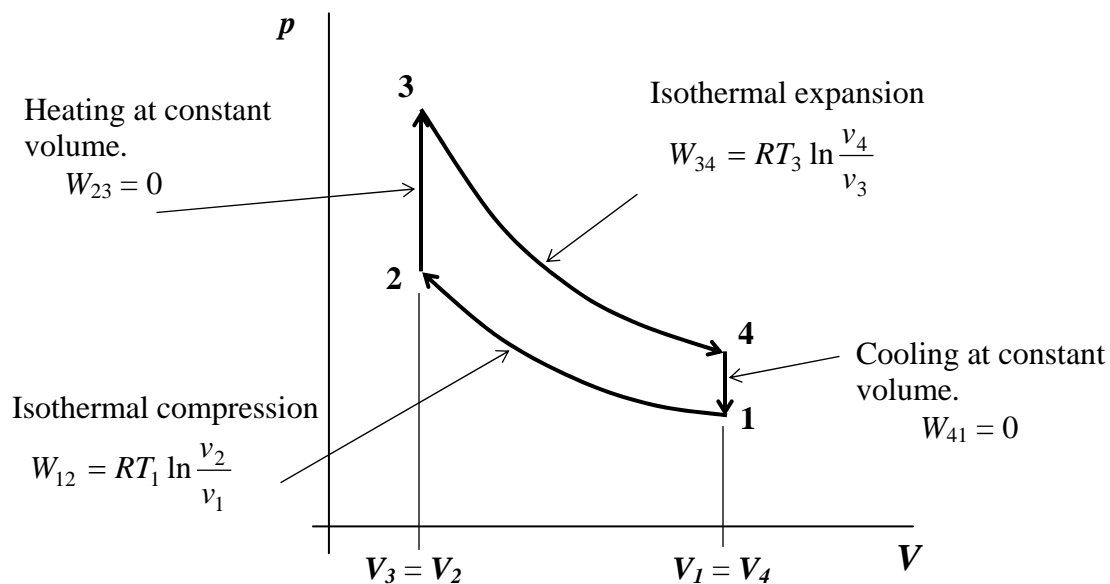
$$W_{net} = mR(T_3 - T_1) \ln \frac{V_1}{V_2}$$

Note that since $T_3 > T_1$, we have $W_{net} > 0$. Thus the net work is positive indicating that the overall effect of the cycle was to have done work on the surroundings.

Note that the area enclosed by the cycle in the $p - v$ diagram represents the net work of the cycle.



Summary of the work calculations



To Apply the **First Law of Thermodynamics** to each of the processes and the cycle we must evaluate the change of internal energy, ΔU , and the heat transfer, Q , for each process. We shall relate the change in internal energy to the change in temperature.

$$\Delta U = f(T)$$

The heat transfer Q will be evaluated from the First Law of Thermodynamics.

$$Q = \Delta U + W$$

To calculate the change in internal energy, ΔU , as a function of temperature we must develop the concept of heat capacity of the fluid. We shall do this next.